Too Many Cooks ...
A New Account of the Earliest Muslim Geodetic Measurements

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Introduction

The measurement of the length of one degree on the meridian by astronomers commissioned by the Abbasid Caliph al-Ma'mūn in Baghdad *ca*. 830 is in one sense well known, and in another, clouded in obscurity. Different versions of the observations are recorded by Ibn Yūnus (*fl*. Cairo *ca*. 990) and al-Bīrūnī (*fl*. Central Asia *ca*. 1025), as well as by various later writers. Particularly those versions in which two groups of astronomers laden with instruments are reported heading off in opposite directions along a meridian in the middle of the desert simply boggle the mind. There is a substantial secondary literature on these reports, including important contributions by C. A. Nallino, S. H. Barani, A. Sayılı, E. S.

Kennedy, R. P. Mercier, and F. J. Ragep.[1] In this paper these well-known accounts will be discussed, but only briefly, for my main purpose is to present a new contemporaneous account of the measurements.

The earliest Muslim latitude and longitude measurements in Mecca and Baghdad, also commissioned by al-Ma'mūn, are less well known, and they too are clouded in obscurity. Until recently they were known only from passing remarks by al-Bīrūnī,[2] and it was Aydın Sayılı who first drew attention to that scholar's brief mention of the simultaneous lunar eclipse observations in the two cities.[3] It was clear that the main purpose of the measurements was to derive the *qibla* at Baghdad, but details were lacking.[4]

In 1985 Dr. Y. Tzvi Langermann of the Hebrew University in Jerusalem published a newly-discovered treatise on both sets of observations by the contemporary astronomer Ḥabash,[5] and I have come across a different

[1] See Nallino, *Scritti*, V, pp. 408-457; Barani, "Muslim Geodesy"; Sayılı, *The Observatory in Islam*, pp. 85-86; Kennedy, *al-Bīrūnī's Taḥdīd*, pp. 131-136; and the recent critical survey of the evidence in Mercier, "Muslim Geodesy", pp. 178-181. On Ragep's astounding findings see n. 67 below. See also the forthcoming volumes of Sezgin, *GAS*, relating to geography. The various places mentioned in this article are shown on a map in Mercier, "Muslim Geodesy", p. 180.

[2] On al-Bīrūnī see the article by E. S. Kennedy in *DSB*. His treatise on mathematical geography has been published by P. Bulgakov, translated by J. Ali, and analyzed by E. S. Kennedy - see the bibliography for details.

[3] Sayılı, *The Observatory in Islam*, p. 85.

[4] For a general survey of the *qibla* problem see King, "Sacred Direction", and the articles "Ḳibla. ii. Astronomical aspects" and "Makka. iv. As centre of the world" in *EI*², reprinted in King, *Studies*, C-IX and C-X, as well as King & Lorch, "Qibla Charts". See also n. 60 below on non-mathematical methods that were widely used in medieval Islam.

[5] On Ḥabash see the articles by W. Hartner in *EI*² and S. Tekeli in *DSB*, and the recent study of his *zīj* in Debarnot, "Ḥabash's Zīj (Istanbul MS)". (The later recension extant in a Berlin manuscript is currently being investigated by Dr. Benno van Dalen (Utrecht and Frankfurt).) On the title of his treatise, see Sezgin, *GAS*, V, p. 276, no. 5, where no manuscripts are listed: the treatise is now published in Langermann, "Ḥabash on Distances".

account by Yaḥyā ibn Aktham,[6] the judge appointed by al-Ma'mūn to oversee the observations. In this paper I shall compare both of these reports in the light of the later accounts of Ibn Yūnus and al-Bīrūnī. Mercier's recent overview of the evidence available to him, which included the treatise of Ḥabash, draws attention to the obvious inconsistencies in the sources. From the outset it must be borne in mind that this newly-discovered report of Yaḥyā ibn Aktham raises yet more questions and casts but little light on the confusion.

In the sequel I use the following notation freely:

- q *qibla*(local direction of Mecca, measured from the local meridian)
- L terrestrial longitude
- ΔL longitude difference from Mecca
- ΔL_B longitude difference between Baghdad and Mecca
- $\Delta\phi$ latitude difference from Mecca
- $\Delta\phi_B$ latitude difference between Baghdad and Mecca
- ϕ latitude
- ϕ_B latitude of Baghdad
- ϕ_M latitude of Mecca.

In fact, q is a trigonometric function of ΔL, ϕ and ϕ_M, but most of the earliest mathematical procedures, including the one used by al-Ma'mūn's astronomers, were approximate, and one popular method used for several centuries involved only ΔL and $\Delta\phi$.[7]

In the translations the distinction between numbers written in words and those expressed in Arabic numerals has been preserved. Numbers expressed in alphanumerical (*abjad*) notation are underlined. Insertions in the text to smooth the flow of the translation, as well as occasional citations of the Arabic original, are in parentheses; restorations are in square

[6] On Ibn Aktham see, for example, the biographical notice (devoid of any reference to the observations commissioned by al-Ma'mūn) in al-Khaṭīb al-Baghdādī, *Ta'rīkh Baghdād*, XIV, pp. 191-204, and also Sayılı, *The Observatory in Islam*, pp. 56 and 69. On his son Muḥammad, who authored a treatise on number theory, see Sezgin, *GAS*, V, pp. 273-274.

[7] See King, "Earliest Qibla Methods and Tables", especially pp. 103-107.

brackets; and my comments in curly brackets.

1. Ibn Yūnus' account

In Chapter 2 of his monumental work, a *zīj* or astronomical handbook with tables,[8] dedicated to the Fatimid Caliph al-Ḥākim and called the *Ḥākimī Zīj*, Ibn Yūnus[9] refers in general terms to the measurement of longitude differences by means of lunar eclipse observations,[10] and then briefly refers to the geodetic measurements:[11]

> Discussion of the (distances in) cubits between places: Sanad ibn ᶜAlī mentioned in a statement of his which I found that al-Ma'mūn ordered Khālid ibn ᶜAbd al-Malik al-Marwarrūdhī and him to measure the amount of one degree of a great circle on the surface of the earth. He said: "We both set off together for this (purpose). (al-Ma'mūn) ordered ᶜAlī ibn ᶜĪsā al-Asṭurlābī and ᶜAlī ibn al-Buḥturī to do the same and they went off in a different direction." Sanad ibn ᶜAlī said: "Khālid ibn ᶜAbd al-Malik and I travelled to (a place) between *W'mxh*[12] and Tadmur (= Palmyra) and there we measured the amount of a degree of a great circle on the surface of the earth. It was fifty-seven miles (*mīl*). ᶜAlī ibn ᶜĪsā and ᶜAlī ibn al-Buḥturī also made measurements and the two of them

[8] A *zīj* is a set of astronomical tables with introductory and explanatory materials. See Kennedy, "*Zīj* Survey", for an introduction, based on about 125 such works, many of which contain several hundred pages of tables and text. The number of known *zīj*es is now closer to 200.

[9] On Ibn Yūnus see my article in *DSB*, and also King, *Studies*, A-IX, pp. 343-345.

[10] See Schoy, "Muslim Geography", pp. 265-267, for a translation of the relevant passage from MS Leiden Or. 143, p. 80.

[11] The text, found in MS Leiden Or. 143, pp. 81-82, was first translated in Caussin, "Ibn Yūnus", pp. 78 and 80, n. 1. The text is published in Barani, "Muslim Geodesy", pp. 8-11, and a new translation is in Mercier, "Muslim Geodesy", p. 179.

[12] The letter *x* here denotes an unpointed carrier in Arabic, which could be read as a *bā'*, *yā'*, *nūn*, etc. See further the commentary to this passage.

found the same as this. The two reports from the two directions (telling of) the two measurements with the same result arrived at the same time."

Aḥmad ibn ᶜAbdallāh, known as Ḥabash, stated in his book in which he mentioned the observations (of the compilers) of the *Mumtaḥan* (*Zīj*) in Damascus, that al-Ma'mūn ordered that one degree of a great circle on the surface of the earth should be measured. He said: "They set off for this (purpose) in the desert of Sinjār, (travelling) until two measurements of the meridian altitude on the same day differed by one degree, and then they measured the distance between the two places, and it was 56 and one-quarter miles, where each mile is four thousand cubits, these being the "black cubits" that were adopted by al-Ma'mūn."

Ibn Yūnus continues with a description of the way in which the distance along the meridian can be measured. One takes two ropes, each 50 cubits long. Lay out the first one in the meridian. Then lay out the second one along the first one, starting at the mid-point of the first. And so on ... (for 56-odd miles!).[13] He concludes with a discussion of measuring rods, describing instruments of square cross-section and 24 times as long as they are wide, adding that these would cost 1000 dirhams if made of silver, or 1000 dinars in gold. Since, according to a 14th-century Egyptian source, Ibn Yūnus was paid 100 dinars a day for his services to the Fatimid Caliph al-Ḥākim,[14] we may speculate that he himself might have had such a rod in gold, if not several!

Commentary

The *Mumtaḥan Zīj* was the major production of the astronomers commissioned by al-Ma'mūn, who were under the direction of Yaḥyā ibn

[13] See Mercier, "Muslim Geodesy", p. 181, for a translation of this passage.

[14] MS Leiden Or. 468, fol. 84v. On the importance of this manuscript see King, *Studies*, B-VII, pp. 250-252. It is now being studied by Margarita Castells of Barcelona.

Abī Manṣūr.[15] This work is available only in a later recension that has still not been studied in detail. Groups of astronomers in Baghdad and Damascus were involved in observations for this work, but it is by no means clear what the input of the two groups was. Ibn Yūnus mentions that Ḥabash was writing about the Damascus observations, but the surviving manuscript of Ḥabash's treatise mentions Damascus only once, when talking about a large equatorial ring made by Khālid al-Marwarrūdhī, "who was in Damascus".[16]

Caussin de Perceval, who first translated the text of this passage, thought that the mysterious place-name *W'mxh* (MS Leiden Or. 143, p. 81, l. 19) might be the ancient Apamea,[17] but the name is nowhere else attested in the early Islamic sources. The name reappears as *W'myh* in the parallel passage in MS London B.L. Or. 3624, fol. 42v, l. 14, of the *Mukhtār Zīj* of Abū 'l-ᶜUqūl (*fl.* Taiz *ca.* 1300),[18] a work relying heavily on Ibn Yūnus. The best-known city with the name Apamea is near Homs and cannot be considered. There is, however, another Apamea due north of Palmyra, near Zeugma.[19] Others have suggested that the word is simply a corruption of (al-)Raqqa,[20] which has the advantage of lying roughly due north of Palmyra and is connected to it by a Roman road, the Via Diocletiana (which runs north-east leaving Palmyra in order to avoid the

[15] On this work see Vernet, "Tabulae Probatae", and Kennedy, "*Zīj* Survey", pp. 132 (no. 51) and 144-147. On its context see Sayılı, *The Observatory in Islam*, pp. 50-87.

[16] Langermann, "Ḥabash on Distances", p. 125.

[17] Caussin, "Ibn Yūnus", p. 80, note a.

[18] On this author and his work see King, *Astronomy in the Yemen*, pp. 30-32, and Varisco, *Yemeni Almanac*, pp. 10-11.

[19] Mercier, "Muslim Geodesy", p. 179, n. 27.

[20] See, for example, Barani, "Muslim Geodesy", p. 28, and Kennedy, *al-Bīrūnī's Taḥdīd*, p. 131. In fact Raqqa is not due north of Palmyra (it lies roughly NNE), but the two cities have the same longitude in various early Islamic geographical tables - see Mercier, "Muslim Geodesy", *loc. cit.*

mountains and then due north to meet the Euphrates just west of Raqqa).[21] But Ibn Yūnus, writing in Cairo, was well aware of the existence of distant Raqqa, not least because his predecessor al-Battānī had worked there.[22] Mercier has suggested a possible misreading of $\theta\epsilon\mu\alpha$, the name of a locality north of Palmyra and just across the Euphrates, as $o\epsilon\mu\alpha$, which could conceivably be rendered *Wāmiya* in Arabic.[23] The problem of this mysterious name is not yet solved.

Two different results, 57 and 56 1/4 miles, are reported for the two different expeditions. Al-Farghānī, a late contemporary of al-Ma'mūn's astronomers, mentioned 56 2/3,[24] which Nallino and Barani have seen as a mean value between these two results that subsequently passed as the "real finding" (Barani) of al-Ma'mūn's scientists.[25] But as we shall see, the more original reports mention a figure of 56 miles. Here the reader should be reminded that these findings were not exactly "new", even in the Islamic context: already al-Fazārī in the 8th century had recorded the circumference of the earth according to "the Indians", namely, 6,600 *farsakhs*, where a *farsakh* is 16,000 cubits. Thus, as al-Bīrūnī points out elsewhere in the *Taḥdīd*, the length of 1° is 18 1/3 *farsakhs*, that is, 55 miles, since one *farsakh* equals three miles.[26] What is new is the organized scientific measurements, but only in so far as we are prepared to believe the reports, for certainly some imaginary elements have been inserted. Also the value 56 1/4 was not new: it can be derived from the 75 miles attributed in numerous sources to al-Khwārizmī, another of al-

[21] *Ibid*.

[22] On al-Battānī see the article by W. Hartner in *DSB*.

[23] Mercier, *op. cit.*; Sédillot proposed Wāsiṭ - see Nallino, *Scritti*, V, p. 429 - but Arabic does not allow of such spelling-mistakes.

[24] Mercier, "Muslim Geodesy", p. 178.

[25] Nallino, *Scritti*, V, pp. 431-432; and Barani, "Muslim Geodesy", p. 31.

[26] See *al-Bīrūnī's Taḥdīd*, text, pp. 211-212, translation, p. 177, commentary, p. 132, as well as Pingree, "al-Fazārī", p. 114.

Ma'mūn's astronomers, by converting from Roman miles to Arab miles.[27]
On the black cubit see the commentary to the text of Ḥabash.

2. Al-Bīrūnī's account

We continue with the references of al-Bīrūnī to the observations, quoting his monumental work on mathematical geography, the *Taḥdīd nihāyāt al-amākin*.[28] The text is brief and no details are given:

> ... The *qibla* of Islam is the Sacred Mosque ... . The latitude of Mecca is known but there are various opinions about the number of minutes in the twenty-second degree, because (?) the astronomers (*al-ḥussāb*) take it to be twenty-one degrees {with no minutes}. It is related that Manṣūr ibn Ṭalḥa al-Ṭāhirī concerned himself with finding its correct magnitude: he determined that this was in excess of (twenty-one degrees) by two-thirds of a degree, which is in agreement with what Ḥabash reported about the latitude measurement commissioned by al-Ma'mūn (*raṣad al-Ma'mūn iyyāhu*). (But) others have claimed that the excess (over twenty-one degrees) is (only) one-third of a degree. The longitude (of Mecca) is also known, because all the reports agree that Manṣūr ibn Ṭalḥa found its longitude to be sixty-seven degrees, which agrees with what Ḥabash al-Ḥāsib stated in his *Kitāb al-Ab^cād wa-'l-ajrām*. (He said) that al-Ma'mūn had organized someone to observe lunar eclipses (at Mecca), and that (that person) {singular} had found a difference of three degrees between its meridian and that of Baghdad. So, if the longitude of Baghdad is seventy degrees, that of Mecca would be sixty-seven degrees. ...

[27] Mercier, "Muslim Geodesy", *loc. cit.*

[28] Bulgakov, ed., *al-Bīrūnī's Taḥdīd*, pp. 209-210. Ali's translation (p. 175) is unreliable here. See also Kennedy's commentary (p. 130).

Later in the same treatise al-Bīrūnī again cites the same work of Ḥabash:[29]

> The observations (commissioned) by al-Ma'mūn (took place) only when he (had) read in the books of the Greeks that the equivalent (*ḥiṣṣa*) of one degree was five hundred *stades*, which is the unit of theirs that they used for measuring distances, and when he did not find amongst the translators sufficient knowledge about its length compared with other known (units). At that time - according to what Ḥabash related on the authority of Khālid al-Marwarrūdhī and a group of scholars of (instrument) construction and expert constructors from amongst the carpenters and brass-workers - (al-Ma'mūn) ordered the construction of the instruments and the selection of a place for this survey. There was chosen a location in the desert of Sinjār in the area (*min ḥudūd*) of Mosul, nineteen *farsakhs* from the city (*min qaṣabatihi*) and forty-three *farsakhs* from Samarra, where they were satisfied that (the ground was) level. They transported the instruments there and they selected a place where they observed the solar altitude at midday. Then two groups set forth (in two different directions). Khālid and a group of surveyors and instrument-makers (*al-massāḥ wa-'l-ṣunnā*ᶜ) headed in the direction of the northern (celestial) pole, and ᶜAlī ibn ᶜĪsā al-Asṭurlābī and Aḥmad ibn al-Buḥturī the surveyor with a(nother) group towards the south (celestial) pole. Each of the two groups observed the altitude of the sun at midday until they found that it had changed by one degree, apart from the change that resulted in the solar declination (*siwa 'l-taghayyur al-ḥādith mina 'l-mayl*). They measured the track on their way out (*fī dhihābihim*) and set up markers (*sihām*, lit., arrows) as they went, and as they returned they investigated (*i*ᶜ*tabarū*) the distance (*al-misāḥa*) for the second time. The two groups met again at the place from which they had set out, and they found that one degree of the terrestrial meridian is equivalent to fifty-six miles. (Ḥabash) claimed that he had heard Khālid dictating that number to the *Qāḍī* Yaḥyā ibn Aktham, so (Ḥabash himself) heard it from the horse's mouth (*fa-'ltaqaṭahu minhu samā*ᶜ*an*).

[29] *al-Bīrūnī's Taḥdīd*: text, pp. 213-214; translation, pp. 178-179; and commentary, p. 133. See also Barani, "Muslim Geodesy", pp. 11-14.

> Abū Ḥāmid al-Ṣaghānī, who heard it from Thābit ibn Qurra, told me the same thing.
> (On the other hand) it is related of al-Farghānī (that he reported) two-thirds of a mile in addition to the (56) miles mentioned above. Similarly I found all of the records confirming these (additional) two-thirds, and I may not attribute that to their having dropped out of the manuscript of the *Kitāb al-Ab^cād wa-'l-ajrām* because Ḥabash derived from that (value) the circumference and diameter of the earth and all of the (planetary) distances. When I investigated (these) I found that they result from fifty-six miles only {that is, without the two-thirds}. It is preferable to imagine that (these different results for the length of one degree) derive from two accounts by two teams. (Certainly) this is a subject surrounded with confusion, which should inspire renewed investigations and observations. Who is prepared to help me with this (project)? ... ...

Elsewhere al-Bīrūnī deals in general terms with the problem of determining longitude differences from simultaneous observations of a lunar eclipse,[30] but does not mention the Abbasid observations again.

Commentary

Manṣūr ibn Ṭalḥa was a philosopher, mathematician, astronomer and music theorist, who died *ca.* 855.[31] Khālid al-Marwarrūdhī was one of the leading astronomers of al-Ma'mūn: he apparently authored a *zīj* that has been lost without a trace.[32]

Of particular interest in this account are the coordinates of the locations in the plain of Sinjār where the two groups of observers set forth in two different directions. This information does not stem from the treatise of

[30] *al-Bīrūnī's Taḥdīd*: see text, pp. 166-169, translation, pp. 129-132, and commentary, pp. 100-101, and my review of the last in *Centaurus* 19 (1974), pp. 320-323, especially p. 322. See also Mercier, "Muslim Geodesy", p. 176.

[31] Sezgin, *GAS*, V, p. 245, and VI, p. 145.

[32] *Ibid.*, V, p. 244, and VI, p. 139.

Ḥabash as it has come down to us - see the next section. What is disappointing is that there is no record of two different results - such as one would expect from any scientific mission - which could then have been compared and used to produce a "final" result. Rather, in this case, the "final" result is presented as a *fait accompli*.

Having briefly discussed these two later accounts, both of which have been known to previous researchers, we are now in a position to investigate the more original accounts.

3. The account of Ḥabash

The first two parts of the treatise of Ḥabash, preserved in a precious 17th(?)-century astronomical compendium given to Rabbi Josef Kafaḥ by a Muslim Yemeni colleague before he left Sanaa for Jerusalem,[33] provide considerably more information - see Arabic text no. 1. The text translates:[34]

> In the Name of God the Merciful and Compassionate, from whom we seek help. *The Book of Sizes and Distances* by Ḥabash ibn ᶜAbdallāh the astronomer (*al-ḥāsib*).
> He said that the Commander of the Faithful, al-Ma'mūn, wanted to know the size of the earth, so he made some investigations about this and found that Ptolemy had stated in one of his books that the circumference of the earth was so many thousand *stades*. Thereupon he asked the interpreters about the meaning of *stades* and they gave different interpretations. [He said of their interpretations:] "They do not dispense with (?) what we wanted to know." {The text is corrupt here.}

[33] This is yet additional proof of the importance of Yemeni manuscripts for the history of Islamic astronomy. See King, *Astronomy in the Yemen*, for a first look at over 100 Yemeni astronomical manuscripts, now supplemented by Langermann, "Bibliographical Notes", based on Jewish sources.

[34] Langermann, "Ḥabash on Distances", pp. 115-116 and 122-123. Some problems in the published text have been fixed, and the translation is my own. I am grateful to Dr. Langermann for photocopies of the relevant sections of the original manuscript.

(Al-Ma'mūn) therefore sent out Khālid ibn ᶜAbd al-Malik al-Marwarrūdhī, ᶜAlī ibn ᶜĪsā al-Asṭurlābī and Aḥmad ibn al-Buḥturī the surveyor together with a group of surveyors and craftsmen including carpenters and brass-workers, to make correctly the instruments which they would need, transporting all of them to a place which they selected in the desert of Sinjār.

Next Khālid and one group headed towards the North Pole of the Banāt Naᶜsh (= Ursa Minor), and ᶜAlī and Aḥmad and a(nother) group headed towards the South (celestial) Pole. They {*i.e.*, the former group ?} continued until they finally found that the (solar) meridian altitude had decreased and changed from what they had found in the place where they had separated, by the amount of one degree, after they had subtracted from that (the change in) solar declination during the time it took (for them) to cover that distance. (As they went) they set up markers (*sihām*), and then they retraced their steps using the markers and checked the distance a second time. They found the amount of one degree of the earth's surface to be fifty-six miles, where one mile is four thousand black cubits, this being the cubit established by al-Ma'mūn for measuring clothes, surveying buildings and reckoning distances between stations on the pilgrimage road (*qismat al-manāzil*).

I heard this (information) which I have mentioned in my book {*sc*. in this book of mine?} from Khālid ibn ᶜAbd al-Malik al-Marwarrūdhī as he was conveying it to the *Qāḍī* Yaḥyā ibn Aktham. Yaḥyā [had been] ordered [by al-Ma'mūn] {the text appears to be corrupt here} to write down for him {sc. for al-Ma'mūn} all that Khālid told him, so he wrote (it) for him. I have written what I heard from Khālid himself.

The Commander of the Faithful al-Ma'mūn - may God be pleased with him - (also) wanted to measure the azimuth of the *qibla*. So he sent out (someone) at the time of a lunar eclipse to measure the longitude between Mecca and Baghdad. (That person) found that the meridian of Mecca was west of that of Baghdad by approximately three degrees. If we modify one degree of the celestial equator {the expression *al-falak al-mustaqīm*, meaning *sphaera recta* or right sphere, is usually used for astronomical ascensions[35] - compare the terminology of Yaḥyā ibn Aktham} we find that the amount on the celestial day-circle corresponding to the parallel of Mecca (*ḥiṣṣatuhā fī [madār] al-falak al-makhṭūṭ ᶜalā ᶜarḍ Makka*) is approximately fifty-six minutes. So if we multiply the distance between the meridians of Mecca and Baghdad, (namely,) three degrees, by 56 minutes where 60 parts are one degree (*juz'*), we find that the corrected longitudinal distance between them on that parallel (*fī 'l-falak al-makhṭūṭ ᶜalā madār ᶜarḍ Makka*) is approximately two degrees and forty-eight minutes. (That same observer ?) also found the latitude of Mecca to be approximately twenty-one degrees and forty-two minutes and that of Baghdad to be approximately thirty-three degrees and twenty-seven minutes. (If) we subtract the latitude of Mecca from that of Baghdad the difference between the two is eleven degrees and forty-five minutes. If we multiply the difference between the two longitudes, namely 2 degrees and 48 minutes, by itself, the result is 7 degrees, 50 minutes and 24 seconds. Half of this is 3 degrees, 55 minutes and 12 seconds, or 235 minutes and 12 seconds. This we divide by 11 degrees and 45 minutes, which is the difference between the two latitudes, [the quotient is 0;20(,1,1). We add this to the 11;45 and obtain] for the distance between Baghdad and Mecca 12 degrees and 5 minutes. This we multiply by 56 miles and obtain approximately six hundred and seventy-six and two-thirds miles as the distance between Baghdad and Mecca as the crow flies (*ᶜala 'l-sahm al-mustawī*). When Khālid ibn ᶜAbd al-Malik al-Marwarrūdhī submitted (this) value of one degree of the earth('s circumference) to al-Ma'mūn, (the Caliph) wanted to check it and so he [sent out] {reading *wajjaha* for *wajada*} someone {singular! - compare the account of Yaḥyā ibn Aktham} to measure the road between Baghdad and Mecca by the shortest route and he {singular!} found it to be seven hundred and twelve miles. So we (?) found that the difference between the distance as the crow flies (*al-sahm alladhī yajrī fī 'l-hawā*) and the measure-ment of the road was thirty-five miles and one-third of a mile. Al-Ma'mūn said that this was not to be regarded as excessive because there must be (in addition to) the flat

[35] See the article "Maṭāliᶜ" in *EI*².

parts, inclines up or down on (the road) amounting to this (difference).

Commentary

In his *Planetary Hypotheses*, Ptolemy stated that the circumference of the earth was 180,000 *stades* (*astādhiyya*).[36]

The determination of terrestrial latitude is best achieved by measuring the altitude of the Pole Star or by measuring the meridian altitudes of the sun. Our text asserts that the second method was used. As John Britton has shown, there are problems inherent in the use of the meridian quadrant which affect the accuracy of the altitude measurements.[37] Furthermore, given the problems associated with obtaining a reasonable result from such a crude procedure, it is hardly necessary to make a correction for the change in solar declination during the time the team moved the fifty-odd miles. The report attributed to Yaḥyā ibn Aktham - see below - states that the observations were made at the summer solstice - when the daily change in declination is a minimum - but adds that the declination at that time is zero, which is clearly absurd. Does the text mean that one can find the meridian by standing perpendicular to the direction of the setting sun at the equinoxes?

The information on the so-called "black" cubit is interesting: W. Hinz has stated in one place that this unit was standardized by al-Ma'mūn and elsewhere that it was not.[38] Now we know from a contemporary source that it was.

It is the account of the simultaneous lunar eclipse observations and latitude measurements in Baghdad and Mecca which adds to the information recorded by al-Bīrūnī. The simultaneous observation of a lunar eclipse in two localities is a very sensitive kind of observation even with

[36] See Goldstein, "Ptolemy's *Planetary Hypotheses*", p. 7.

[37] See Britton, "Ptolemy on the Obliquity", for details.

[38] See Hinz, *Maße und Gewichte*, pp. 60-61, and the same author's article "Dhirāᶜ" in *EI*², as well as other sources cited in Barani, "Muslim Geodesy", pp. 44-45. See also Mercier, "Muslim Geodesy", p. 177, n. 13, pointing to errors made by Hinz.

sophisticated instruments, and it is particularly difficult to obtain good results when the longitude difference is so small. Problems include the determination of the exact moment of a particular eclipse phase and the determination of the time at that moment. An error of 1° in the measurement of the altitude of a star at one end could of itself lead to an error of the same order in the time estimate, and hence in the longitude difference. The details of the necessary observations are recorded by Ibn Yūnus and al-Bīrūnī.[39] Ḥabash unfortunately gives no information of consequence on the simultaneous eclipse observations; he simply reports that the astronomers measured the distance between the meridian of the two localities as 3°, and that they then "adjusted" this to 2;48°. Indeed, the length of the arc corresponding to a longitude difference of ΔL on a parallel of latitude through Mecca is $\Delta L \cos \phi_M$ and $\mathrm{Cos}\ \phi_M / R = 0;56$ ($R = 60$), so that the 3° reduces to 2;48°.[40]

The latitudes found for Mecca and Baghdad, *viz*. 21;42° and 33;27°, are new to the modern literature, but again no information is provided on the way they were derived. (The accurate values are 21;26° and 33;21°.) Al-Bīrūnī records that Manṣūr ibn Ṭalḥa found $\phi_M = 21;40°$, and this is clearly rounded from 21;42°. The latter value is not attested to in any known source, but the approximation 21;40° was widely used over the centuries. The value 33;27° for ϕ_B is not attested elsewhere; the approximation 33;25° was, however, widely used. (The former value is attested as a latitude for Damascus,[41] but it is not known who first derived it.) See below on other values used for ϕ_M and ϕ_B.

Notice that the latitude difference (33;27° - 21;42°) is correctly derived as 11;45°. In the report attributed to Yaḥyā ibn Aktham, however, 12;15°

[39] Ibn Yūnus' account is translated in Schoy, "Muslim Geography", pp. 265-267. On al-Bīrūnī's discussion see n. 30 above.

[40] See King, "Earliest Qibla Methods and Tables", pp. 90-91.

[41] See King, *Studies*, A-X, p. 78, on the tables for time-keeping for Damascus prepared in the first half of the 14th century by al-Mizzī. It is also used on an astrolabic quadrant for Damascus dated 735 H [= 1334/35] (International Instrument Checklist No. 5008, now preserved in the British Museum, London, inventory no. 62 12-27 1), on which see Mayer, *Islamic Astrolabists*, p. 44, *sub* ʿAlī ibn al-Shihāb.

is used - see below. (The difference between the modern values is 11;55°.) To find the hypotenuse of a right-angled triangle on the sphere (where the longitude difference is measured on a small circle), Ḥabash not unreasonably applies Pythagoras' Theorem for a plane triangle. With Langermann, I have restored a line in the translation where Ḥabash uses the standard approximate rule:

$$\sqrt{(a^2 + b^2)} \approx a + b^2 / 2a \qquad (b << a^2)$$

for taking the square root, a rule known in ancient Babylonia, Egypt, India and China.[42] Thus he uses:

$$\sqrt{[(11;45)^2 + (2;48)^2]} \approx 11;45 + (2;48)^2 / (2 \times 11;45)$$

Ḥabash's results for both the calculated distance Baghdad - Mecca, namely, 12;5° = 676 2/3 miles, and for the measured distance, 712 miles, are different from those reported by Yaḥyā ibn Aktham - see below. (Note that the figure of 712 miles is confirmed by al-Bīrūnī.[43]) See below on the fact that Ḥabash presents no further information on the *qibla* at Baghdad.

4. The account of Yaḥyā ibn Aktham

I now present some new material, taken from MS Princeton Yahuda 4657 (fols. 71v-72r), a copy datable to about 1500 of an early-13th-century Egyptian treatise on folk astronomy by an unidentified author named Sirāj al-Dunyā wa-'l-Dīn.[44] This account of the observations, recorded by the *Qāḍī* Yaḥyā ibn Aktham, is often at variance with that recorded by Ḥabash

[42] van der Waerden, *Ancient Geometry and Algebra*, pp. 46-47,168 and 216. For similar approximations in Islamic sources see Youschkevitch, *Mathématiques arabes*, p. 77, corresponding to Juschkewitsch, *Mathematik im MA*, p. 194; and Berggren, *Episodes*, pp. 48-53.

[43] *al-Bīrūnī's Taḥdīd*: text, p. 234, and translation, p. 208.

[44] On this manuscript see *Princeton Catalogue*, no. 4983, and also *Cairo ENL Survey*, no. C4.

discussed above. A translation follows - see Arabic text no. 2:

> Now that I have finished my statement on the most widely used calendars, I should like to mention some aspects of geometry (*al-handasa*) relating to the size (*dawr*) of the earth.
>
> Chapter. The motive which inspired al-Ma'mūn to (promote) observations, taken from the *Qāḍī* Yaḥyā ibn Aktham. (He reported): "The Commander of the Faithful al-Ma'mūn became very excited about knowing the size of the earth. So he asked me and I told him that the astronomers familiar with geometry (*al-muhandisūn min aṣḥāb al-najāma*) had knowledge of these matters. So he summoned Khālid ibn ᶜAbd al-Malik al-Marwarrūdhī, Yaḥyā ibn Abī Manṣūr, ᶜAlī ibn ᶜĪsā and Aḥmad ibn al-Buḥturī, and they chose a group of brass-workers and carpenters (to assist them in making instruments). Al-Ma'mūn questioned them on the way to proceed and they answered him in unison that it was easy. (They would do) this by facing (*annahum tawajjahū*) ...
>
> ... {there appears to be a lacuna in the text} for a while (*waqtan*) when the sun was entering the sign of Cancer (*nuzūl al-shams ra's al-saraṭān*) {*i.e.*, at the time of the summer solstice}, for at this time the sun has no declination {!! - read: no change in declination}.
>
> They headed for the desert between Tadmur (= Palmyra) and Raqqa and observed the celestial pole (!) until they had accurately determined (*ḥaqqaqū*) its altitude. Next they moved (*tawajjahū*) in a straight line from north to south parallel to the meridian until the altitude of the pole had changed by one degree and then they measured the distance between the two localities (where the observations had been made). They found that one degree on the surface of the earth was (equal to) fifty-six miles, where one mile is four thousand "black" cubits (*dhirāᶜ bi-'l-sawād*), (one of which) is six *qaḍba*s, one of which is four *aṣbaᶜ*s, one of which is the width of six barley corns laid side by side in a row. Then they multiplied 56 by 360 which is the total circumference of the meridian (*dawr al-samā'*): the result was twenty thousand one hundred and sixty miles, which is the measure of the circumference of the earth (*dawr al-samā'*). (This) they divided by 3 and one-seventh, and the diameter of the earth came to 6,414 miles and 3 fifths {actually, 6/11} of a mile. Then al-Ma'mūn told them that he would like to know the azimuth of the

qibla and the distance between Baghdad and Mecca, and they replied that (they could do) this at the time of a lunar eclipse. So they {he?} sent out (*anfadhū*) a group of observers to Mecca to determine the precise time of the beginning and end of the lunar eclipse. Those who had stayed in Baghdad did the same. They {*sc.* the two groups} found that the meridian of Mecca was west of that of Baghdad by three degrees. They corrected (*ᶜaddalū*) these degrees by the right ascensions {!! *bi-maṭāliᶜ al-falak al-mustaqīm* - compare the terminology of Ḥabash} and they found them to be (equal to) two degrees and 48 minutes, which is the "corrected" longitude between Mecca and Baghdad.
They (also) found the altitude of the (celestial) pole at Mecca to be 21 degrees and at Baghdad to be 33 degrees {note: no minutes are given for either value}, so that the latitude of Baghdad is 12 degrees and 15 [minutes] north of that of Mecca. When they multiplied the longitude difference by itself (the result) was 7 degrees (*daraja*) and 50 minutes and two-fifths of a minute {*i.e.*, 7;50,24}, and when they multiplied the latitude difference by itself (the result) was 150 units (*juz'*) and [three minutes] {text: one-third of a unit} and fifty [seconds] {text: units} {*i.e.*, 150;3,50 - note that the square of 12;15 is 150;3,45}. When they added the two quantities and took the square root of the sum the result was 12 degrees and [30 minutes] {text: 20} to a close approximation. And when they multiplied that by 56 miles the result was 700 and 3 miles and one-half and one-quarter of a mile. {See the commentary on this calculation.}
Al-Ma'mūn wanted to check the calculations made by Yaḥyā {*sc.* Yaḥyā ibn Abī Manṣūr} and his colleagues, so he sent out someone to measure the road to Mecca, and they {plural! - compare the account of Ḥabash} found that the (number of) miles between Baghdad and Mecca by the shortest and straightest road was 700 and 10 miles, which was about 6 miles more than the results of the calculation. al-Ma'mūn pronounced that the calculation was more accurate and that the difference was due to the depression (*istifāl*) of the *wādī*s and the elevation (*irtifāᶜ*) of (hills on) the surface of the earth. When he had become convinced of this, he became excited to know the maximum distance of the moon from the surface of the earth, so he asked them about this and they answered ....

.... ."

Commentary

There are clearly some corruptions of the text, which, as it stands, does not reflect well on the *Qāḍī*'s competence to participate in the observational program. For example, it is not absolutely clear from the text that he knew the difference between an equinox and a solstice. If the sun was indeed at the summer solstice, then the daily change in its declination would be negligible even in more sophisticated observations. This con tradicts the statement of Ḥabash that the astronomers made a correction for the change in solar declination inbetween the two decisive solar altitude measurements. The next points to note are first that this text states that the observations were made by heading for the desert between Palmyra and Raqqa, and second that there is no mention whatsoever of Sanad ibn ᶜAlī.

This text mentions that the altitude of the celestial pole in Mecca was found to be 21° and in Baghdad, 33°. Yet the latitude difference stated is 12;15°, and the ensuing distance calculations are based on that value (not 11;45° as reported by Ḥabash). Since we have no information on the way in which the values were obtained, there is little more to say on them, although it may be useful to remark on other early values of these two parameters.

Excursus: Early values of the latitudes of Mecca and Baghdad

In addition to the values 21;0° and 21;40° for ϕ_M that are mentioned in our sources, we may cite various others used in the 9th and 10th centuries. Ptolemy's value of 22° for ϕ_M (for "Makoraba")[45] was used by the 10th-century astronomer Abū 'l-Wafā' (who used 33;25°for ϕ_B and 3° for ΔL_B).[46] The identity of the astronomers who derived the value 21;30° for ϕ_M within a few years of the observations commissioned by al-Ma'mūn is unknown. Likewise, we do not know who determined ϕ_M as 21;20° (the

[45] Kennedy & Kennedy, *Islamic Geographical Coordinates*, p. 225.

[46] Kennedy, "Abū'l-Wafā' on the Distance Baghdad-Mecca", p. 200.

accurate value is 21;26°).[47] This was used in one of the first tables displaying the *qibla* as a function of terrestrial coordinates, which was prepared in Baghdad in the 9th or perhaps the 10th century.[48]

We have already encountered the values 33° and 33;27° for ϕ_B in our sources. In various early Abbasid works, some to be associated with al-Khwārizmī, a contemporary of these observations, the value 33° is also used for ϕ_B (the accurate value is 33;20°).[49] Yet in al-Khwārizmī's and also al-Battānī's geographical tables the value given is 33;9°.[50] This distinctive value reappears on one of the plates of of an astrolabe made in 11th-century al-Andalus.[51] We need not assume that such a value was actually measured: it may have been calculated form the length of longest daylight.[52] The more accurate value 33;25° was used, for example, by al-Nayrīzī in Baghdad *ca.* 900,[53] and is also found in the geographical tables of Suhrāb, *ca.* 930.[54] The value value 33;21° appears to have been derived by Ibn al-Aᶜlam, a 10th-century Baghdad astronomer of some

47 King, "Earliest Qibla Methods and Tables", pp. 126-127 and 129.

48 *Ibid.*, pp. 118-126.

49 King, "al-Khwārizmī", p. 2.

50 Kennedy & Kennedy, *Islamic Geographical Coordinates*, p. 55.

51 Namely, the astrolabe made by Ibrāhīm ibn al-Sahlī dated 478 H [= 1085/86] (International Instrument Checklist No. 121, preserved in the Staatliche Kunstsammlungen in Kassel, inventory no. A38) - see Mayer, *Islamic Astrolabists*, p. 52. The latitude 33;9° is also stated to serve Damascus.

52 The length of longest daylight chosen could have been 14;15 hours, the obliquity of the ecliptic that of Ptolemy, 23;51,20°. (Muslim astrolabists used this latter parameter for centuries, unswayed by the more up-to-date values found by the Abbasid astronomers and their successors.)

53 Schoy, "al-Nayrīzī über die Qibla", p. 57.

54 Kennedy & Kennedy, *Islamic Geographical Coordinates*, p. 55.

renown.[55] It is used in several tables in the anonymous recension of the *Mumtaḥan Zīj*, the only available version of the major production of the astronomers of al-Ma'mūn.[56]

Returning to the text, it is not without interest to follow the calculations recorded by Yaḥyā of the distance Baghdad - Mecca. These do not correspond to those reported by Ḥabash, but it seems as though the latter were made by Ḥabash himself. The squares of the sides of the (plane) right-angled triangle of which this is the hypotenuse are given as 7;50,24 and 150;3,[45] (text has 50″). Their sum is 157;54,9. Using the standard approximation, the square root of this is:

$$\sqrt{(144 + 13;54,9)} \approx 12 + 13;54,9 / (2 \times 12)$$
$$\approx 12 + 0;34,45 \approx 12;34 \text{ (by truncation)} \approx 12;30 \text{ (text has 12;20).}$$

Now we multiply 12;34 by 56: the product is 11,43;44, which is indeed about 703 + 1/2 + 1/4, as stated in the text. Note, however, that the results of al-Ma'mūn's final survey of the road between Mecca and Baghdad as reported here are also at variance with what was recorded by Ḥabash.

The reader will observe that no value is presented for the *qibla* at Baghdad. Clearly, though, the method used was equivalent to:

$$\sin q = \Delta L \cos \phi_M / \sqrt{[(\Delta\phi)^2 + (\Delta L \cos \phi_M)^2]} .$$

55 On Ibn al-Aʿlam see Sezgin, *GAS*, V, p. 309, and VI, pp. 215-216, and on his lost astronomical tables see Kennedy, "Ibn al-Aʿlam", Mercier, "Ibn al-Aʿlam" and Tihon, "Alim".

His derivation of ϕ_B is described in a note on the flyleaf of an early copy of the *Zīj-i Īlkhānī* which passed through Christie's of London in 1986. The text translates: "Note (*fā'ida*): al-Sharīf {the standard epithet of Ibn al-Aʿlam} found that the highest (solar) altitude was 80;12° and the lowest 33;6°. From this it was established that the obliquity of the ecliptic was 23;33°, the latitude of Baghdad 33;21° and the altitude of (the sun at) the equinox(es) 56;39°. Copied from the handwriting of the author."

56 See MS Escorial ar. 927, fols. 64r, 64v and 93v (pp. 125, 126 and 184 of the "facsimile" published in Frankfurt in 1986, which does not accurately reproduce the original manuscript).

This corresponds to one described in an early Abbasid text.[57] Its justification is simple: distances on the sphere are considered in a plane. And, in particular, distances on the parallel of latitude of Mecca are modified by a factor of cos ϕ_M = Cos ϕ_M / R (where R = 60).

For Ḥabash's values 2;48° and 12;5° for the numerator and the denominator, I compute 13;24° for *q*. For Ibn Aktham's values 2;45° and 12;34°, I compute 13;38°. (The value 13;13° for Baghdad is specifically mentioned in the instructions to an Abbasid *qibla* table, but the table is based on ϕ_M = 21;20°.[58])

The geographical parameters derived from these observations (ϕ_M = 21;40° and ΔL = 3°), but now with the better value ϕ_B = 33;25°, were used by al-Nayrīzī (*fl.* Baghdad *ca.* 900) in a numerical example appended to his treatise on the determination of the *qibla* by spherical trigonometry.[59]

5. Concluding remarks

It should be borne in mind that before these observations were conducted to establish the mathematically-computed *qibla*, the Muslims in the province of al-ᶜIrāq had been using astronomical alignments or cardinal directions for the *qibla*. A popular *qibla* direction for al-ᶜIrāq was winter sunset, chosen in order to be "facing" the north-east wall of the Kaᶜba.[60]

[57] King, "Earliest Qibla Methods and Tables", p. 102. See also Ahmedov *et al.*, "Istanbul Manuscripts", on the source.

[58] King, "Earliest Qibla Methods and Tables", pp. 127-129.

[59] Schoy, "al-Nayrīzī über die Qibla", p. 57.

[60] On the use of astronomical alignments for the *qibla* see King, *Studies*, C-X-XIII (where C-X is a reprint of the article "Makka. iv. As centre of the world" in *EI²*), and, more recently, *idem*, "Orientations".

It is not only for the history of Islamic architecture that the *qibla* is important. In an Arabic account of the supernova of 1006 A.D. the direction of the supernova is defined in terms of the "*qibla* of al-ᶜIrāq" (see Goldstein, "Supernova of 1006", p. 107). But which *qibla* is being referred to here? Winter sunset, a popular traditional direction, or the *qibla* determined by al-Ma'mūn's astronomers? Or yet another direction?

Others preferred due south, following the practice of the Prophet in Medina. In the year 847 the mosque in the new Abbasid city of Samarra was apparently (that is, according to Herzfeld and Sarre) laid out at about 15° W of S,[61] perhaps as a result of the observations described above. Surely other religious sites could be identified in Baghdad with an orientation of about 13° W of S. But very little reliable information on the orientations of Islamic religious architecture in Iraq is currently available, and little more should be said until they are measured properly.

As Mercier has already pointed out, all of these reports about the expeditions must be regarded with a certain amount of scepticism. He has expressed doubt that the Palmyra expedition ever took place, and his arguments are, given the evidence, sound. Without wishing to do injustice to Sanad ibn ᶜAlī, Mercier suspects that we are dealing with a legendary report based on a possible earlier (that is, pre-Islamic) expedition. But the main cause of scepticism remains the fact that the results are too good to be true, if indeed the procedures involving measuring solar meridian altitudes were used. Not least is the problem presented by the size of the solar disc. Again, an error in one minute in the solar altitude would yield an error of one mile in the distance, and measuring the solar altitude to the nearest minute is something that one might have been able to do with a good instrument in Baghdad, but anticipating that one could achieve the same accuracy in the wilderness during an expedition of several days requires considerable imagination. Small wonder, then, that not long thereafter someone thought of simply measuring the distance between Baghdad and Samarra, and the difference in solar meridian altitudes in the two places. Unfortunately, however, Samarra is by no means due north of Baghdad. The report of this minor expedition, cited by al-Bīrūnī from Abū 'l-Faḍl al-Harawī, confirmed the result 56 2/3, and was rightly regarded by al-Bīrūnī as rather fishy.[62]

Some later Muslim scholars ignored the results of al-Ma'mūn's

[61] Herzfeld & Sarre, *Archäologische Reise*, III, pl. XIX. (Note that in pl. XX no orientation is given.).

[62] *al-Bīrūnī's Taḥdīd*: text, pp. 212-213; translation, pp. 177-178; and commentary, pp. 132-133.

expeditions altogether. Thus, for example, the celebrated early-14th-century geographer-prince Abū 'l-Fidā' preferred Ptolemy's value of 66 2/3 miles per degree.[63] A report such as the one by Yaḥyā ibn Aktham would certainly have encouraged serious people to favour such a reliable authority from Antiquity (though see below).

Our problem is that we do not have access to the original reports by the astronomers who participated in the expedition(s). Were there really two different expeditions, one in the desert north of Palmyra that started from Damascus and another in the desert of Sinjār that started from Baghdad? Even the latter, which is reported to have taken the astronomers 200 km into the desert, must have been a major undertaking. And it would have been utter folly to schlepp instruments and personnel 500 km across inhospitable terrain from Baghdad to the Syrian desert. We are still, however, confronted with the report by Sanad ibn ᶜAlī, which is early enough that one cannot explain it away.

Other reports of different astronomical observations (mainly of conjunctions and eclipses, equinoxes and solstices) conducted by the Abbasid astronomers in the 9th century, such as we find quoted in the writings of Ibn Yūnus and al-Bīrūnī, are impressive by any standards and are not marred by the inconsistencies found in these various reports of the geodetic measurements. What has happened here confirms the adage: "Too many cooks spoil the broth". Ibn Yūnus made a similar criticism of al-Ma'mūn's team-work, stating that one good man could achieve more on his own.[64] He had, of course, himself in mind. Alas he did not tap his patron al-Ḥākim for a geodetic expedition up the Nile. Al-Bīrūnī, who was more convinced of the need for new geodetic measurements, did conduct the necessary research programme and was able to derive a reasonable value without involving two teams of astronomers - and one superfluous *qāḍī* completely innocent of what they were doing - wandering about in the desert.[65]

[63] Mercier, "Muslim Geodesy", p. 176, n. 12, and pp. 177-178.

[64] Caussin, "Ibn Yūnus", pp. 34-40.

[65] See the summary in Mercier, "Muslim Geodesy", pp. 182-184.

One may wonder why in later mathematical geography Muslim astronomers preferred to use the less accurate Ptolemaic value of 66 2/3 miles for one degree. For example, it is Ptolemy's value that is used on 17th-century Persian maps of the world centred on Mecca, from which one can simply read off the *qibla* and the distance to Mecca for any locality in the Islamic commonwealth.[66] The explanation is probably that it was thought that the parameter 66 2/3 actually was the result of al-Ma'mūn's astronomers. Even the great 13th-century polymath Naṣīr al-Dīn al-Ṭūsī, in his influential *Tadhkira fī ᶜilm al-hay'a*, attributes this value to them.[67] Thus the efforts and results of al-Ma'mūn's astronomers were not only not properly recorded at the time, these results were also not properly transmitted for posterity.

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[66] On these remarkable maps (one discovered in 1989 and another in 1995) see Lorch & King, "Qibla Charts", pp. 198-200 and 202; my article "Samt" [= direction] in *EI*², with corrections in the article "Ṭāsa" [= magnetic compass]; and King, *World-Maps*. A detailed study is in preparation.

[67] See Ragep, *al-Ṭūsī's* Tadhkira, I, pp. 310-314, and II, pp. 501-510. Ragep has shown that the value is recorded already in a report by Muḥammad ibn Mūsā ibn Shākir (d. 873).

النصوص العربية

١: مخطوطة الربان كفاح في القدس (ص. ١-٣ من رسالة حبش)

بسم الله الرحمن الرحيم وبه نستعين كتاب الأجرام والأبعاد لحبش بن عبد الله الحاسب قال إن أمير المؤمنين المأمون أحب أن يعرف مقدار الأرض فبحث عن ذلك فوجد بطليموس (!) يذكر في بعض كتبه أن دور الأرض كذا وكذا ألف اسطادنيوس[1] فسأل المفسرين عن عن تفسير الاسطادنيوس[2] فاختلفوا في تفسيره هذا ...[3] لا يغني[4] عما نريد فوجه خالد بن عبد الملك المروروذي[5] وعلي بن عيسى الأسطرلابي وأحمد بن البحتري الذارع مع جماعة من الذراع والصناع من النجارين والصفارين لتصحيح ما يحتاجون إليه من الآلات وحملهم بأجمعهم إلى موضع اختاروه من برية سنجار[6] ثم توجه خالد وطائفة معه قطب بنات نعش الشمالي وتوجه علي وأحمد وطائفة معهم إلى جهة القطب الجنوبي فمضوا حتى انتهوا إلى أن وجدوا غاية ارتفاع نصف النهار قد زال وتغير عن ارتفاع نصف النهار الذي وجدوه في الموضع الذي افترقوا منه مقدار درجة واحدة بعد أن نقص من ذلك مقدار ميل الشمس في مدة مسافة الطريق في ذهابهم ونصبوا السهام ثم رجعوا أيضا على تلك السهام وامتحنوا الذرع ثانية

[1] بغير نقط في الأصل.

[2] بغير نقط في الأصل.

[3] إنه يجوز أن تنقص كلمة هنا.

[4] في الأصل: يني.

[5] في الأصل: المروزودي.

[6] في الأصل: سيحان.

فوجدوا مقدار درجة واحدة من وجه الأرض ستة وخمسين ميلا والميل أربعة آلاف ذراع بالأسود وهو الذراع الذي وضعه المأمون لذرع الثياب ومسح البناء وقسمة المنازل وسمعت هذا الذي ذكرته في كتابي من خالد بن عبد الملك المروروذي[7] يحدث به القاضي يحيى بن أكثم[8] فأمر يحيى أن يكتب له جميع ما ذكره خالد فكتب له وكتبته أنا من خالد سماعا وإن أمير المؤمنين المأمون رضي الله عنه أحب أن يمسح سمت القبلة فوجه في وقت كسوف قمري ليمتحن الطول بين مكة وبغداد فوجد نصف نهار مكة غربيا عن نصف نهار بغداد بثلث درج بالتقريب فإذا[9] عدلنا درجة واحدة من الفلك المستقيم نجد حصتها في مدار[10] الفلك المخطوط على عرض مكة ست وخمسين دقيقة بالتقريب فإذا ضربنا البعد بين نصف نهار مكة ونصف نهار بغداد ثلث درج في ٥٦ دقيقة وجعلنا كل س جزءا نجد البعد المعدل في الطول بين نصف نهار مكة ونصف نهار بغداد في الفلك المخطوط على مدار عرض مكة درجتين وثماني وأربعين دقيقة بالتقريب ووجد أيضا عرض مكة أحد وعشرين درجة واثنان وأربعون دقيقة بالتقريب وعرض بغداد ثلث وثلثون درجة وسبع وعشرين دقيقة بالتقريب فنقصنا عرض مكة من عرض بغداد فبقي الفضل بين العرضين أحد عشر درجة وخمسة وأربعين دقيقة فإذا ضربنا الفضل بين الطولين وهو ب درج مح دقائق في نفسه يجتمع ٧ درج و ٥٠ دقيقة ٢٤ ثانية ونصفه ٣ درج و ٥٥ دقيقة و ١٢ ثانية يكون ذلك ٢٣٥ دقيقة و ١٢ ثانية نقسمه على ١١ درجة و ٤٥ دقيقة وهو الفضل بين العرضين فبلغ مسافة الطول بين بغداد ومكة ١٢ درجة و ٥ دقائق ضربناه في ٥٦

[7] في الأصل: المروزودي.

[8] في الأصل: اكتم.

[9] في الأصل: فاد.

[10] في الأصل: مقدار.

ميلا فبلغ المسافة بين بغداد ومكة على السهم المستوي ستمائة وستة وسبعين ميلا وثلثي ميل بالتقريب وقد كان المأمون عندما رفع إليه خالد بن عبد الملك مقدار درجة واحدة من الأرض أحب أن يمتحن ذلك فوجد من ذرع الطريق بين بغداد ومكة على أقرب الطرق فوجده سبع مائة واثني عشر ميلا فوجدنا الخلاف بين السهم الذي يجري في الهوى وبين ذرع الطريق خمسة وثلثين ميلا وثلث ميل فقال المأمون هذا ما لا يستكثر لأن الطريق يجب فيه العوادل والصعود والهبوط مقدار هذا ...

٢: مخطوطة برينستن ٤٦٥٧ (٤٩٨٣)، ق ٧١و - ٧٣و

... ولما فرغنا من ذكر التواريخ المشهورة أردنا أن نذكر طرفا من علم الهندسة في دور الأرض فصل السبب الباعث للمأمون على الرصد المنقول عن القاضي يحيى بن أكثم[1] إن أمير المؤمنين سمت همته إلى معرفة دور الأرض فسألني عن ذلك فقلت إن للمهندسين من أصحاب النجامة عندهم علم ذالك فأحضر خالد بن عبد الملك المروروذي ويحيى بن أبي منصور وعلي بن عيسى وأحمد بن البحتري[2] واختاروا[3] من الصفارين والنجارين جماعة فقال لهم المأمون كيف السبيل إلى ذلك فقالوا بأجمعهم هذا سهل وذلك أنهم توجهوا وقتا عند نزول الشمس رأس السرطان إذ ليس لها ميل هناك وقصدوا البرية الذي بين تدمر والرقة ورصدوا القطب حتى حققوا ارتفاعه ثم توجهوا على خط مستقيم من الشمال إلى الجنوب موازيا لدائرة نصف

[1]في الأصل: اكتم.

[2]في الأصل: التجري.

[3]في الأصل: واختارو.

النهار حتى تغير ارتفاع القطب مقدار درجة واحدة ثم مسحوا ما بين الموضعين فوجدوا مقدار درجة واحدة من وجه الأرض ستة وخمسين ميلا والميل أربعة ألف (!) ذراع بالسواد وهو ست قبضات القبضة أربع أصابع عرض الإصبع إلى الإصبع ست شعيرات مصفوفة بطون بعضها إلى بعض فضربوا نو في شس الذي هو دور السماء فخرج من الضرب عشرون ألف ميلا ومائة ميل وستون ميلا وهو مساحة دور الأرض فقسموا على ٣ وسبع فخرج قطر الأرض ٦٤١٤ ميلا و ٣ أخماس ميل فقال لهم المأمون إني أحب أن أعرف سمت القبلة وبعد ما بين بغداد ومكة فقالوا نعم إذلك (إذ ذلك) وقت خسوف القمر ثم انفذوا جماعة من أهل الرصد إلى مكة فحققوا وقت خسوف القمر ووقت تمام انجلائه وكذالك فعلوا (!) المقيمون ببغداد وقت خسوف القمر أيضا فوجدوا نصف نهار مكة غربيا عن نصف نهار بغداد بثلاث درج فعدلوا هذه الدرجات[4] بمطالع الفلك المستقيم فوجدوها درجتين و مح دقيقة وهو الطول بين مكة وبغداد معدلا ووجدوا ارتفاع القطب بمكة كا درجة وببغداد لج فعرض بغداد شمال عن مكة يب درجة ويه فإذا ضربوا ما بين الطولين في نفسه كان ز درج ون دقيقة وخمسا دقيقة وإذا ضربوا ما بين العرضين في نفسه كان ذلك قن جزء وثلث جزء وخمسون جزء فإذا جمعوهما وأخذوا جذر المجتمع يخرج يب درجة و ك بأقرب تقريب فإذا ضربوا ذلك في نو ميلا تجتمع (!) في ذ ميل و ٣ أميال ونصف وربع ميل وكان المأمون يحب أن يمتحن ما حسبه يحيى وأصحابه فوجه من ذرع طريق مكة فوجدوا الأميال بين بغداد ومكة على أقصر الطرق وأقربها استواء ذ ميل وي أميال فزاد الذرع على ما خرج بالحساب و أميال بالتقريب فقال المأمون الحساب أصدق وهذا التفاوت من استفال الأودية وارتفاع وجه الأرض فلما سكنت نفسه إلى ذلك سمت همته إلى أن يعرف أبعد بعد القمر عن وجه الأرض فسألهم عن ذلك فقالوا ...

[4]في الأصل: الدرجة.

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