

---

# “Scaling Down Downside Risk with Inter-Quantile Semivariances”

Uribe J

---



Institut de Recerca en Economia Aplicada Regional i Pública  
*Research Institute of Applied Economics*

**Universitat de Barcelona**

Av. Diagonal, 690 • 08034 Barcelona

---

WEBSITE: [www.ub.edu/irea/](http://www.ub.edu/irea/) • CONTACT: [irea@ub.edu](mailto:irea@ub.edu)

---

The Research Institute of Applied Economics (IREA) in Barcelona was founded in 2005, as a research institute in applied economics. Three consolidated research groups make up the institute: AQR, RISK and GiM, and a large number of members are involved in the Institute. IREA focuses on four priority lines of investigation: (i) the quantitative study of regional and urban economic activity and analysis of regional and local economic policies, (ii) study of public economic activity in markets, particularly in the fields of empirical evaluation of privatization, the regulation and competition in the markets of public services using state of industrial economy, (iii) risk analysis in finance and insurance, and (iv) the development of micro and macro econometrics applied for the analysis of economic activity, particularly for quantitative evaluation of public policies.

IREA Working Papers often represent preliminary work and are circulated to encourage discussion. Citation of such a paper should account for its provisional character. For that reason, IREA Working Papers may not be reproduced or distributed without the written consent of the author. A revised version may be available directly from the author.

Any opinions expressed here are those of the author(s) and not those of IREA. Research published in this series may include views on policy, but the institute itself takes no institutional policy positions.

## *Abstract*

---

We propose a risk-management strategy for portfolio allocation based on volatility scaling. The strategy involves decomposing realized volatility according to the magnitude and sign of a given return and, then, using part of the realized variance to design volatility-scaled versions of traditional portfolios. By applying our method to four risk-portfolios (namely, market, small minus big, high minus low, and winners minus losers), we show that scaling according to an appropriate criterion (i.e. the realized volatility of the largest negative returns) increases the profitability of the original strategies, while it simultaneously reduces other risks related to market crashes. The better economic performance of our method – the inter-quantile semivariance model – lies in its better adjustment to the market liquidity of our statistics, and more accurate modeling of the risk-return relationship and of the asymmetric impacts on consumption, production and asset prices, generated by a different fragment of the market realized variance.

*JEL classification:* G11, G12, C21, C58.

*Keywords:* asset pricing, risk decomposition, realized volatility, semivariances, volatility scaling, volatility forecasting, liquidity shocks.

Jorge M. Uribe: Riskcenter, University of Barcelona, Spain, Department of Economics, Universidad del Valle, Colombia. E-mail: [jorge.uribe@ub.edu](mailto:jorge.uribe@ub.edu)

## 1. Introduction

Barroso and Santa-Clara (2015) propose managing the risk of the momentum strategy by scaling the long-short portfolio by its realized volatility and targeting a strategy with constant volatility. This strategy ensures a considerably better economic performance compared to that provided by the unscaled version (in fact, it almost doubles the Sharpe ratio) and reduces the probability of momentum crashes. Barroso and Santa-Clara's approach is particularly convenient because it relies only on past data and, thus, its use is feasible in real time (especially given that it does not suffer from look-ahead bias)<sup>1</sup>. Here, we propose a new volatility-scaling strategy in which we decompose monthly realized volatility according to the quantiles of the return distribution in each period, using only part of the realized volatility to scale the original portfolios. In this way, we are able to construct scaled portfolios that exhibit higher economic profitability and lower exposure to higher order risks (related to the third moment of the distribution of gains). We test our strategy using four traditional risk-factor portfolios: market, small minus big, high minus low, and momentum. The method is shown to deliver higher average return and Sharpe ratios than the earlier proposal and to reduce more significantly higher order risks related to the third moment of the distribution of gains and losses. Furthermore, we estimate the transaction costs of our method for the case of momentum and show them to be considerably lower than those associated with the volatility scaling method proposed by Barroso and Santa-Clara (2015), and, on occasions, they are even lower than the transaction costs of the original unscaled momentum.

We refer to our method as inter-quantile semivariances (IQS). IQS nests traditional realized variance (Andersen et al., 2010) and semivariance estimators that discriminate only between positive and negative returns (Barndorff-Nielsen et al., 2010). This generality allows us to explore which fragments of the return distribution provide better hedging against negative market fluctuations (while maximizing profitability). The attractiveness of IQS models lies in their better forecasting of future returns, and in the fact that, first, they

---

<sup>1</sup> Volatility scaling has been used by other authors, including Moskowitz et al. (2012), Baltas and Kosowski (2013) and Kim et al. (2016) for different purposes.

reflect more accurately illiquidity shocks to the market<sup>2</sup> and, second, they better capture the nonlinear dynamics of macroeconomic series after a market volatility shock is observed<sup>3</sup>.

To the best of our knowledge, this paper is the first to decompose the realized variance (RV) according to the underlying quantiles of the return distribution. By so doing, we identify the fragment of the distribution that should be employed to construct the scaled versions of the portfolios. Our method delivers higher Sharpe ratios when applied to risk-factor portfolios: c. 1.31-1.71 times higher than the original portfolios, and c. 0.93-1.96 times higher than the volatility-scaled portfolios that employed all the returns. Moreover, our strategy consistently increases the skewness of the portfolios (i.e. shifting it from a negative to positive direction), compared to Barroso and Santa-Clara's (2015) RV-scaled alternative. This is important because negative skewness is related to market crashes, which is especially relevant in the case of momentum portfolios (Daniel and Moskowitz, 2016). A raw winners-minus-losers momentum strategy exhibits a skewness of -2.34, while the RV-scaled version of this strategy shifts it up to -0.49. Our strategy goes further, as it produces a scaled portfolio with a skewness of virtually 0, while, simultaneously, preserving the better Sharpe ratio attained by the scaling-strategy almost unaltered. Our strategy presents the additional advantage of reducing the transaction costs of rebalancing compared to the RV scaling strategy. That is, while the average monthly turnover of Barroso and Santa-Clara's (2015) strategy is 78.9% for our sample period, our method generates a monthly average turnover of 65.7%.

A second contribution of our study can be assigned to that nascent strand of the literature which seeks to analyze the distinctive impacts of 'good' and 'bad' volatility shocks on asset prices and the macroeconomy (Patton and Sheppard, 2015; Segal et al., 2015; Amaya et al., 2015; Bollerslev et al., 2017; Bollerslev et al., 2018). In general, these studies document how the asymmetric effects of such shocks are priced by the market, both in the cross-

---

<sup>2</sup> The ways in which liquidity is associated with volatility and, thus, priced by the market, have been extensively documented. See, for example, Stoll (2000), Pastor and Stambaugh (2003), Corwin and Schultz (2012), Bao and Pan (2013), Chung and Chuwonganant (2014), Abdi and Rinaldo (2017), among others.

<sup>3</sup> Nonlinear effects of this kind have been extensively analysed in, for example, the context of flight-to-safety scenarios between bonds and stocks (Vayanos, 2004; Adrian et al., 2016); flight-to-quality that arises from a complex relationship between funding and market liquidity (Brunnermeier and Pedersen, 2009); and endogenous amplification mechanisms of volatility shocks under financial distress in the macroeconomy (Brunnermeier and Sannikov, 2014).

section and the time-series of stock returns. Unlike these earlier works, here we focus on the asymmetries that exist in the macroeconomic reactions to good and bad volatility shocks from the stock market. We are able to do so using our IQS models. In turn, this helps improve the performance of the IQS models used in the scaling procedure.

Our main results highlight that using only part of the observed portfolio returns to construct the semivariances employed in the scaling process provides better hedging against market fluctuations, and leads to a better economic performance. Specifically, we show that when we use only the largest negative (returns) shocks (below the 33rd percentile) to construct the scaling series, we are able to reduce the probability of crashes and, at the same time, to attain higher Sharpe ratios than those achieved with extant alternatives. This is equivalent to our targeting directly the downside risk of the portfolio when we rebalance it according to the weights implied by the selected IQS. These results are in line with the main findings reported, for example, by Schneider (2016), where after decomposing market returns into factors related to the realized variance, realized loss aversion and higher order risk aversion, the greatest impact on equity premia originates from the factor associated with loss aversion.

The rest of this paper is organized as follows. In section two, we introduce the IQS model, which is our main tool. In the third section, we describe our main data sources and present summary statistics for the portfolios analyzed. In the fourth section, we compare the performance of our methods with respect to their closest counterparts in the literature, both from static and time-varying perspectives. In section five, we outline our economic intuition regarding the kind of shocks and asymmetries that underlie the better adjustment of the IQS models to the data (in terms of liquidity, predictability of returns, consumption, production and asset prices). In section six, we conclude.

## **2. Inter-quantile semivariances - IQS**

By way of background to the methods proposed herein, consider the traditional RV estimator as presented, for instance, by Andersen et al. (2010). The RV estimator of the log asset prices  $Y$  can be expressed as follows:

$$RV = \sum_{j=1}^n \left( Y_{t_j} - Y_{t_{j-1}} \right)^2, \quad (1)$$

where  $0 = t_0 < t_1 < \dots < t_n = 1$  are the times at which prices are available. This has been shown to be a useful methodology for estimating and forecasting conditional variances for risk management and asset pricing<sup>4</sup>. Here we construct inter-quantile semivariances, estimated using only certain fragments of the returns distribution. That is, while the estimates of a traditional RV use all the observations available within a period  $[0,1]$ , to estimate an IQS model we first classify each return according to the empirical return quantiles within each period  $t$ , and then separate the observations into several categories as follows:

$$\begin{aligned}
IQS^1 &= \sum_{j=1}^{t_j \leq 1} (Y_{t_j} - Y_{t_{j-1}})^2 \mathbf{1}_{Y_{t_j} - Y_{t_{j-1}} \leq q_t^1} \\
IQS^2 &= \sum_{j=1}^{t_j \leq 1} (Y_{t_j} - Y_{t_{j-1}})^2 \mathbf{1}_{q_t^1 < Y_{t_j} - Y_{t_{j-1}} \leq q_t^2} \\
&\quad \vdots \\
IQS^S &= \sum_{j=1}^{t_j \leq 1} (Y_{t_j} - Y_{t_{j-1}})^2 \mathbf{1}_{q_t^{S-1} < Y_{t_j} - Y_{t_{j-1}} \leq q_t^S} \\
IQS^{S+1} &= \sum_{j=1}^{t_j \leq 1} (Y_{t_j} - Y_{t_{j-1}})^2 \mathbf{1}_{Y_{t_j} - Y_{t_{j-1}} > q_t^S}
\end{aligned} \tag{2}$$

where  $\mathbf{1}_y$  is an indicator function taking a value of 1 if argument  $y$  is true,  $q_t^s$  is a quantile of the distribution of returns in period  $t$ , and  $s = [1, 2, \dots, S]$  is the number of quantiles, such that  $S + 1$  is the number of categories that constitutes a model. For instance, if  $q_t^s$  is a *percentile*, then we end up with 100 IQS time-series (categories), as in this case  $s = [1, 2, \dots, 99]$ , and the case is labeled an IQS 100 model. If  $q_t^s$  is a *quartile*, then we end up with four IQS series, as here  $s = [1, 2, 3]$ , and the case is labeled an IQS 4 model.

In IQS models, each series measures that part of the RV that is *due to shocks of a certain magnitude and sign*. For example, in the case of an IQS 4 model,  $IQS^1$  corresponds to extreme negative returns, while  $IQS^{S+1=4}$  corresponds to extreme positive returns. On the other hand, both  $IQS^2$  and  $IQS^3$  measure medium-sized returns, i.e. just below and just above the median, respectively. One advantage of this procedure is that, in common with other semivariances, it sums up to the traditional RV statistic:

$$RV = IQS^1 + IQS^2 + \dots + IQS^S + IQS^{S+1} . \tag{3}$$

---

<sup>4</sup> See Liu et al. (2015) and references therein.

Notice as well that the traditional RV estimator can be seen as an IQS 1 model, that is, an inter-quantile semivariance model with just one category ( $S = 0$ ). This procedure is, likewise, closely related to other popular realized semivariance (RS) estimators, such as the one advanced by Barndorff-Nielsen et al. (2010). The RS estimator separates shocks using the sign of the returns rather than the quantiles. To understand how this estimator is related to our proposal consider an IQS 2 model given by the following two equations:

$$\begin{aligned} IQS^1 &= \sum_{j=1}^{t_j \leq 1} r_{t_j}^2 \mathbf{1}_{r_{t_j} \leq q_t^1} \\ IQS^2 &= \sum_{j=1}^{t_j \leq 1} r_{t_j}^2 \mathbf{1}_{r_{t_j} > q_t^1} \end{aligned} \quad (4)$$

where  $r_{t_j} = Y_{t_j} - Y_{t_{j-1}}$ . Naturally, in this case, the quantile of interest that separates the two categories is the median. If we denote the median of the log-returns within each time interval  $[0,1]$  as  $r_t^M \equiv q_t^1$  and we subtract it from the returns within each time interval we obtain:

$$\begin{aligned} \overline{IQS}^1 &= \sum_{j=1}^{t_j \leq 1} \bar{r}_{t_j}^2 \mathbf{1}_{\bar{r}_{t_j} \leq 0} \\ \overline{IQS}^2 &= \sum_{j=1}^{t_j \leq 1} \bar{r}_{t_j}^2 \mathbf{1}_{\bar{r}_{t_j} > 0} \end{aligned} \quad (5)$$

where  $\bar{r}_{t_j} = r_{t_j} - r_t^M$ . In this case, the IQS 2 model fitted to the normalized returns  $\bar{r}_{t_j}$  is equivalent to the RS estimator proposed by Barndorff-Nielsen et al. (2010), given by:

$$\begin{aligned} RS^- &= \sum_{j=1}^{t_j \leq 1} r_{t_j}^2 \mathbf{1}_{r_{t_j} \leq 0} \\ RS^+ &= \sum_{j=1}^{t_j \leq 1} r_{t_j}^2 \mathbf{1}_{r_{t_j} > 0} \end{aligned} \quad (6)$$

In empirical exercises, the median of daily financial returns is generally indistinguishable from zero, so that  $r_{t_j} \approx \bar{r}_{t_j}$ , and, therefore, RS can be seen as a special case of IQS (with two categories). In such cases, we have that  $IQS^1 \approx RS^-$  and  $IQS^2 \approx RS^+$ . This is convenient because, as Barndorff-Nielsen et al. (2010) point out, traditional RV models are silent about the asymmetric behavior of jumps, which is important for example when estimating downside or upside risks. Unlike RS models, our method relies on a more general risk decomposition than that allowed by discriminating between good and bad volatility measures. Moreover, it allows us to explore whether different magnitudes of the shocks contain information that can contribute to better risk management and pricing.

### 3. Data

Our calculations of RV and IQS models rely on Kenneth French's data library ([http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html)). We retrieved (value-weighted) winners minus losers (WML), small minus big (SMB), high minus low (HML) and market (RMRF) portfolios daily from March 1927 to December 2017. The calculations in section 5 employ the series of aggregate and innovation liquidity from August 1962 to December 2017 made publicly available on Robert Stambaugh's home page ([http://finance.wharton.upenn.edu/~stambaug/liq\\_data\\_1962\\_2017.txt](http://finance.wharton.upenn.edu/~stambaug/liq_data_1962_2017.txt)). The stock level data used to estimate the turnover of the momentum strategy come from Wharton's CRSP database and consist of the universe of NYSE, AMEX, and NASDAQ stocks, with share codes 10 or 11, from December 1925 to December 2017. The data used in the FAVAR calculations made in section 5.3 constitute monthly observations of 128 macro-variables (from January 1959 to December 2017). These variables include indicators of output and income, labor market, housing, consumption, orders and inventories, money and credit, interest and exchange rates, prices and the stock market. They can be downloaded free of charge at <https://research.stlouisfed.org/econ/mccracken/fred-databases/monthly/2017-12.csv> and a full description of the data is available at [https://research.stlouisfed.org/econ/mccracken/fred-databases/Appendix\\_Tables\\_Update.pdf](https://research.stlouisfed.org/econ/mccracken/fred-databases/Appendix_Tables_Update.pdf). These data have been made publicly available and are fully described by McCracken and Ng (2016).

Table 1 presents summary statistics of the risk portfolios. As can be seen, there are significant variations across these portfolios in terms of average returns, standard deviation, skewness and kurtosis, which serves to guarantee a certain level of generality in the application of the methods proposed here.

**Table 1. Performance of four traditional portfolios:** The table shows value-weighted market portfolio (RMRF), small minus big (SMB), high minus low (HML) and winners minus losers (WML, 9th minus 1st deciles sorted according to prior winners) monthly statistics. All statistics are computed with monthly returns. The table shows the maximum, the minimum one-month return, the mean average excess return (annualized), the (annualized) standard deviation, excess kurtosis, skewness, and (annualized) Sharpe ratio. The sample runs from 1926:01 to 2017:12.

Portfolio	Maximum	Minimum	Mean	Standard deviation	Kurtosis	Skewness	Sharpe ratio
RMRF	38.85	-29.13	7.92	18.55	7.84	0.19	0.43
SMB	36.70	-17.28	2.58	11.11	19.46	1.94	0.23
HML	35.46	-13.28	4.60	12.13	19.03	2.17	0.38
WML	26.16	-77.02	14.09	27.02	17.46	<b>-2.34</b>	0.52

#### 4. Scaling down market crashes

In what follows we use our recently introduced tools to construct volatility-scaled (zero investment and self-financing) strategies and we compare the economic performance of IQS-scaled portfolios with the performances of their original unscaled versions and scaled portfolios that employ realized variances. To do so, we follow Barroso and Santa-Clara (2015) who propose momentum-trading strategies that minimize risk, using RVs. They construct a scaled version of the original WML portfolio, which uses a target level of volatility and estimates of the RV of momentum to reduce portfolio risk, while maintaining a high average return. They use the following formula:

$$r_{WML^*,t} = \frac{\sigma_{\text{target}}}{\hat{\sigma}_{t-1}} r_{WML,t}, \quad (7)$$

where  $r_{WML,t}$  is the unscaled momentum,  $r_{WML^*,t}$  is the scaled or risk-managed momentum,  $\hat{\sigma}_t$  is the RV of momentum and  $\sigma_{\text{target}}$  is a constant corresponding to the target level of volatility. Unlike these authors, we do not focus solely on momentum strategies (we also consider other portfolios) and we generalize the scaling device in the numerator in equation (7), taking into account the magnitude and sign of the observed returns by means of the IQS models. In other words, we replace  $r_{WML^*,t}$  with  $r_{i^*,t}$ , and  $\hat{\sigma}_{t-1}$  with  $\hat{\sigma}_{i,t-1}^j$ , where  $i =$

$\{Market, SMB, HML, WML\}$  and  $j = \{IQS^l 1, IQS^l 2, IQS^l 3\}$  and the superscript  $l$  stands for the lowest semivariance in each IQS model.

#### 4.1. *IQS-scaling and economic performance*

To construct our scaled portfolios we use an IQS category for a given month to scale the one-month-ahead portfolio returns. In this way, we guarantee that the scaling strategy is feasible in real time. We use the idiosyncratic IQS of each portfolio in the scaling process, following the extant literature, and also because the idiosyncratic volatilities convey distinctive information. This can be seen for instance when calculating the Pearson correlation across the low semivariances (below the median) of the four portfolios being analyzed. The highest correlation is between the market and SMB semivariance (equal to 0.79) and the lowest is between SMB and WML (equal to 0.35)<sup>5</sup>. Our main results are presented in Figure 1 and Table 3. These include the densities and economic performance of the unscaled raw portfolios, the IQS 1 (RV)-scaled portfolios, as in Barroso and Santa-Clara (2015), and the scaled portfolios using IQS 3.

Figure 1 shows the densities of the original portfolios (solid red line), together with the densities of the IQS 1-scaled portfolios (dotted blue line) and the densities of the IQS 3-scaled portfolios (solid black line). The differences between the three portfolio return distributions are notable. While the original and the RV-Scaled (IQS 1) portfolios exhibit a larger concentration of returns around the mean, indicating a higher ‘peakedness’ of the distributions<sup>6</sup>, the IQS 3 models reduce their mass around this fragment. Moreover, the IQS 3 models exhibit a higher probability mass in the right tail of the distributions than those shown by the original and the IQS 1-scaled portfolios (see Table 2 for further details). The IQS 3 strategy allows a larger variation of the returns, but it also shifts the distribution away from the losses tail more significantly than is the case with the IQS 1-scaling strategy. This allows it not only to reduce the probability of crashes but also to ensure larger economic gains on average. In general, IQS 3-scaling allows the investor to reduce the

---

<sup>5</sup> All the correlations are reported in Table A7 of the online Appendix.

<sup>6</sup> We differentiate between ‘peakedness’ and kurtosis, the latter referring to the tails of the distribution. There is a tendency to confuse the two concepts; however, here, we follow the clarification offered by Westfall (2014).

higher order risks associated with the return distributions, while maintaining the attractiveness of higher average returns and higher Sharpe ratios.

**Figure 1: Volatility-scaled portfolio densities.** In the figure the solid red lines correspond to the densities (kernels) of the daily distribution of gains for the market (RMRF), small minus big (SMB), high minus low (HML) and winners minus losers (WML) portfolios. The figure also shows two managed versions of each portfolio. The dotted blue line plots the returns of the portfolios scaled using the realized volatility (RV), which corresponds to IQS 1. The solid black line plots the returns scaled using the IQS 3 model. The plots are shown for the period 1927:03 -2017:12.

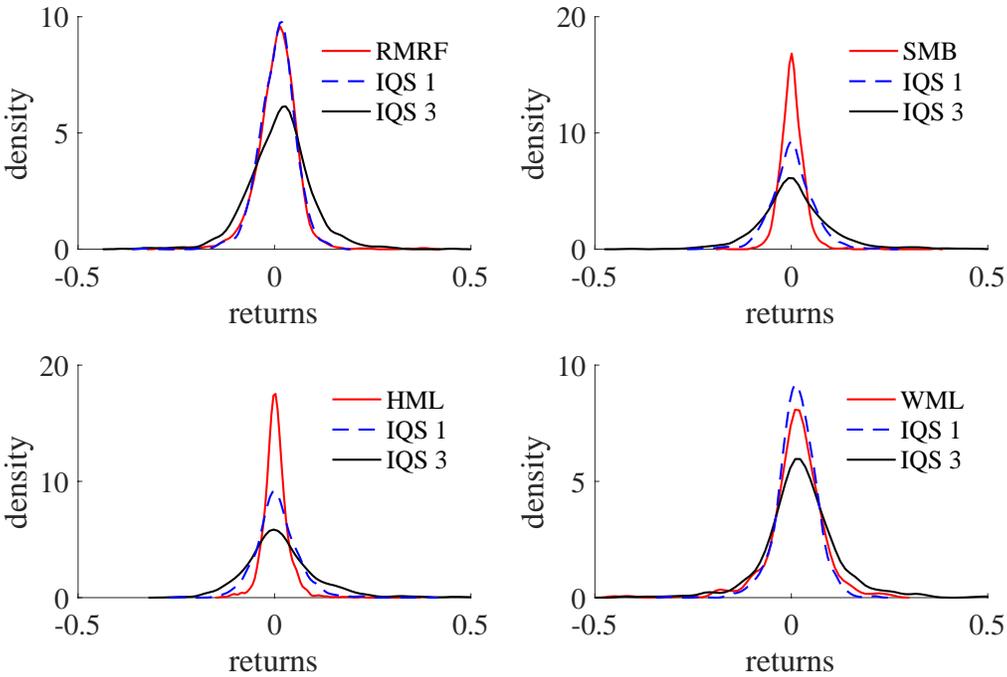


Table 2 shows the economic performance of the scaled and unscaled portfolios. The maximum, minimum, mean, standard deviation, kurtosis and skewness statistics for each portfolio within the sample period. In the last two columns we also present the Sharpe ratio of each portfolio and the information criteria, which contrast the performance of the original portfolios against that of their scaled counterparts, in each case. Note that for the RMRF, SMB, and HML portfolios the highest Sharpe ratios correspond to the IQS 3-scaled portfolios. Only in the case of the WML portfolios does the highest Sharpe ratio correspond to the IQS 2-scaled portfolio (although it is very similar to the IQS 3 ratio).

If we focus our attention on the WML portfolio, several outcomes illustrate the advantages of decomposing risk as proposed herein. In line with Barroso and Santa-Clara (2015), RV-scaling increases the Sharpe Ratio of the original WML portfolio substantially (from 0.52 to 0.96). This strategy also reduces skewness from -2.34 to -0.49. Yet, when we scale the original portfolio using just a fragment of the RV series, as in the IQS scaling strategies, the improvement in terms of skewness (reduction of momentum crashes) is even more notable. The IQS 3 strategy shifts skewness to 0.01. That is, the IQS strategies make gains just as likely as losses in the WML returns distribution, which is a considerable advantage over the original portfolio and the RV-scaled portfolio. This occurs because IQS 3 strategies give more weight to the variance associated with negative returns (the case of IQS 2) or with even larger negative returns (the case of IQS 3). Thus, they allow a greater variability of the distribution of gains, consistent with higher average returns, but they shift the entire distribution to the right, biasing the distribution towards the gains tail.

These improvements in terms of skewness are consistently associated with a considerable increment in the portfolios' average returns. The increment is 15.47 percentage points for the IQS 3-scaling strategy compared to that of the original WML portfolio and 13.13 percentage points compared to that of the IQS 1-scaled portfolio. The volatility of these IQS 3-scaled portfolios is also higher, but this volatility occurs in a more convenient fragment of the distribution, i.e. one that is more prone to gains than to losses.

**Table 2. Economic performance of the scaling strategies:** The table compares the moments of scaled and unscaled versions of traditional portfolios. Specifically, it shows the performance of four traditional portfolios: the value-weighted market (RMRF), small minus big (SMB), high minus low (HML) and winners minus losers (WML) portfolios. All statistics were computed with monthly returns. The table shows the maximum, the minimum one-month returns, the mean average excess return (annualized), the (annualized) standard deviation of each factor, excess kurtosis, skewness, (annualized) Sharpe ratios relative to the unscaled versions and the information criteria for each scaling function against the unscaled versions. In the case of RMRF, SMB and HML the shocks below the median were used to scale the original raw portfolios, while the 33rd percentile was used in the case of WML. The sample runs from 1927:02 to 2017:12. The target volatility was 12% annualized (although this has no direct consequences for our results).

Portfolio	Maximum	Minimum	Mean	Standard deviation	Kurtosis	Skewness	Sharpe ratio	Information criterion
<b>Market</b>								
Unscaled	38.85	-29.13	7.89	18.57	7.82	0.19	0.42	-
IQS 1-Scaled	16.06	-32.97	8.44	16.09	3.16	-0.61	0.52	0.18
IQS 2-Scaled	33.91	-38.10	15.54	26.62	2.40	-0.16	<b>0.58</b>	0.25
IQS 3-Scaled	45.30	-38.60	16.11	27.59	2.89	<b>-0.06</b>	<b>0.58</b>	0.24
<b>SMB</b>								
Unscaled	36.70	-17.28	2.61	11.13	19.41	1.94	0.23	-
IQS 1-Scaled	23.63	-22.93	2.93	18.77	1.87	0.07	0.16	-0.14
IQS 2-Scaled	64.11	-39.50	9.75	31.83	4.74	0.85	0.31	0.11
IQS 3-Scaled	72.55	-42.32	11.55	34.68	6.01	<b>1.10</b>	<b>0.33</b>	0.14
<b>HML</b>								
Unscaled	35.46	-13.28	4.55	12.13	19.10	2.17	0.37	-
IQS 1-Scaled	38.07	-23.19	7.87	18.59	3.35	0.64	0.42	0.08
IQS 2-Scaled	51.73	-26.33	18.21	31.37	4.29	1.24	<b>0.58</b>	0.32
IQS 3-Scaled	71.37	-26.73	19.28	33.07	6.06	<b>1.47</b>	<b>0.58</b>	0.31
<b>WML</b>								
Unscaled	26.16	-77.02	14.01	27.04	17.43	-2.34	0.52	-
IQS 1-Scaled	21.44	-31.10	16.35	17.11	2.85	-0.49	0.96	0.70
IQS 2-Scaled	46.88	-59.50	27.65	31.19	4.75	-0.33	<b>0.89</b>	0.51
IQS 3-Scaled	55.77	-60.17	29.48	33.53	5.63	<b>0.01</b>	0.88	0.48

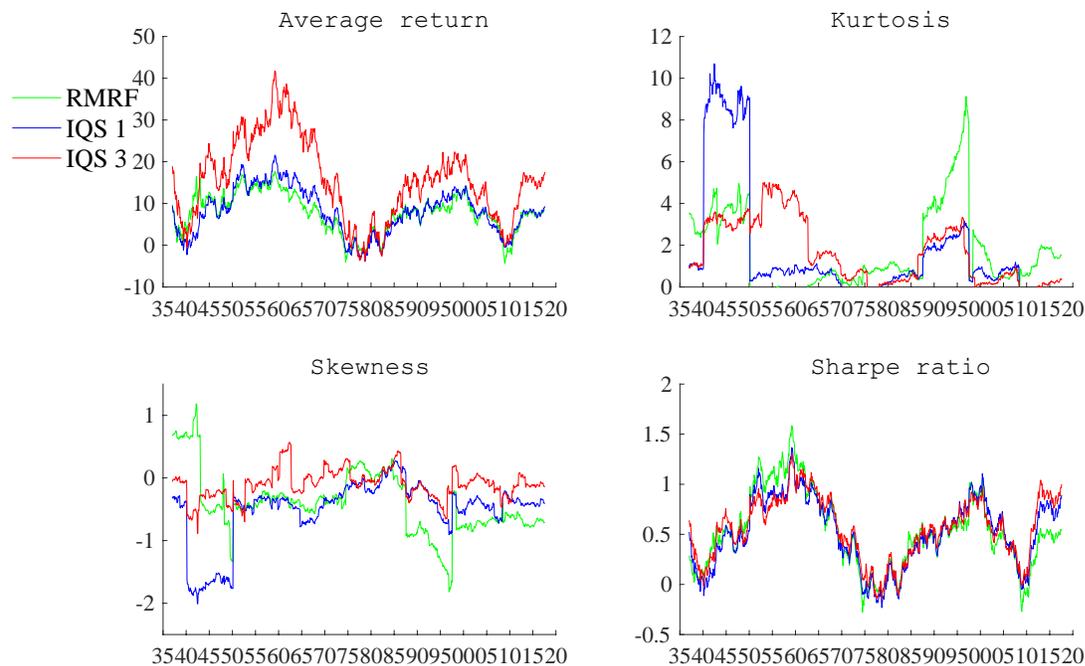
#### 4.2. Time-varying economic performance

In this section, we compare the time-varying performance of the IQS 3-scaling strategies with those of the RV-scaling and the plain portfolios. In Figure 2 we present the results for the market portfolio, while in Figures A2, A3 and A4 in section C of the Appendix we

present the results for the SMB, HML, and WML portfolios. The first plot in Figure 2 corresponds to the average return, the second (top right) to kurtosis, the third (bottom left) to skewness and the fourth (bottom right) to the Sharpe ratio. Economic performance is measured using ten-year rolling windows, so that each observation corresponds to a date on the horizontal axis and was constructed with information for this specific month and the previous 119 observations. The first date in the plot is April 1932 and the last is December 2017.

The first regularity in time and across different portfolios is that IQS 3-scaled portfolios have, in general, a higher average return than the original and RV-scaled portfolios. This difference is as high as 20 percentage points (pp) in the case of the market portfolio from the late 50s to the late 70s.

**Figure 2. Time-varying performance of scaling strategies (market portfolio):** The figure shows the average return, kurtosis, skewness and Sharpe ratio of the unscaled market portfolio, IQS 1 (RV), and IQS 3 scaled portfolios from April 1932 to December 2017. Each point in the plot was constructed using a ten-year rolling window.



The gap between the average return produced by the IQS 3-scaling strategies and that of the raw portfolios had been reduced considerably by the end of the sample for all the portfolios. In the last ten years the gap was around 7 pp with respect to the market-based portfolio.

For the end of the sample, the Sharpe ratio of the IQS 3-scaling strategy for the market portfolio (0.999) almost doubles that of the plain portfolios (0.543) and it is higher than that of RV-scaled portfolios (0.892) as well. More importantly, these higher Sharpe ratios, which are a regular feature of the IQS-scaling strategies, are accompanied by lower kurtosis and positive skewness significantly shifted to the right compared to the plain market portfolio and the RV-scaled portfolio. Similar patterns are documented in Figures A2, A3 and A4 in the appendix for SMB, HML and WML portfolios.

#### 4.3. *Transaction costs*

One issue related with scaling methods is whether time-varying weights generate an increase in turnover that can potentially offsets the benefits of the strategy after transaction costs. This is a relevant concern for momentum portfolios that present more frequent and sharper rebalancing compared to other portfolios in our analysis. To consider this, we compute the turnover of momentum and the IQS-managed versions presented before from stock-level data on returns and firm size from December 1925 to December 2017. The turnover of a leg of the momentum portfolio is given by:

$$x_t = 0.5 \times \sum_i^{N_t} |w_{i,t} - \tilde{w}_{i,t-1}|, \quad (8)$$

where  $w_{i,t}$  is the weight of the stock  $i$  in in the leg of the portfolio at time  $t$ ,  $N_t$  is the number of stocks in the portfolio's leg at time  $t$ ,  $r_{i,t}$  is the return of asset  $i$  at time  $t$ , and  $\tilde{w}_{i,t-1}$  is the weight in the trading as follows:

$$\tilde{w}_{i,t-1} = \frac{w_{i,t-1}(1+r_{i,t})}{\sum_i^{N_t} w_{i,t-1}(1+r_{i,t})}. \quad (9)$$

The turnover of the WML strategy corresponds to the sum of the turnover of the short and the long legs. In the case of the risk-managed portfolio, we compute the turnover for each leg as follows:

$$x_t = 0.5 \times \sum_i^{N_t} \left| \frac{w_{i,t}}{L_t} - \frac{\tilde{w}_{i,t-1}}{L_{t-1}} \right|, \quad (10)$$

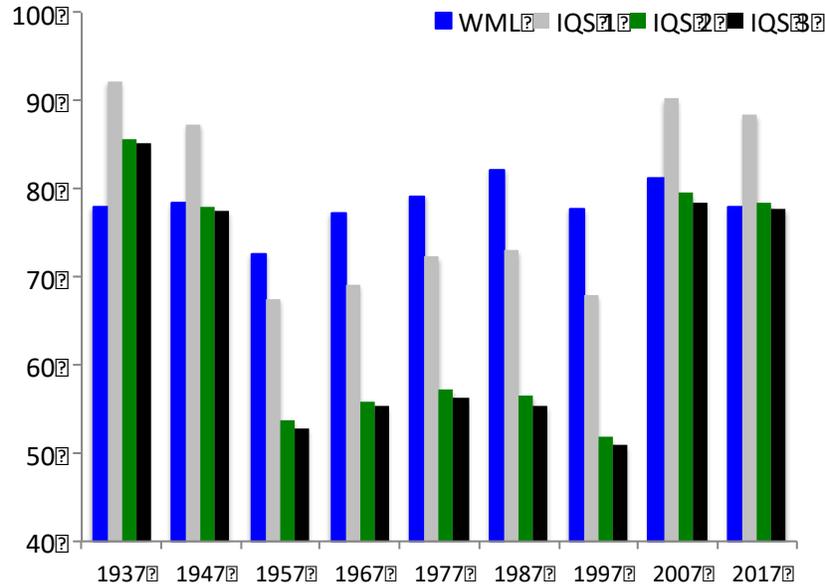
where  $L_t = \sigma_{target} / \hat{\sigma}_t$ .  $L_t$  is therefore constructed for each IQS model associated with the WML strategy. Whenever the turnover of a given month exceeds 1, we set it to 1. In this way, we obtain a similar turnover for momentum and RV-scaled momentum to that reported by Barroso and Santa-Clara (2015). That is, while their estimates of the turnover are 74 and 75% from Mar 1951 to Dec 2010, respectively, for momentum and RV-scaled momentum, our estimates for the same period are 78.6 and 73.8%, respectively, for these same two strategies.

Using the full sample, the turnover of momentum, IQS 1-, IQS 2-, and IQS 3- scaled momentum are 78.3, 78.9, 66.5 and 65.70%, respectively<sup>7</sup>. Figure 3 shows the evolution of the turnover series for the four strategies in ten-year windows from 1928-1937 to 2008-2017. As can be observed, IQS 2 and IQS 3 always generate a lower turnover than the IQS 1 model, which corresponds to the traditional volatility scaling proposed by Barroso and Santa-Clara (2015). The turnover of the weighting strategies naturally increases in the decades that include a crisis in the stock market. In contrast, when there is no crisis, the volatility scaling reduces the transaction costs of the original momentum strategy.

---

<sup>7</sup> If turnover is not restricted to a maximum of 1 each month, then these numbers increase but the ordering of the IQS-scaled models is preserved as follows: 121.6, 92.9 and 91.2% for IQS 1, 2 and 3, respectively.

**Figure 3. Original momentum and IQS-scaled momentum turnover.** The figure shows the average monthly turnover of momentum, IQS 1, 2 and 3 scaled momentum strategies. Each bar corresponds to the average within a period of ten years starting from 1928-1937 and ending 2008-2017.



## 5. Economic and market determinants of risk asymmetries

In this section we explore the role of i) liquidity shocks, ii) risk-return predictability, and iii) two macroeconomic indicators (consumption and industrial production) and asset prices in explaining the better performance of IQS models for managing portfolio risk.

### 5.1. Market liquidity and volatility risk, the role of the asymmetries

Liquidity is a natural candidate for asymmetric associations with IQS categories, because the economically and statistically significant role of volatility driving market illiquidity is well known<sup>8</sup>, and also because the relationship between volatility and liquidity is likely to be nonlinear. There are several factors, including funding liquidity, flight-to-safety, flight-to-quality and traders' capital minimum thresholds (among others), that impact the way in which volatility propagates to liquidity (Brunnermeier and Pedersen, 2009) and which are essentially related to the sorts of nonlinearities that IQS statistics address in practice.

<sup>8</sup> See for instance Stoll (2000), Bao and Pan (2013) and Chung and Chuwonganant (2014).

The role of liquidity-related factors in explaining asset prices has been extensively studied in the literature<sup>9</sup>. Prominent recent examples of asset pricing studies that consider liquidity factors are Pastor and Stambaugh (2003), Hasbrouck (2009), Corwin and Schultz (2012), Mancini et al. (2013) and Abdi and Ranaldo (2017). In general, the theory predicts that both the level of liquidity and liquidity risks are priced, while empirical studies find that the effects of liquidity on asset prices are statistically significant and economically relevant, and that expected stock returns are related cross-sectionally to the sensitivities of returns to fluctuations in aggregate liquidity<sup>10</sup>.

This strong and negative relationship between market volatility and illiquidity is documented here in Table 3 and Figure 4. In Table 3 we show the results of the regressions of the two measures of liquidity on IQS low and high (semivariances above and below the median in the IQS 2 model) and on total volatility, for the four portfolios under analysis. We found the negative relationship between volatility and IQS-variances always to be statistically significant while it is asymmetric for the market and SMB portfolios, with a notably better adjustment (measured by  $R^2$ ) of the IQS-low category compared to the IQS-high and total volatility. In the case of HML and WML, although the liquidity measures are also statistically significant, the model adjustment is considerably lower and, consequently, there is not much asymmetry in the response of liquidity to the IQS high and low categories.

---

<sup>9</sup> Amihud et al. (2005) provide a survey of this relationship.

<sup>10</sup> Market illiquidity is related to such factors as exogenous transaction costs, demand pressures and inventory risks. At the aggregate level, most liquidity measures are constructed as averages of individual liquidity measures – see Pastor and Stambaugh (2003) and Corwin and Schultz (2012), for instance.

**Table 3. Market liquidity and IQS models:** The table shows the slope coefficients, t-statistics, p-values and R-squared statistics for univariate regressions of the aggregate liquidity and innovation liquidity measures proposed by Pastor and Stambaugh (2003) on the total monthly variance (IQS 1), and two categories of an IQS 2 model, namely, the inter-quantile variance constructed with returns above the median (IQS high) and below the median (IQS low). The sample runs from August 1962 to December 2017, which is the longest span available for the liquidity measures on Robert Stambaugh’s home page. The IQS models were estimated using four traditional portfolios: the value-weighted market (RMRF), small minus big (SMB), high minus low (HML) and winners minus losers (WML). The IQS standard deviations were annualized.

Model	Aggregate Market Liquidity				Innovation Market Liquidity			
	<i>coefficient</i>	<i>t-statistic</i>	<i>p-value</i>	<i>R</i> <sup>2</sup>	<i>coefficient</i>	<i>t-statistic</i>	<i>p-value</i>	<i>R</i> <sup>2</sup>
<u>Market</u>								
IQS 1-total	-0.35	-6.07	<0.01	21.51	-0.26	-4.52	<0.01	14.27
IQS 2-low	-0.45	-7.00	<0.01	23.30	-0.37	-5.74	<0.01	19.06
IQS 3-high	-0.41	-6.00	<0.01	14.27	-0.26	-3.77	<0.01	6.64
<u>SMB</u>								
IQS 1-total	-0.57	-5.45	<0.01	13.03	-0.46	-4.31	<0.01	10.39
IQS 2-low	-0.68	-5.39	<0.01	13.65	-0.58	-4.65	<0.01	12.27
IQS 3-high	-0.56	-3.39	<0.01	5.99	-0.40	-2.43	<0.01	3.88
<u>HML</u>								
IQS 1-total	-0.38	-4.97	<0.01	7.63	-0.28	-3.41	<0.01	5.20
IQS 2-low	-0.49	-4.86	<0.01	6.41	-0.35	-3.16	<0.01	3.99
IQS 3-high	-0.45	-4.88	<0.01	6.70	-0.35	-3.46	<0.01	4.91
<u>WML</u>								
IQS 1-total	-0.11	-4.89	<0.01	6.00	-0.07	-2.72	<0.01	2.90
IQS 2-low	-0.12	-4.62	<0.01	4.61	-0.07	-2.09	0.02	1.65
IQS 3-high	-0.17	-5.18	<0.01	6.28	-0.13	-3.41	<0.01	4.11

To explore the relationship between IQS models and liquidity further, in line with Brunnermeier and Pedersen (2009), we examine the relationship between the traders’ funding liquidity and market liquidity. These authors establish a link between market volatility and liquidity (i.e. volatility is a state variable impacting market liquidity and risk premia). This relationship is the product of interactions between traders, who in all cases must respect their own capital budget for leveraging and covering margins. In the

framework devised by these authors, we can identify several nonlinear effects in relation to liquidity: for example, liquidity can suddenly dry up, it has commonality across assets (thus, it is likely to be related to market crashes) and it is subject to flight-to-quality considerations. All these effects permeate the relationship between liquidity and volatility, and motivate the nonlinearities featuring it<sup>11</sup>.

In Figure 4 we explore some of these nonlinearities. In the left-hand panel we show the regression and correlation between the two variables (aggregate and innovation liquidity, on the one hand, vs. IQS categories, on the other). In this case we include only those observations recorded when liquidity was among the lowest 10% in the sample. In the right-hand panel we do the same but here we only include the observations recorded when liquidity was among the highest 10% in the sample. The sample runs monthly from August 1962 to December 2017.

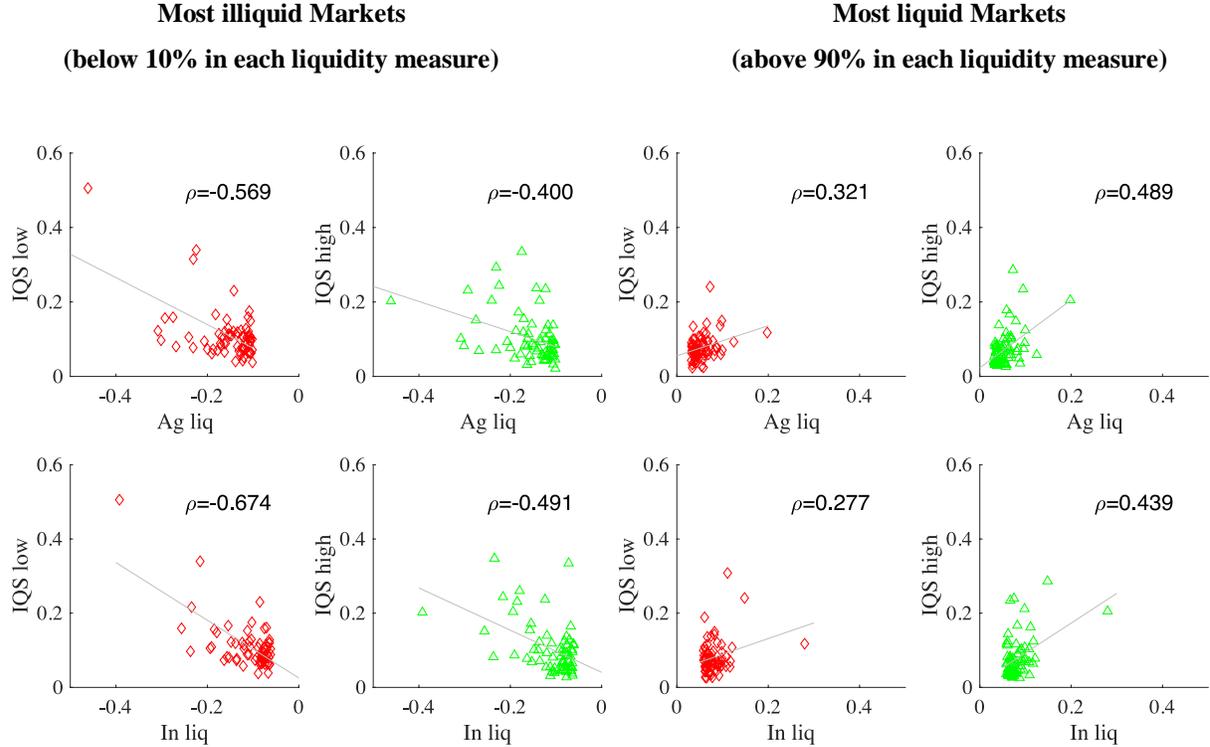
The main results of this exercise can be summarized as follows: i) The relation between volatility and liquidity is highly nonlinear. It is strong and negative when liquidity is low and positive and weak when liquidity is high. ii) When liquidity is low this relationship is better captured by *IQS low* than by *IQS high*. This is evidenced by negative correlations between *IQS low* and market liquidity, on the one hand, and innovation liquidity, on the other, of -0.57 and -0.67, respectively, which contrasts with correlations of -0.40 and -0.49, respectively, in the case of *IQS high*. This is important because it informs us that when volatility impacts liquidity the most, *IQS low* captures liquidity better and, as such, it better isolates scaled portfolios from market liquidity fluctuations<sup>12</sup>.

---

<sup>11</sup> A recent study by Amiram et al. (2016) explores other forms of nonlinearities with a link between volatility and liquidity. The authors found that this relationship is mainly driven by the jump component of the volatility process (rather than by the softer continuous time constituent). It could be argued that these jumps are more closely related to market crashes than to regular market periods and, therefore, we should expect nonlinearities in the sort of volatility-liquidity relationship documented in Figure 5.

<sup>12</sup> We corroborated these results (available upon request) using quantile regressions (Koenker and Bassett, 1978). The negative relationship with volatility is more significant for the lowest levels of liquidity. This is reversed when the market is more liquid, becoming insignificant (or even positive) in some cases for the highest levels of liquidity.

**Figure 4. Market liquidity in high and low liquidity environments vs. the two categories of an IQS 2 model:** The figure plots the *aggregate liquidity* and the *innovation liquidity* measures proposed by Pastor and Stambaugh (2003) against the two categories of an IQS 2 model, namely the inter-quantile variance constructed with market (RMRF) returns above the median (IQS high) and below the median (IQS low). The left-hand panel corresponds to the estimates using observations for the 10% of the sample at which liquidity was lowest. The right-hand panel corresponds to the estimates using observations for the 10% of the sample at which liquidity was highest. The sample period runs from August 1962 to December 2017.



## 5.2. Stock returns predictability and asymmetric effects

We can rewrite equation (7) solving for current returns and generalizing it to any portfolio  $i$  as follows:

$$r_{i,t} = \frac{r_{i,t}^*}{\sigma_{\text{target}}} \hat{\sigma}_{i,t-1}, \quad (11)$$

where  $r_{i,t}$  is the current return on portfolio  $i$ ,  $r_{i,t}^*$  is the scaled-portfolio return and  $\hat{\sigma}_{i,t}$  is the volatility of portfolio  $i$ . Notice that volatility scaling is related to the predictive power of lagged volatility on current portfolio returns. Indeed, we can rewrite (11) as  $r_{i,t} = f_t(\hat{\sigma}_{i,t-1})$  to emphasize a non-linear relationship between risk and return, throughout the time varying function  $f_t(\cdot)$ . This nonlinearity has recently been explored by Adrian et al.

(2016) and by Ghysels et al. (2014). The former find that indeed the lagged-equity volatility (measured as VIX) has a nonlinear and significant relationship with current stock and bond portfolio returns.

We used a nonlinear model, in line with McAleer and Medeiros (2008) in the forecasting exercise of this section. This framework is particularly well suited to our purposes. It allows us to condition risk-slopes (i.e. the forecasting betas of the risk measures) on the level of total volatility, and to interpret the results as arising from two extreme regimes (high and low volatility). The model assumes that the transition between the regimes is smooth, but it also includes abrupt switches between the states as a special case.

We observe in Table 4 that the null of non-significance is rejected in most cases, with the exception of the SMB portfolio, and in Table 5 we observe that in most cases the lowest semivariances of an IQS 2 model work better for forecasting future returns than the full-realized variance provided by the IQS 1 model.

**Table 4. Forecasting power of IQS-Models:** The table shows the F-statistics and their corresponding p-values for four forecasting models of portfolio returns (market, small minus big, high minus low and winners minus losers). The dependent variable is the next month return, and the regressor variable is the current variance at the lowest category of a given IQS model (plus an intercept). In all cases we used a non-linear smooth-transition specification developed by McAleer and Medeiros (2008) and we set the total monthly realized variance of each portfolio as the transition variable. All the models have the same number of parameters, as we allow for two extreme regimes in each case (high and low volatility). The sample runs from 1927:02 to 2017:12

Model	Market		SMB		HML		WML	
	<i>F</i>	<i>P-value</i>	<i>F</i>	<i>P-value</i>	<i>F</i>	<i>P-value</i>	<i>F</i>	<i>P-value</i>
IQS 1	0.51	0.80	1.47	0.19	4.40	0.00	6.59	0.00
IQS 2	2.79	0.01	1.09	0.37	15.28	0.00	7.07	0.00
IQS 3	2.75	0.01	1.04	0.40	14.73	0.00	7.08	0.00

**Table 5. Forecasting power of IQS-Models using AIC and BIC statistics:** The table shows the AIC and BIC statistics for the four IQS models. In all cases we used a non-linear smooth-transition specification and set the total monthly realized variance of each portfolio as the transition variable. All the models have the same number of parameters in which the dependent variable is the next month return, and the regressor variable is the current IQS-variance in the lowest category (plus an intercept). We allow for two extreme regimes, in line with McAleer and Medeiros (2008), and the parameters transit smoothly in-between through a logistic function. The lowest AIC and BIC statistics, for each portfolio, corresponding to an IQS model with at least two categories, are in bold. IQS 1 is the traditional RV- estimate. The sample runs from 1927:02 to 2017:12.

Model	Market		SMB		HML		WML	
	<i>AIC</i>	<i>BIC</i>	<i>AIC</i>	<i>BIC</i>	<i>AIC</i>	<i>BIC</i>	<i>AIC</i>	<i>BIC</i>
IQS 1	-6342.1	-6372.1	<b>-7464.0</b>	<b>-7494.0</b>	-7292.1	-7322.0	-5557.6	-5587.5
IQS 2	<b>-6355.7</b>	<b>-6385.7</b>	-7461.7	-7491.7	<b>-7354.4</b>	<b>-7384.4</b>	-5560.3	-5590.3
IQS 3	-6355.5	-6385.5	-7461.4	-7491.4	-7351.4	-7381.3	<b>-5560.4</b>	<b>-5590.4</b>

### 5.3. *Macroeconomic dynamics of good and bad market shocks*

Here we explore how the asymmetry captured by IQS models in market returns is related to economic activity, consumption and asset prices. If different semivariance categories of the IQS model generate different fluctuations in economic state variables, which in turn determine the equity premia, we would expect these different categories to exhibit a distinctive performance when employed to scale traditional portfolios. This exercise is grounded in the theory developed by Brunnermeir and Sannikov (2014), which emphasizes the nonlinear nature of financial shocks on the real economy, as well as in the study undertaken by Segal et al. (2015), which identifies the different ways in which uncertainty shocks can find their way to the real and financial markets.

We opted to estimate a factor-augmented vector autoregression (FAVAR) as opposed to a traditional VAR<sup>13</sup>. We treated all the factors in the model as unobservable in nature, and we

<sup>13</sup> Segal et al. (2015) used a traditional VAR (1) with few variables and annual observations: expected consumption growth; good, bad or total volatility; and a macroeconomic indicator (or asset price) on which they seek to evaluate the effect of volatility. Their exercise differs from ours in several respects: First, they estimate the shocks that arise from the *economic activity* series (industrial production), while in our case the shocks originate from the market portfolio. Second, they use annual data and we use monthly data. Unfortunately, the number of

only treated the (respective) IQS series and the (respective) macroeconomic series (either consumption, industrial production, or asset prices) as being observable. Our approach rests both theoretically and methodologically on Bernanke et al. (2005). Thus, in line with their recommendations, we estimated the unobservable factors in a first step via principal components, using a data rich environment. Then, we orthogonalized the factors with respect to the IQS-volatility series and estimated traditional VAR models that consist of the orthogonalized unobservable factors, the IQS series (either low or high) and the macro series (either consumption, industrial production, or the SP500 series).

To identify the structural shocks to IQS and, thus, the dynamics of consumption and industrial production after a shock to the IQS categories, we also adopted a fairly eclectic approach. We used a partial identification strategy in which the macro series are assumed contemporaneously exogenous with respect to the IQS series, and the IQS series are contemporaneously endogenous with respect to the macro indicator. The rest of the system was left unidentified, as we are interested only in the responses of consumption, industrial production, and asset prices to the IQS shocks<sup>14</sup>. We included ten unobservable factors in our estimations, which account for about 50% of the variability in the system, and, following Bai and Ng's (2002) information criteria, we determined the number of static factors<sup>15</sup>. Following AIC and FPE criteria, we constructed our model with four lags. All the variables are stationary before entering the system.

Our main results are summarized in Figure 5, which shows the dynamic responses of consumption (a, b), industrial production (c, d), and asset prices (e, f), after a shock to the two categories of an IQS 2 model (low and high) and to the total RV (IQS 1). Panels a, c and e compare these dynamic responses, while panels b, d and f show the responses to the IQS low category and their respective 84% bootstrapped confidence intervals.

The impact of volatility on consumption is negative and statistically significant. The lowest point in the reduction of consumption following a financial volatility shock is reached 4

---

observations obtained with annual data is very small and so the impulse-response functions are statistically insignificant (both for real and financial shocks), making interpretation difficult. For this reason, we opted for a monthly frequency. Third, and finally, in our set we control for omitted variables, which are of great concern in macro-empirical exercises, and so we opted for the FAVAR model.

<sup>14</sup> See Watson (2008) for a summary of this partial identification scheme.

<sup>15</sup> In line with the test proposed by Bai and Ng (2007), the ten static factors were found to correspond to three underlying dynamic factors.

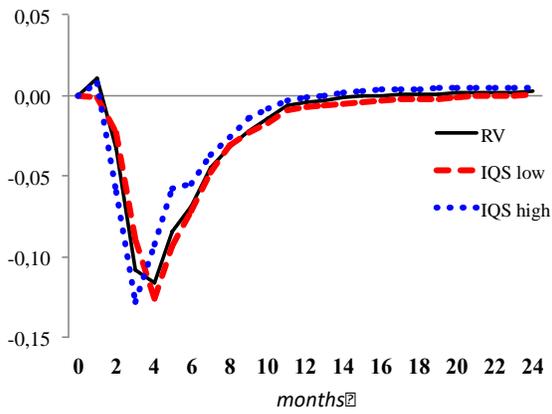
months after the shock, while it takes 11 months for the system to return to its original position. This holds for all of the IQS measures. However, the response of the system to an IQS low shock is more persistent than that to an IQS high shock.

In the case of industrial production we also find notable differences. The impact of an IQS low shock is more pronounced and, once again, more persistent than that of an IQS high shock. The latter also presents a ‘rebound’ effect: that is, after an original deterioration in production following an IQS high shock, a positive response is observed during the second and fifth months, and only afterwards does the system stabilize. This response is not found in the case of an IQS low shock (bad news), which remains consistently negative and significant.

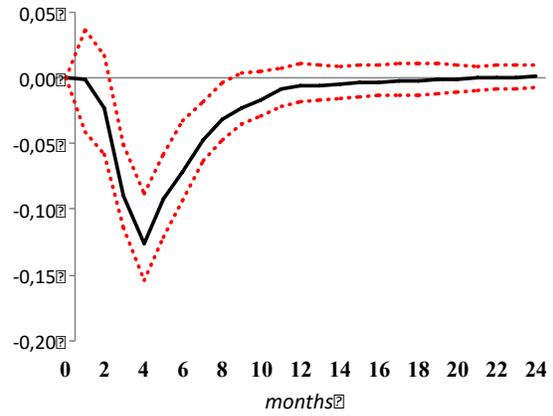
Finally, the asymmetric effects on asset prices are even more evident. The impact of an IQS low shock is most pronounced four months after the shock, when it is also statically significant, and is also more persistent than that generated by good volatility shocks.

These results highlight the asymmetric nature of risk and the effects on the real economy, which are better captured by IQS models than by traditional variances.

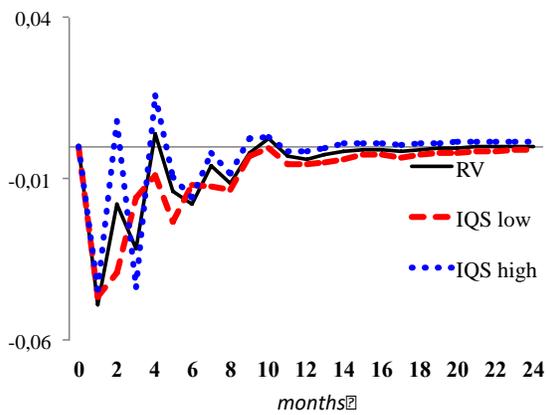
**Figure 5. Dynamic response of consumption and economic activity to IQS low and high shocks:** The figure shows the dynamic response of consumption, industrial production (as a proxy for economic activity) and asset prices after a shock to the market-realized variance, the low category of an IQS-2 model (constructed using returns below the monthly median), and the high category of an IQS-2 model (constructed using returns above the monthly median). Panels a, c and e compare the three responses, while panels b, d and f show the responses of the variables to the IQS-low shocks and their 0.84 bootstrapped confidence intervals. The shocks were estimated using a FAVAR model with ten (orthogonalized) unobservable factors, and the corresponding series (either consumption, industrial production or SP500), together with a measure of IQS volatility (either IQS 1, IQS low or IQS high). The unobservable factors in the FAVAR models were obtained via principal components from a panel of 128 macroeconomic variables described by McCracken and Ng (2016). The monthly sample spans January 1959-December 2017. All the variables were transformed so as to be stationary when entering the system.



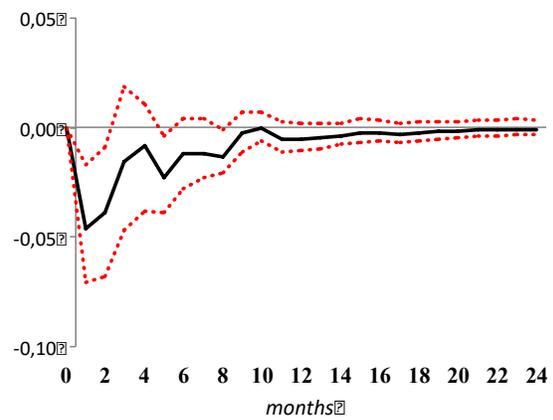
(a) Consumption response



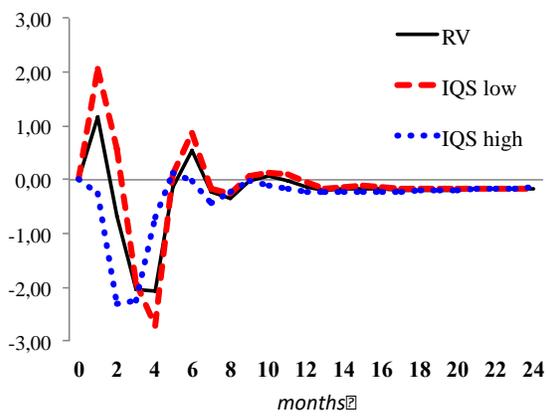
(b) Consumption response



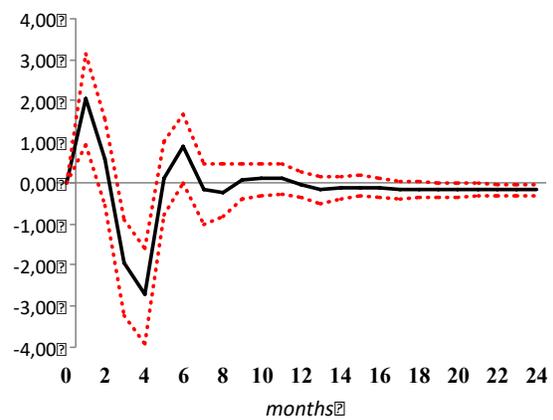
(c) Production response



(d) Production response



(e) Asset price (SP500) response



(f) Asset price (SP500) response

## 6. Conclusions

We have shown that using a fragment of past realized variances to scale traditional portfolios – specifically market, SMB, HML and WML – helps reduce higher order risk, including skewness and market crashes, while simultaneously maximizing the Sharpe ratio and the average return. These findings hold when we compare the economic performance of the IQS-scaling proposed here with both the unscaled and the RV-scaled portfolios.

We investigated whether the enhanced performance of the inter-quantile semivariances models is related to a better forecasting power of future returns and found that, indeed, the better a certain fragment of the realized volatility predicts future returns, the better it performs when used to scale traditional portfolios.

The better economic performance of the IQS-scaling strategies also holds from a time-varying perspective, that is, for 10-year intervals. Basically, IQS-scaling produces higher average returns than the plain and RV-scaled portfolios. These greater returns are also associated with a lower probability of crashes, measured by the distribution's skewness.

Our main results stress the asymmetric nature of risk. In this regard, we have been able to identify the main economic forces behind these asymmetries, which are better captured using the IQS methods proposed here. Specifically, we have explored the relationship between liquidity and the different categories of inter-quantile semivariances. Moreover, our empirical study has enabled us to document nonlinearities in the relationship between volatility and liquidity, which are relevant for asset pricing and risk management. Liquidity is strongly associated with volatility, but even more so with 'bad' volatility, as measured here by the lowest category of the IQS model. As the relationship between volatility and liquidity grows stronger (during episodes of low liquidity), the difference between the effects generated by the two categories of the IQS models also become more evident and significant.

A second determinant of the better performance of the IQS scaling strategies is the fact that IQS low category shocks exhibit greater and more negative impacts on economic activity and consumption than total or good volatility shocks. Here, we have also undertaken an examination of macroeconomic responses to good and bad volatility shocks using FAVAR

models. We measured good and bad volatility shocks as the two categories of an IQS 2 model fitted to the value-weighted market portfolio returns. Both consumption and industrial production react significantly more to bad volatility shocks than to good volatility.

## References

- Abdi, F., Rinaldo, A. 2017. A simple estimation of bid-ask spreads from daily close, high, and low prices. *The Review of Financial Studies*, 30, 4437-4480.
- Adrian, T., Boyarchenko, N., 2012. Intermediary leverage cycles and financial stability. Federal Reserve Bank of New York, Staff Reports 567.
- Adrian, T., Crump, R., Vogt, E., 2016. Nonlinearity and flight to safety in the risk-return trade-off for stocks and bonds. Federal Reserve Bank of New York, Staff Reports 723.
- Aït-Sahalia, Y., Jacod, J., 2012. Analyzing the spectrum of asset returns: Jump and volatility components in high frequency data. *Journal of Economic Literature* 50,1007-1050.
- Amaya, D., Christoffersen, P., Jacobs, K., Vasquez, A., 2015. Does realized skewness predict the cross-section of equity returns? *Journal of Financial Economics* 118, 135-167.
- Amiram, D., Cserna, B., Levy, A. 2016. Volatility, liquidity, and liquidity risk. Columbia Business School Working Paper, October 15 2016.
- Amihud, Y., Mendelson, H., Pedersen, L.H., 2005. Liquidity and asset prices. *Foundations and Trends in Finance* 1, 269-364.
- Andersen, T.G., Bollerslev, T., Diebold, F.X., 2010. Parametric and Nonparametric Volatility Measurement. In *Handbook of Financial Econometrics: Volume 1: Tools and Techniques*, edited by Yacine Aït-Sahalia and Lars Peter Hansen, 67–137. Amsterdam and Boston: Elsevier, North-Holland.
- Bai J., Ng, S., 2002. Determining the number of factors in approximate factor models. *Econometrica* 70, 191–221.
- Bai, J., NG., S., 2007. Determining the number of primitive shocks in factor models. *Journal of Business and Economic Statistics* 25, 52–60.

- Baltas, A., Kosowski, R., 2013. Momentum strategies in futures markets and trend-following funds. European Financial Data Institute – French Finance Association (EUROFIDAI-AFFI) paper. Finance Meeting, Paris, France, December 2012.
- Bansal, R., Yaron, A., 2004. Risks for the long run: A potential resolution of asset pricing puzzles. *Journal of Finance* 59, 1481–1509.
- Bao, J., Pan, J., 2013. Bond illiquidity and excess volatility. *Review of Financial Studies* 26, 3068–3103.
- Barndorff-Nielsen, O.E., Kinnebrock, S., Shephard, N., 2010. Measuring downside risk - realised semivariance. In: Bollerslev, T., Russell, J., Watson, M. (Eds.), *Volatility and Time Series Econometrics: Essays in Honor of Robert F. Engle*, Oxford University Press, Oxford, pp. 117–136.
- Barroso, P., Santa-Clara, P., 2015. Momentum has its moments. *Journal of Financial Economics* 116, 111–120.
- Bernanke, B., Boivin, J., Elias, P., 2005. Measuring the effects of monetary policy: a factor-augmented vector autoregressive (FAVAR) approach. *Quarterly Journal of Economics* 120, 387–422.
- Bloom, N. (2009). The impact of uncertainty shocks. *Econometrica* 77, 623–685.
- Bollerslev, T., Hood, B., Huss, J., Pedersen, L.H., 2017. Risk everywhere: Modeling and managing volatility. Available at SSRN: <http://dx.doi.org/10.2139/ssrn.2722591>
- Bollerslev, T., Zhengzi, S., Zhao, B., 2018. Good volatility, bad volatility, and the cross-section of stock returns. Available at SSRN: <http://dx.doi.org/10.2139/ssrn.2565660>
- Brunnermeier, M., Pedersen, L.H., 2009. Market liquidity and funding liquidity. *Review of Financial Studies* 22, 2201–2238.
- Brunnermeier, M., Sannikov, Y. 2014. A macroeconomic model with a financial sector. *American Economic Review* 104, 379–421.
- Campbell, J. Y., Cochrane, J. H., 1999. By force of habit: A consumption-based explanation of aggregate stock market behavior. *Journal of Political Economy* 107, 205–251.
- Campbell, J. Y., Giglio, S., Polk, C., 2013. Hard times. *Review of Asset Pricing Studies* 3, 95–132.
- Carrodus, M.L., Giles, D.E.A., 1992. The exact distribution of  $R^2$  when the regression disturbances are autocorrelated. *Economics Letters* 38, 375–380.

- Chen, H., Joslin, S., 2012. Generalized transform analysis of affine processes and applications in finance. *Review of Financial Studies* 25, 2225–2256.
- Christensen, K., Oomen, R., Podolskij, M., 2010. Realised quantile-based estimation of the integrated variance. *Journal of Econometrics* 159, 74-98.
- Chung, K.H., Chuwonganant, C., 2014. Uncertainty, market structure, and liquidity. *Journal of Financial Economics* 113, 476-499.
- Corwin, S., Schultz, P., 2012. A simple way to estimate bid-ask spreads from daily high and low prices. *The Journal of Finance* 58, 719-759.
- Cramer, J.S., 1987. Mean and variance of  $R^2$  in small and moderate samples. *Journal of Econometrics* 35, 253-266.
- Daniel, K., Moskowitz, T.J., 2016. Momentum crashes, *Journal of Financial Economics* 122, 221-247.
- Davidson, R. MacKinnon, J., 1981. Several tests for model specification in the presence of alternative hypotheses. *Econometrica* 49, 781-793
- Davidson, R. MacKinnon, J., 1993. *Estimation and inference in econometrics*. New York, Oxford University Press.
- Duffie, D., Pan, J., Singleton, K., 2000. Transform analysis and asset pricing for affine jump-diffusions. *Econometrica* 68, 1343–1376.
- Duong, D., Swanson, N.R., 2015. Empirical evidence on the importance of aggregation, asymmetry, and jumps for volatility prediction. *Journal of Econometrics* 187, 606-621.
- Engle, R.F., Siriwardane, E.N., 2016. Structural GARCH: The Volatility-Leverage Connection. Harvard Business School Working Paper Series 16-09.
- Ghysels, E., Guérin, P., Marcellino, M., 2014. Regime switches in the risk–return trade-off. *Journal of Empirical Finance* 28, 118–138.
- Giglio, S. Dew-Decker, I., 2016. Asset pricing in the frequency domain: theory and empirics. *Review of Financial Studies* 29, 2029-2068.
- Grundy, B., Martin, J., 2001. Understanding the nature of the risks and the source of the rewards to momentum investing. *Review of Financial Studies* 14, 29–78.
- Harding, D., Pagan, A., 2006. Synchronization of cycles. *Journal of Econometrics* 132, 59-79.

- Hasbrouck, J., 2009. Trading costs and returns for US equities: the evidence from daily data. *Journal of Finance* 64, 1445–1477.
- Hillebrand, E., Medeiros, M.C., 2016. Nonlinearity, breaks, and long-range dependence in time series models. *Journal of Business and Economic Statistics* 34, 23-41.
- Hillebrand, E., Medeiros, M.C., Xu, J., 2013. Asymptotic theory for regressions with smoothly changing parameters. *Journal of Time Series Econometrics* 5, 133-162.
- Jacod, J., Li, Y., Mykland, P.A., Podolskij, M., Vetter, M., 2009. Microstructure noise in the continuous case: The pre-averaging approach. *Stochastic Processes and their Applications* 119, 2249–76.
- Kim, A.Y., Tse, Y., Wald, J.K., 2016. Time series momentum and volatility scaling. *Journal of Financial Markets* 30, 103-124.
- Koenker, R., Bassett, G., 1978. Regression quantiles. *Econometrica* 46, 33–50.
- Liu, L., Patton, A., Sheppard, K., 2015. Does anything beat 5-minute RV? A comparison of realized measures across multiple asset classes. *Journal of Econometrics* 187, 293-311.
- Mancini, L., Ranaldo, A., Wrampelmeyer, J., 2013. Liquidity in the foreign exchange market: measurement, commonality, and risk premiums. *Journal of Finance* 68, 1805-1841.
- Martin, I., 2013. The Lucas orchard. *Econometrica* 81, 55–111.
- McAleer, M., Medeiros, M.C., 2008. A multiple regime smooth transition Heterogeneous Autoregressive model for long memory and asymmetries. *Journal of Econometrics* 147, 104-119.
- McCracken, M.W., Ng, S., 2016. FRED-MD: A monthly database for macroeconomic research. *Journal of Business and Economic Statistics* 34, 574-589.
- Moskowitz, T., Ooi, Y., Heje Pedersen, L., 2012. Time series momentum. *Journal of Financial Economics* 104, 228–250.
- Pastor, L., Stambaugh, R.F., 2003. Liquidity risk and expected stock returns. *Journal of Political Economy* 111, 642-685.
- Patton, A., 2011. Volatility forecast comparison using imperfect volatility proxies. *Journal of Econometrics* 160, 246-256.
- Patton, A., Sheppard, K., 2015. Good volatility, bad volatility: Signed jumps and the persistence of volatility. *Review of Economics and Statistics* 97, 683-697.

- Schneider, P., 2016. An anatomy of equity premium. Swiss Finance Institute Research Paper 15-61.
- Segal G., Shaliastovich, I, Yaron, A., 2015. Good and bad uncertainty: Macroeconomic and financial market implications. *Journal of Financial Economics* 117, 369-397.
- Sornette, D., Andersen, J.V., Simonetti, P., 2000. Portfolio theory for ‘fat tails’, *International Journal of Theoretical & Applied Finance* 3, 523-535.
- Stoll. H.R., 2000. Friction. *The Journal of Finance* 55, 1479-1514.
- Vayanos, D., 2004. Flight to quality, flight to liquidity, and the pricing of risk. Working Paper 10327, National Bureau of Economic Research.
- Watson, M., 2008. What’s new in econometrics: time series. NBER Summer Institute, July 2008.
- Westfall, P.H., 2014. Kurtosis as Peakedness, 1905–2014. R.I.P. *American Statistician* 68, 191-195.



Institut de Recerca en Economia Aplicada Regional i Pública  
*Research Institute of Applied Economics*

**Universitat de Barcelona**

Av. Diagonal, 690 • 08034 Barcelona

---

**WEBSITE:** [www.ub.edu/irea/](http://www.ub.edu/irea/) • **CONTACT:** [irea@ub.edu](mailto:irea@ub.edu)

---