Stochastic PDEs and their approximations

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Introduction

Situation of interest: "Crossover" between two distinct scaling regimes.

Examples:

1. Interface motion in 2D (parameter: stability difference, e.g. external magnetic field / temperature)
2. Phase coexistence (crossover between Ising-type behaviour and free-field type behaviour)
3. Diffusing particle killed by environment (parameter: strength of absorption)

Heuristic equations describing the dynamics: simple looking "normal form" nonlinear Stochastic PDEs.

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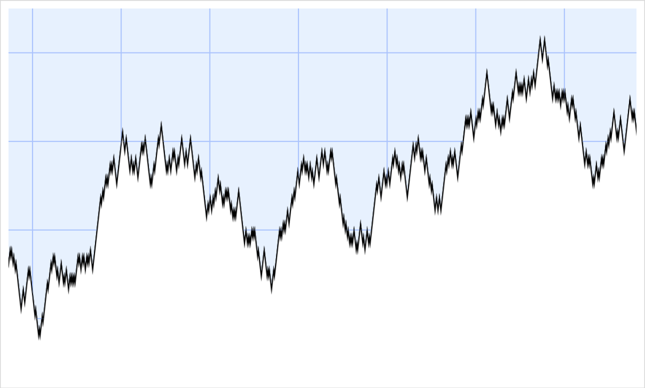
Situati  ng
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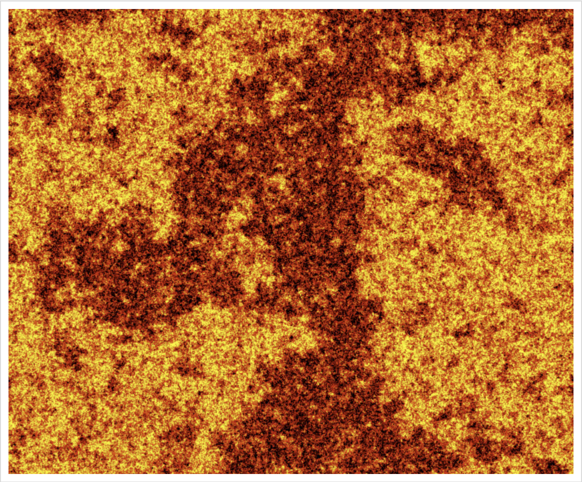

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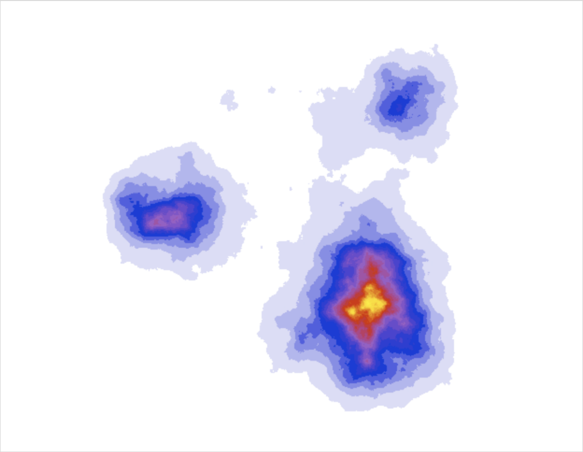

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Interesting "normal form" equations

Previous examples give rise to the following equations:

$$\partial_t h = \partial_x^2 h + (\partial_x h)^2 + \xi - C\ , \qquad \text{(KPZ; } d = 1)$$

$$\partial_t \Phi = -\Delta\big(\Delta\Phi + C\Phi - \Phi^3\big) + \nabla\xi\ . \qquad (\Phi^4;\ d = 2,3)$$

$$\partial_t u = \Delta u + u\,\eta + Cu\ , \qquad \text{(cPAM; } d = 2,3)$$

Here ξ is space-time white noise (think of i.i.d. Gaussians at every space-time point) and η is spatial white noise.

KPZ: universal model for weakly asymmetric interface growth.

Φ^4: universal model for phase coexistence near criticality.

cPAM: universal model for weakly killed diffusions.

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Well-posedness problem

Problem: Products are ill-posed:

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In general $(f, \xi) \mapsto f \cdot \xi$ well-posed on $\mathcal{C}^\alpha \times \mathcal{C}^\beta$ if and only if $\alpha + \beta > 0$.

One has $\xi \in \mathcal{C}^{-\frac{d}{2}-1-\kappa}$ and $\eta \in \mathcal{C}^{-\frac{d}{2}-\kappa}$ for every $\kappa > 0$.

Expectation: $h \in \mathcal{C}^{\frac{1}{2}-\kappa}$, $\Phi \in \mathcal{C}^{-\kappa}/\mathcal{C}^{-\frac{1}{2}-\kappa}$, and $u \in \mathcal{C}^{1-\kappa}/\mathcal{C}^{\frac{1}{2}-\kappa}$.
Consequence: Needs to take $C = \infty$.

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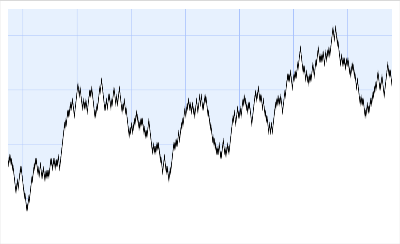

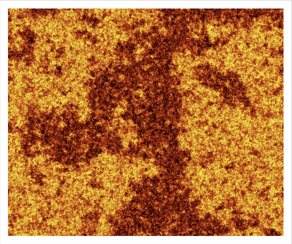

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Well-posedness results

Write ξ_ε for mollified version of space-time white noise. Consider

$$\partial_t h = \partial_x^2 h + (\partial_x h)^2 - C_\varepsilon + \xi_\varepsilon\ , \qquad (d=1)$$

$$\partial_t \Phi = -\Delta\big(\Delta\Phi + C_\varepsilon \Phi - \Phi^3\big) + \nabla\xi_\varepsilon\ , \qquad (d=2,3)$$

(Periodic boundary conditions on torus / circle.)

Theorem (H. '13): There are choices $C_\varepsilon \to \infty$ so that solutions converge to a one-parameter family of limits independent of the choice of mollifier. (The constants do depend on that choice.)

Theorem (H. & Matetski '15, Zhu & Zhu '15, Gubinelli & Perkowski '16, Matetski & Cannizzaro '16, H. & Erhard '17): Various approximation schemes converge to same families of limits.

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General result

Joint with Y. Bruned, A. Chandra, I. Chevyrev, L. Zambotti.

Consider a system of semilinear stochastic PDEs of the form

$$\partial_t u_i = \mathcal{L}_i u_i + G_i(u, \nabla u, \ldots) + F_{ij}(u)\xi_j \;, \qquad (\star)$$

with elliptic $\mathcal{L}_i$ and stationary random (generalised) fields ξ_j that are scale invariant with exponents for which $(\star)$ is subcritical.

Then, there exists a canonical family $\Phi_g\colon (u_0, \xi) \mapsto u$ of "solutions" parametrised by $g \in \mathfrak{R}$, a finite-dimensional nilpotent Lie group built from $(\star)$. Furthermore, the maps Φ_g are continuous in both of their arguments (in law for ξ).

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Toy Example

Recall: function $\eta \Rightarrow$ distribution $\varphi \mapsto \int \eta(x)\varphi(x)\,dx$.
Try to define distribution "$\eta(x) = \frac{a}{|x|} - c\delta(x)$" for $a, c \in \mathbf{R}$.

Problem: Integral of $1/|x|$ diverges, so we would like to to set "$c = \infty$" to compensate!

Sequence of approximations of type $\frac{a}{|x|+\varepsilon} - c_\varepsilon \delta(x)$ that converge:

$$\eta_\chi^\varepsilon(\varphi) = a \int_{\mathbf{R}} \frac{\varphi(x) - \chi(x)\varphi(0)}{|x| + \varepsilon}\,dx - c\varphi(0)\;,$$

for any smooth compactly supported cut-off χ with $\chi(0) = 1$.
Yields canonical two-parameter family $(c, a) \mapsto \eta_{a,c}$ of models as $\varepsilon \to 0$, but no canonical "choice of origin" for c.

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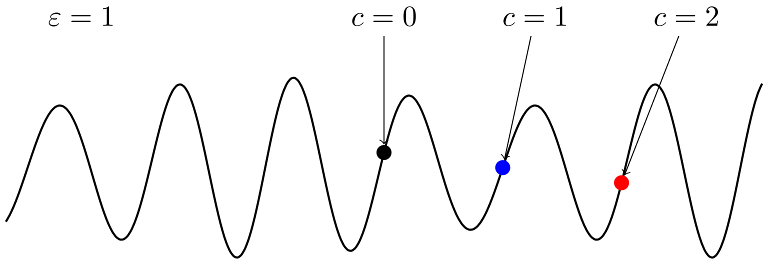
Cartoon

Example depicting a possible behaviour of a family of solutions parametrised by one single parameter c.



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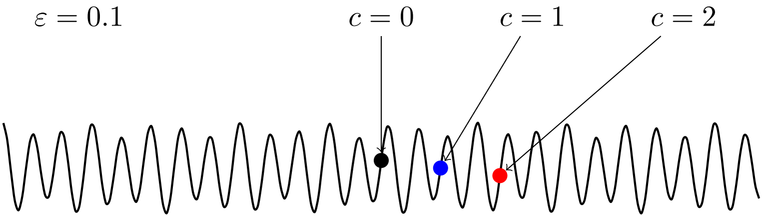

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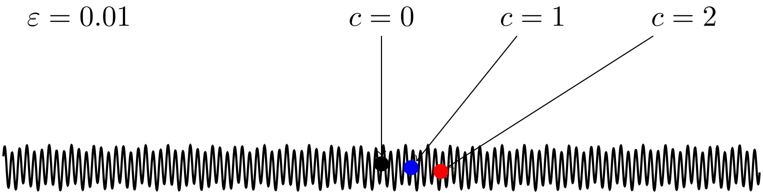

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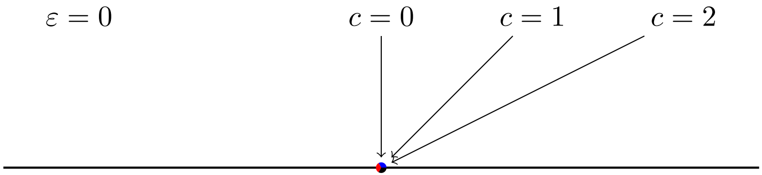

Interesting fact (universality)

Rather counterintuitively, the more singular the limit becomes, the more stable it is! (As a family of possible limits.)

Example: for "nice enough" even $F \colon \mathbf{R} \to \mathbf{R}$, consider

$$\partial_t h = \partial_x^2 h + \varepsilon^{-1} F(\sqrt{\varepsilon}\partial_x h) + \xi_\varepsilon - C_\varepsilon \;.$$

Same symmetries and scaling as the KPZ equation (since $|\partial_x h| \approx \mathcal{O}(\varepsilon^{-1/2})$), but quite a different equation.

Theorem (H., Quastel '15, H., Xu '17): As $\varepsilon \to 0$, there is a choice of C_ε such that h converges to solutions to (KPZ).

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Construction of solutions

Problem: Solutions are not smooth.

Insight: What is "smoothness"? Proximity to polynomials; we know how to multiply these...

Idea: Replace polynomials by a (finite / countable) collection of tailor-made space-time functions / distributions with similar algebraic / analytic properties. Depends on the realisation of the noise and class of equations, but not on "details" of the equation. (Values of constants, initial condition, boundary conditions, etc.)

Amazing fact: If we choose the objects replacing polynomials in a smart way, these very singular solutions are "smooth"!

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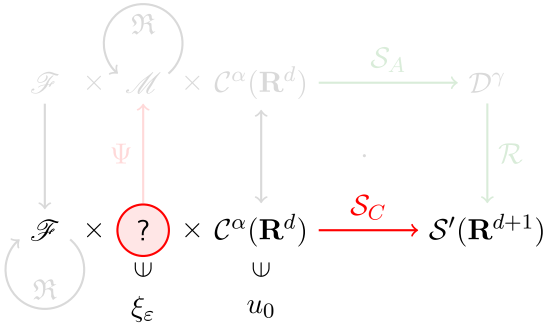
General picture

Method of proof: Build objects for the following diagram:



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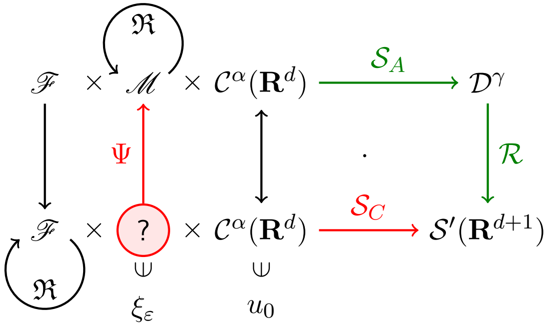

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$\mathcal{S}_A$: Abstract fixed point: locally jointly continuous!

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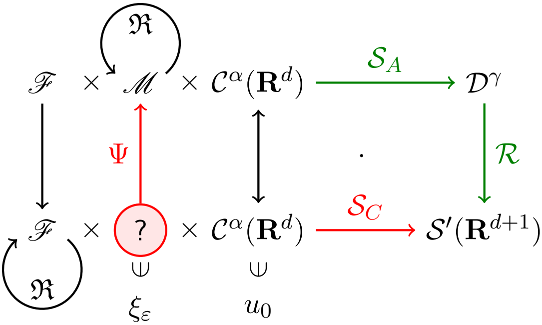

Strategy: find $M_\varepsilon \in \mathfrak{R}$ depending on the law of ξ_ε in such a way that $\xi_\varepsilon \mapsto M_\varepsilon \Psi(\xi_\varepsilon)$ becomes continuous (in law) for a suitable topology.

Discrete approximations

Want some framework allowing to analyse various discretisations of these SPDEs: fully discrete, semi-discrete, various types of grids, various types of noises, etc.

Remark: Variants on "Stability + Consistency $\Rightarrow$ Convergence" don't apply: not clear what either even means in this case...

Important: Adaptive grids seem counterproductive: breaks stationarity of "Taylor monomials" which is essential for renormalisation procedure to be canonical. Also, solutions tend to be "equally bad" everywhere.

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Joint work with D. Erhard.

1. Construct discretisation dependent “black box” encoding the properties of the discretisation at small scales. (Space of distributions + collection of seminorms.)
2. Build a discrete version of the “Taylor monomials” used to model the solution and show that they converge to their continuous counterparts.
3. Combine both ingredients to show convergence of the discrete approximation. This part is discretisation independent. Adjust reg. struc. to use black box to control objects at small scale.

Step 2. requires a suitable discretisation-dependent choice of renormalisation constants.

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2. Build a discrete version of the "Taylor monomials" used to model the solution and show that they converge to their continuous counterparts.
3. Combine both ingredients to show convergence of the discrete approximation. This part is discretisation independent. Adjust reg. struc. to use black box to control objects at small scale.

Step 2. requires a suitable discretisation-dependent choice of renormalisation constants.

Philosophy

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And finally...

Thank you for your attention!