We investigated the event-related brain potentials elicited by arithmetical operations whose solution requires direct memory retrieval or non-retrieval strategies. The problem size effect—the increment in reaction time for arithmetical problems with large operands—appears to be due to the selective use of non-retrieval procedures, and studies with event-related potentials have demonstrated an amplitude modulation of a late positive slow wave (range between 400–800 ms) related to the problem size effect. Two arithmetical operations (additions and subtractions) and three levels of problem size (adding or subtracting 2, 4 or 6) were used. We found an amplitude modulation of the late positive slow wave in subtractions, where non-retrieval procedures are mainly used. This amplitude modulation was not evident in additions, where direct retrieval strategies are believed to be used. Our results suggest that the problem size effect is related to non-retrieval procedures of calculation. NeuroReport 17:357–360 © 2006 Lippincott Williams & Wilkins.

Keywords: addition, calculation, event-related potential, mental arithmetic, subtraction

Introduction
The problem size effect is a well-established phenomenon in mental arithmetic research. It refers to the observation that reaction time increases and accuracy decreases when arithmetical problems are presented with large operands. For example, operations such as 9 + 6 are solved more slowly and less accurately than operations such as 3 + 4. This phenomenon has been reported for simple addition, subtraction, multiplication and division, and in both production and verification tasks (for a review, see [1,2]). The problem size effect is attributed to differences in the accessibility of results: smaller problems are more frequently encountered in educational and natural settings than larger ones, and are therefore solved more efficiently.

Several explanations have been proposed to account for the problem size effect [3], but recent research suggests that it may be attributed to differences in the strategy implemented to solve the arithmetical problems. LeFevre et al. [4] proposed that the use of non-retrieval procedures is one factor that contributes to the problem size effect. In their experiment, participants were instructed to solve problems of addition and to describe how they had solved them. The authors found that small problems were mainly solved by direct retrieval procedures, whereas larger problems were mainly solved by non-retrieval procedures (some examples of non-retrieval procedures that can be used in additions are counting—e.g., the problem 7 + 3 can be solved by counting from 7 to 10: 7, 8, 9, 10—or transformation—e.g., 6 + 5 equals 6 + 4 = 10 plus 1 makes 11). These authors also found that the problem size effect was evident when participants reported using non-retrieval procedures, but was reduced when direct retrieval procedures were used. These authors concluded that direct retrieval procedures are fast, accurate and obligatory, and require minimal cognitive load, so that the problem size effect does not appear. In contrast, non-retrieval procedures are thought to be slower and less accurate, and are susceptible to the problem size effect.

This explanation, based on the type of problem-solving strategy used, accounts for the developmental changes observed in the problem size effect. The effect is greater in children than in adults [5], and it is well known that children mainly use non-retrieval procedures whereas adults mainly use retrieval procedures [4,6].

Event-related potential (ERP) studies of brain activity have reported a late positive slow wave, which starts at about 400 ms after stimuli, functionally related to mental arithmetical calculation [7,8]. Other studies have shown evidence of a modulation of the positive slow wave amplitude associated with the problem size effect in additions and subtractions [9] and in multiplications [10,11]: the amplitude increases linearly as the operand size is increased. As a consequence, it has been suggested that the amplitude of this late positive wave constitutes a brain signature of the problem size effect.
The purpose of the present study was to test whether the type of procedure that participants select to solve an arithmetic problem – direct retrieval or non-retrieval – affects the amplitude modulation of this late positive wave. According to LeFevre et al. [4], the problem size effect will be present if non-retrieval strategies are used, but will be reduced, or absent, if direct retrieval is used. In the present experiment, sequences of five even numbers were presented and the pattern of ERPs elicited by the fourth number was analyzed. Sequences consisted of consecutive additions or subtractions of 2, 4 or 6. Thus, three increments or reductions were used to manipulate the problem size effect. Addition and subtraction were selected to manipulate the strategy of processing in mental arithmetic. For addition, sequences of even numbers were chosen because these are relatively well practiced and automatic operations that are thought to rely mainly on a relatively rapid retrieval from long-term memory. Subtraction was chosen because it is generally agreed that adults’ mental subtraction seems to rely more heavily on non-retrieval procedures [12–14].

We hypothesized an amplitude modulation of the late positive slow wave for subtractions, dependent on the size of the reduction, but a uniform ERP pattern for additions. We expected participants to rely mainly on non-retrieval strategies for problem solution in subtractions and on simpler retrieval from long-term memory in additions.

**Materials and methods**

**Study participants**

Sixteen healthy volunteers with no history of neurological or psychiatric disorder were recruited (10 women; mean age 25.06 years, range 20–38 years; 15 right-handed). All were university students and had normal or corrected-to-normal visual acuity. Participants were informed of the details of the experiment and gave written informed consent to participate.

**Stimuli and procedure**

The stimuli were sequences of five Arabic numerals, which were constructed in the following way: even numbers were selected to begin each sequence, and the same constant quantity (2, 4 or 6) was consecutively added or subtracted. The fifth number presented completed the sequence either correctly or incorrectly.

Sequence presentation was controlled by the STIM 2.0 software (NeuroScan Inc., Herndon, Virginia, USA). Numbers were presented in white against a black background, and subtended a visual angle of 1.76° vertically and 1.10° (for one digit stimuli) or 2.42° (for two digits stimuli) horizontally.

Participants were seated in an electrically shielded, sound-attenuating room at a distance of 130 cm from the display screen, whose center was at eye level. In order to familiarize the participants with the procedure, a series of practice trials (similar to those used in the recording session) were administered before the recording period. The training period finished when one of the following learning criteria was reached: (1) the participant correctly answered the first 10 consecutive trials or (2) the participant correctly answered 90% of trials. When the training period was over, the recording period started. Each trial consisted of a sequence of five Arabic numerals that were presented successively on the screen. The numbers remained on the screen for 1000 ms and, after the presentation of the last number of the sequence, an asterisk was shown for 500 ms. The interstimuli interval was 1500 ms. Participants were asked to judge whether the last number in the sequence was correct or incorrect by pressing one of two response buttons (the responding fingers were counterbalanced across participants). They were required to wait until the asterisk cue, which informed the participants that the sequence had finished, before responding. Participants were advised to blink during the presence of the asterisks or the rest messages in order to reduce the probability of eye movements in the critical epochs. A message indicating a 30-s rest period appeared on the screen after 12 trials and a 5-min rest break was allowed halfway through the experimental trials. In contrast with the training session, during the recording session no feedback was given in the case of incorrect responses.

All participants were tested on 432 trials, 72 for each experimental condition. Twelve blocks of 36 trials were presented to every participant, and the type of trials and their order of appearance were controlled within each block. A block included six trials of each type presented pseudorandomly, so that no more than three increasing sequences or three decreasing sequences could appear consecutively.

**Recording and data analysis**

An electroencephalogram (EEG) was recorded with the SynAmps/SCAN 3.0 hardware and software (NeuroScan Inc.) from 31 tin electrodes mounted in a commercial electrocap (Electro-Cap International, Eaton, Ohio, USA). Nineteen electrodes were positioned according to the 10–20 International System: three electrodes were placed over midline sites at Fz, Cz and Pz locations, along with eight lateral pairs of electrodes over standard sites on frontal (FP1/FP2, F7/F8, F3/F4), central (C3/C4), temporal (T3/T4, T5/T6), parietal (P3/P4) and occipital (O1/O2) positions. Two electrodes were placed at Fpz and Oz, and 10 electrodes were placed halfway between the following additional locations: frontocentral (FC1/FC2), frontotemporal (FT3/FT4), centroparietal (CP1/CP2), temporoparietal (TP3/TP4) and mastoids (M1/M2). The common reference electrode for EEG measurements was placed on the tip of the nose. EEG and electrooculogram (EOG) channels were continuously digitized at a rate of 500 Hz by a SynAmp amplifier (5083 model, NeuroScan Inc.). A band-pass filter was set from 0.05 to 30 Hz, and electrode impedance was always kept below 5 kΩ. For monitoring eye movement and blinks, FP1, FP2, FPz (for the vertical EOG) and an electrode placed at the external canthus of the right eye (for the horizontal EOG) were used.

The percentage of correct responses for additions and subtractions was analyzed with a Friedman test, taking the variable increment (2, 4 and 6) as the within-subjects factor. Then, whenever the Friedman test was significant, Wilcoxon T-tests were used to perform paired contrasts. Reaction times were not recorded in this experiment because participants were asked to respond after an asterisk mark appeared on the screen in order to avoid a contamination of the ERP by response-related activity.

Analysis of the electrophysiological response was carried out on the fourth number of the sequence (this number was selected because at this point of the sequence participants...
were assumed to be implicitly calculating the arithmetical operation that would allow them to follow the sequence. Firstly, epochs for every participant in each experimental condition were averaged relative to a prestimulus baseline that was made up of the 100 ms of activity preceding the epoch of interest. Secondly, trials with artifacts (voltage exceeding $\pm 750\mu$V in any channel) and those with response errors were excluded from the ERP average. The mean number of epochs included in each ERP average varied between 51.5 and 53.8 for the various types of stimuli used. Finally, ERPs were quantified as mean amplitude measures in the 500–800 ms latency window following the onset of the fourth number of the sequence, which was the stimulus of interest.

A $2 \times 3 \times 3 \times 5$ repeated-measures analysis of variance (ANOVA) was performed on the ERP amplitude at 15 electrodes (F7, F3, Fz, F4, F8, T3, C3, Cz, C4, T4, T5, P3, Pz, P4 and T6). The factors taken were operation type (addition or subtraction), increment (2, 4 or 6), frontality (frontal, central and parietal) and laterality (five levels from left to right). The Greenhouse–Geisser correction for sphericity departures [15] was applied when appropriate. The F value, the uncorrected degrees of freedom, the probability level following correction and the $\epsilon$ value are reported. Tests of simple effects were calculated in the presence of a significant interaction (differences were considered significant at $P<0.05$), and planned comparisons were conducted to test the effect of the variable increment. Topographic maps were plotted using the EEProbe 3.1 program (ANT Software BV, Enschede, The Netherlands).

### Results

The correct rate in additions showed significant differences between adding 2, 4 and 6 ($P<0.004$). Participants made significantly more hits when they added 2 than when they added 4 ($P<0.017$) or 6 ($P<0.002$). Their performance was not significantly different when they added 4 and 6. Means and standard errors (in parenthesis) for correct rate in additions were 92.8 (8.1), 88.8 (10.2) and 88 (10) for adding 2, 4 and 6, respectively. A similar pattern of results was found for the correct rate in subtractions. The overall analysis showed differences between the three increments ($P<0.002$) and participants performed better when they subtracted 2 than when they subtracted 4 ($P<0.002$) or 6 ($P<0.003$). Again, there were no differences between subtracting 4 and 6 ($P>0.05$). Means and standard errors (in parenthesis) for correct rate were 92.8 (8.1), 87.9 (9.7) and 85.9 (12) for subtracting 2, 4 and 6, respectively.

Grand-average ERPs for the fourth number in the addition and subtraction sequences at Fz, Cz and Pz are shown in Fig. 1a. We see an amplitude modulation of the late positive slow wave for subtractions – though there is no difference between subtracting 4 and 6 – and a uniform ERP pattern for additions. Scalp topography maps in Fig. 1b show that the amplitude modulation of the positive slow wave in subtractions is largest at centroparietal sites.

The repeated-measures ANOVA performed on the 500–800 ms window supports these observations. The interaction operation $\times$ increment [$F(2,30)=3.57$, $P=0.04$, $\epsilon=0.77$] confirmed that the effect of the increment depends on the arithmetical operations. To analyze this effect,
separate ANOVAs were carried out for additions and for subtractions. Results showed that the increment × frontality effect was statistically significant for subtractions \(F(4,60)=4.77, P=0.01, \eta^2=0.54\) but addition displayed no significant increment effect \((P<0.05)\). A more detailed analysis was then performed for subtraction. First, tests of simple effects showed that the increment effect was only present at parietal sites \(F(2,30)=4.48, P=0.02, \eta^2=0.85\). Second, planned comparisons revealed that the slow positive effect was significantly more prominent for \(-4\) and \(-6\) than for \(-2\) \((P<0.05)\) at parietal sites. No difference in voltage between subtracting 4 and subtracting 6 was observed.

Discussion
The aim of the present study was to assess whether the type of strategy that people use to solve an arithmetical problem – direct retrieval or non-retrieval procedures – would affect the late positive slow wave, which has been related to mental arithmetical calculation [7–11]. Our experimental design makes it possible to separate direct retrieval procedures from non-retrieval procedures by presenting additions of even numbers (which are well practiced, require minimum cognitive load and are thus thought to rely mainly on retrieval strategies) and subtractions (which are considered to rely more heavily on reconstructive strategy-based processing [12–14]). We also manipulated the problem size effect by presenting increments or reductions of 2, 4 and 6. According to LeFevre et al. [4], the problem size effect appears to be due to the selective use of non-retrieval procedures, and recent studies with ERPs have demonstrated an amplitude modulation of the late positive slow wave related to the problem size effect [9–11]. In summary, we expected to find an amplitude modulation of the late positive slow wave in subtractions but not in additions.

As expected, no amplitude modulation of the positive slow wave was found for additions in response to the problem size, but a modulation was found for subtractions, where it is generally agreed that non-retrieval procedures are mainly used [12–14]. These results are in agreement with those reported by Jost et al. [16]. They studied the effects of problem size on ERPs and concluded that the problem size effect is not only due to differences in the activation of the correct result but rather it is caused by the use of different solution strategies. Our results are also in agreement with previous studies with reaction time [4] in which the problem size effect has been related to the use of non-retrieval calculation procedures. Although the positive wave was larger when people were asked to subtract 4 and 6 than when they were asked to subtract 2, contrary to expectations, no difference in voltage was found between subtracting 4 and subtracting 6. This unexpected result may be due to the fact that subtracting 4 and 6 are operations that present similar degrees of difficulty. This explanation is supported by the hit rate data that showed that participants performed better when they subtracted 2 than when they subtracted 4 or 6. Moreover, in previous ERP research [9], we reported a similar result: with the operations –2, –3 and –4, differences in voltage and hit rate were only found between –2 and –3, and –2 and –4. In that study, we suggested that –3 and –4 might present similar difficulty.

Conclusion
Our findings are consistent with previous research that has suggested that the problem size effect depends on strategy choice. As expected, an amplitude modulation of the positive slow wave was found in subtractions, where non-retrieval procedures are assumed to be mainly used, but no modulation was found in additions, where retrieval procedures are mainly used. This suggests that in a production task people might be able to solve simple addition with no need to pass through the magnitude representation of the quantity, whereas to solve subtractions people would need it. This conclusion supports Dehaene and Cohen’s triple-code model [17]. Moreover, the positive slow wave reported in the present experiment has a parietal topography; it has been suggested elsewhere that the parietal lobe contributes to the representation of numerical quantity [17].

References