Problem size effect in additions and subtractions: an event-related potential study

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Abstract

The psychophysiological basis of problem size effect in the arithmetical processing of additions and subtractions was studied with event-related brain potentials (ERP). Subjects were presented sequences of seven numbers, and ERPs elicited to the sixth number were analyzed. Two variables were manipulated: operation type (addition and subtraction) and problem size (by adding or subtracting 2, 3 or 4). Results showed two phases in the ERP pattern: an early phase, appearing to reflect automatic processing involved in stimulus identification, and a positive slow wave, believed to be a computing indicator of the subsequent calculation. The amplitude of this positive slow wave was modulated by the problem size (the more problem size, the larger the amplitude), suggesting that the amplitude of this slow wave indexes the activation of the cerebral network underlying problem size effect.

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Numerical cognition has been extensively investigated by behavioral methods, but the more recent use of psychophysiological measures, such as ERPs, PET, and fMRI, has provided some evidence on the neurobiological basis of arithmetical processing. By recording ERPs, the present study aims at providing some neurobiological foundations for one of the most widely reported behavioral effect in the literature on mental arithmetic, i.e., the problem size effect.

The term “problem size effect” (or “problem difficulty effect”) refers to the phenomenon that reaction times (RTs) are longer when arithmetical problems are presented with larger operands, and hence with larger answers [1,4]. For example, 3 + 4 generally is solved more quickly and accurately than 8 + 4. The robustness of this effect has been widely demonstrated: it holds for simple addition, subtraction, multiplication and division; it is evident using both RTs and error rates; it is obtained both in production and verification tasks; and it characterizes performance across the entire span of ages [1]. The problem size effect has been attributed to differences in accessibility of results, due to differences in the frequency of problem usage. Smaller problems are encountered more frequently and therefore are more likely to have high memory strength relative to large problems.

Few studies have recorded ERPs systematically in mental calculation tasks. With regard to the mental process of calculation, some recent studies have presented evidence that an early portion of the ERPs reflects physical identification of the number and that a late positive slow wave is functionally related to exact mental arithmetical calculation [5,6]. Other studies reported a modulation of the positive slow wave amplitude associated with the problem size effect in simple multiplications: the easier the arithmetical operation, the larger its amplitude [10,11]. These results suggest that a positive slow potential reflects mental processes essential to calculation prior to obtaining the result of the operation, and that the amplitude modulation of this wave may be a brain signature of the problem size effect.
In the present study, sequences of seven numbers were sequentially presented and the pattern of ERPs elicited by the sixth number was analyzed. The type of sequence was manipulated by presenting six different operations: +2, +3, +4, −2, −3, and −4. Three increments were selected in order to manipulate the problem size effect, and addition and subtraction were chosen to study whether it is possible to extend the results obtained by Pauli et al. in multiplications: the lesser the problem size, the more negative the slow wave.

We hypothesized that two phases of arithmetical processing would be differentiated in the ERP pattern. Firstly, as the early portion of ERPs is considered to be a reflection of the physical identification of the number and its meaning, no differences between conditions were expected up to 250 ms post-stimulus (because it is the reflection of a common mental process). Moreover, as the N1-P2 complex occurring in this latency range is associated to stimulus identification [5,6], we expected this ERP complex to be present in all the conditions. Second, in order to follow the sequence, the subject was expected to start to compute the new operation in order to predict the following number in the sequence. Therefore, as the calculation has been associated with a positive slow wave whose amplitude depends on the problem size, we hypothesized that a positive slow wave of different amplitude would follow the N1-P2 complex. Because simple additions and subtractions were used in the present study, we expected to find a slow wave modulation similar to that reported by Pauli et al. in simple multiplications: the lesser the problem size, the more negative the slow wave.

Fourteen volunteers (11 women; age 20–33 years) participated in the experiment. All participants had normal or corrected to normal visual acuity, and gave written informed consent to participate.

Sequences of seven Arabic numbers were used as stimuli, and were constructed according to the following rules: consecutively adding or subtracting 2, 3 or 4 to or from the previous number. The seventh number presented completed the sequence either correctly or incorrectly.

Sequence presentation was controlled by STIM 2.0 software (NeuroScan Inc.). Numbers were shown in white on a black background, and subtended a visual angle of 1.63° vertically and 0.8° (for one-digit stimuli) or 1.63° (for two-digit stimuli) horizontally.

Each trial consisted of a sequence of seven Arabic numbers presented sequentially on the screen. Numbers remained on the screen for 1000 ms and the inter-stimulus interval was 1500 ms. After the last number of the sequence, an asterisk was shown for 1000 ms. The function of this asterisk was to give the participant the following information: (1) the sequence had finished; (2) blinking was allowed; and (3) a motor response was required in order to indicate if the seventh number of the sequence was correct or incorrect. The inter-trial interval was 1500 ms. During the recording session, subjects were instructed to relax and to look at the center of the screen. They were also instructed to avoid blinking: brief resting periods after each trial were announced with an asterisk appearing on the screen.

Two recording sessions were held on different days. This procedure was chosen because participants were tested on 576 trials—96 for each experimental condition—and it was necessary to avoid tiredness. In each session, 12 blocks of 24 trials were presented to each participant. A message indicating a 30 s rest appeared on the screen after 12 trials, and there was a 5 min rest in the middle of the recording session. The order of appearance of the trials was controlled and no more than three addition sequences or three subtraction sequences appeared consecutively.

ERPs were recorded and analyzed with the SCAN 3.0 software (NeuroScan, Inc., Herndon, VA, USA). EEG was recorded from seven tin electrodes mounted in a commercial electro-cap (Electro-Cap International, Eaton, OH, USA) and positioned according to the 10–20 International System: C3, C4, P3, P4, Cz, Fz, and Pz. Connected ear lobes served as reference, and an equidistant point between Fpz and Fz was used as location of the ground electrode. To monitor eye movement and blinks, four additional Ag/AgCl electrodes were used: two for the VEOG recording placed above and below the right eye, and two for the HEOG recording placed at the two external canthi.

EEG and EOG channels were continuously digitized at a rate of 250 Hz by a SynAmp™ amplifier (5083 model, NeuroScan, Inc.). A band pass filter was set from 0.05 to 30 Hz, and electrode impedance was always kept below 5 kΩ.

Epochs for each subject in each experimental condition were averaged relative to a pre-stimulus baseline that was made up of the 100 ms of activity preceding the epoch of interest. The sixth number of the sequence was chosen because it was considered that subjects at this sequence’s point would have discovered the arithmetic rule. Trials with artifacts (voltage exceeding ±50 μV in any channel) and trials with incorrect behavioral responses were excluded from the ERP average (21% of trials were rejected for these reasons; however, as subjects were tested on 96 trials for each experimental condition, enough trials were retained at each average wave).

ERPs were analyzed by computing the mean amplitude in the 100–150 ms window, the 175–225 ms window, and in 100 ms windows from 300 to 800 ms post-stimuli. A $2 \times 3 \times 7$ repeated-measures analysis of variance (ANOVA) was performed to analyze electrophysiological data, taking as factors operation type (addition or subtraction), increment (2, 3, or 4), and location (C3, C4, P3, P4, Fz, Cz, Pz, and Fz). Repeated-measures ANOVAs were carried out with the Greenhouse–Geisser correction for sphericity departures, which was applied when appropriate. The $F$-value, the probability level following correction, and the $η^2$ strength of association index [8] are reported. Whenever a main effect reached significance, post hoc contrasts using a non-posted error term were calculated [7]; in order to analyze significant interactions simple effects were calculated. In both cases, the
Hochberg approach was used to control for the increase in type I error. Finally, trend analyses were performed to try to fit a linear trend between the variables voltage and increment.

As for the analysis of behavioral data, the correct response rate for the variable increment (2, 3, or 4) in additions and subtractions was analyzed with the Friedman test. Then, whenever the Friedman test reached significance, Wilcoxon T-tests were employed to perform paired contrasts.

Analysis for correct response rate showed no significant differences between adding 2, 3 or 4 (p < 0.387): mean and standard error (in parenthesis) for correct rate were 96.6 (0.8), 94.2 (1.9) and 95.5 (1.5), respectively. Although it was expected to find differences between the three types of additions, the absence of differences can be due to a ceiling effect [2]. However, when the operation to be performed was a subtraction, differences for the correct response rate were found (p < 0.04). Paired contrasts showed significant differences for correct rate between subtracting 2 and 3 (p < 0.01) and subtracting 2 and 4 (p < 0.03). There was no difference in correct rate between subtracting 3 and 4. The mean and standard error were 97.5 (0.58), 92.8 (1.8) and 94.4 (1.5) for subtracting 2, 3 and 4, respectively.

Grand average ERPs for the sixth number in the addition and subtraction sequences are shown in Fig. 1. In the addition sequences, there were no differences between the three types of increment until 250 ms post-stimulus. Thereafter, a positive wave appeared which increased in amplitude with larger numbers. Amplitude differences in the positive slow wave were present as a function of the increment: the larger the increment, the larger the voltage. With regard to subtractions, differences for the correct response rate were found (p < 0.04). Paired contrasts showed significant differences for correct rate between subtracting 2 and 3 (p < 0.01) and subtracting 2 and 4 (p < 0.03). There was no difference in correct rate between subtracting 3 and 4. The mean and standard error were 97.5 (0.58), 92.8 (1.8) and 94.4 (1.5) for subtracting 2, 3 and 4, respectively.

The ANOVA results for the effects that reached statistical significance (p-values less or equal to 0.05) are summarized in Table 1. The main effect of the variable increment reached statistical significance in the windows from 300 to 800 ms. Trend analyses showed that a linear trend could be adjusted between increment and voltage in all the windows analyzed (all p-values < 0.006). As for the interactive effects, the type of operation modulated the increment effect in the 300–400 and 500–600 windows, as was shown by the significant Operation × Increment interaction. In order to perform a more detailed analysis of this interactive effect, trend analyses were calculated for addition and for subtraction separately. For addition, a linear trend was adjusted in the windows from 300 to 800 ms: the larger the increment, the more positive the voltage (all p-values < 0.009). For subtraction, no trend could be adjusted in these windows, either linear or quadratic. Nevertheless, post hoc contrasts showed that there were no differences between subtracting 3 or 4 in the 300–800 intervals, although there was a difference between these two subtractions and subtracting 2 (p-values less or equal to 0.001): the voltage was less positive for −2 than for −3 and −4.

Finally, statistical significance was reached for the Operation × Location interaction in the 500–600, 600–700 and 700–800 ms windows. Simple effect analysis showed no differences between addition and subtraction at any location in the 700–800 window. However, simple effect analysis showed differences between addition and subtraction at P3 and Pz in the 500–700 ms interval: the voltage was more positive for subtraction than for addition.

As expected, two phases for the arithmetical processing in sequences were differentiated on the ERP pattern: stimulus identification, and calculation to predict the following number on the sequence.

In the first phase, no differences between conditions were found up to approximately 250 ms post-stimulus. This finding replicated those of other studies [5,6], in which a N1-P2 complex was associated with the identification of the stimulus. This encoding phase lasted around 250 ms, in agreement with the classical ERP pattern connected to visual stimuli recognition [3].

The second phase observed in the ERP pattern could be attributed to the cognitive processing of calculation. At this point, a positive wave starting at 250 ms and lasting at about 1000 ms was observed. Previous research has suggested that a positive slow wave is associated with the computation of the arithmetical operation [5,6,10,11], so the positive slow wave reported in the present study may be considered to be an index of the calculation process. Another finding supports this conclusion. In the present experiment an amplitude modulation of the positive slow wave was associated to the specific operation to be performed (+2, +3, +4, −2, −3, −4). When the computation in question was an addition, a linear trend between increment and voltage could be fit in the 300–800 ms interval. This result may be explained in terms of problem size effect because it agrees with previous research where a similar modulation of a slow wave was reported. Pauli et al. [10] presented simple multiplication problems with three different levels of problem size and found a differentiation in the slow wave amplitude associated with the level of problem difficulty. These authors also manipulated practice by presenting the same operations in three different sessions, and it was found that the amplitude differences in the slow wave diminished with practice. In contrast, when a new series of problems were introduced, the differences in the slow waves depending on the level of difficulty of the problem again became observable. These results allowed Pauli et al. [10] to relate the amplitude modulation of the slow wave with the problem size effect, which reduces by practice. Our results provide new evidence to support Pauli et al.’s claim [10], and enables us to generalize the result to additions. Moreover, we found that voltage and problem size fit a linear function when subjects have to perform simple additions.

Contrary to expectations, the increment effect in subtractions differed from the increment effect in additions. Again, a slow positive wave was present when the operation to be performed was a subtraction, but in this case there was no amplitude difference between −3 and −4, though both elicited a more positive slow wave than that elicited by −2. Although
this was an unexpected result, it can be explained because problem size effect is thought to be influenced by the frequency with which the problem is encountered in daily life [1]. Simple problems receive more practice, consequently, while $-3$ and $-4$ can be considered tasks of similar difficulty, or of similar frequency in daily life, $-2$ is clearly a more frequent operation. This explanation can be supported by the results of the analysis of correct response rate. In this analysis, it was showed that there were no differences in correct rate for $-3$ and $-4$, and that both operations showed less correct rate than $-2$. Therefore, these results can be attributed to the fact that $-3$ and $-4$ are more difficult operations than $-2$, and that both operations had similar difficulty.

Table 1

<table>
<thead>
<tr>
<th>Time (ms)</th>
<th>Increment</th>
<th>Operation × Increment</th>
<th>Operation × Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>100–150</td>
<td>0.386</td>
<td>0.028</td>
<td>0.749</td>
</tr>
<tr>
<td>175–225</td>
<td>2.970</td>
<td>0.186</td>
<td>1.501</td>
</tr>
<tr>
<td>300–400</td>
<td>9.501</td>
<td>0.001**</td>
<td>2.242</td>
</tr>
<tr>
<td>400–500</td>
<td>9.549</td>
<td>0.002**</td>
<td>2.512</td>
</tr>
<tr>
<td>500–600</td>
<td>10.327</td>
<td>0.001**</td>
<td>4.453</td>
</tr>
<tr>
<td>600–700</td>
<td>9.499</td>
<td>0.001***</td>
<td>2.297</td>
</tr>
<tr>
<td>700–800</td>
<td>7.327</td>
<td>0.009***</td>
<td>1.417</td>
</tr>
</tbody>
</table>

* Sphericity assumed.
* Sphericity assumed.
** Greenhouse-Geisser correction.
* 0.05 ≥ $p$ > 0.01.
** 0.01 ≥ $p$ > 0.001.
*** $p$ ≤ 0.001.
Although the analysis of correct rate could justify the absence of amplitude differences between −3 and −4, the fact that no differences in correct rate but differences in amplitude were found in additions could make this argument questionable. However, a possible explanation for the absence of correct rate differences in additions can be drawn in terms of a ceiling effect. The three types of addition might be too easy to be discriminated by the correct response rate. Reaction time would have been a useful measure in order to have more evidence concerning this point; however, the present design was optimized to control for the possible response effect over the ERPs, so that measurement of reaction time was not recorded.

Finally, addition and subtraction also differed at P3 and Pz in the 500–700 ms windows; the voltage was more positive for subtraction than for addition. This voltage difference can be considered as new evidence for a positive slow wave related to arithmetical processing. Because adding a number is easier than subtracting a number, the amplitude differences between addition and subtraction may be an index of problem difficulty. The fact that this difference was located at parietal sites agrees with previous research, which reported the contribution of the parietal cortex during general arithmetical processing tasks [9,12].

In summary, the present study provides new evidence to confirm and extend on previous research about the psychophysiological basis of the problem size effect in arithmetical processing. Previous research with simple multiplications has reported evidence concerning a positive slow wave related to mental arithmetic; whose amplitude seems to be an index of the problem size effect. Two main conclusions can be drawn from the present study. First, the positive slow wave can be taken as a direct electrophysiological correlate of the calculation itself. Second, the amplitude modulation as an index of the problem size effect extends to additions and, to some degree at least, to subtractions.

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