COMPARATIVE ADVANTAGE ACROSS GOODS AND PRODUCT QUALITY

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This version: September 2008

Abstract: This paper analyzes the connection between country specialization across goods and country specialization within goods along the quality dimension. It builds a model that introduces quality differentiation and firm heterogeneity into the Dornbusch, Fischer and Samuelson (1977) framework. Country market shares across goods are continuous and decreasing in comparative costs. Within each industry, (*i*) the highest quality is produced by the country with the absolute advantage in the industry; (*ii*) the lowest quality is produced by the country with the lowest wage; (*iii*) each country's average quality is decreasing in its comparative costs in the industry and increasing in its wage level. The model is consistent with previously documented facts and with the specific empirical motivation being provided: it is shown for some illustrative goods that exporter *revealed comparative advantage*, conditional on exporter income per capita, is positively correlated with unit value of exports (unit value being interpreted as a proxy for quality).

Keywords: Vertical differentiation, Quality margin, Extensive margin, Comparative advantage, North-South trade. JEL: F10, L16.

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* Universidad de Murcia and IVIE. A previous version of this paper circulated under the title "Product Quality and Trade", March 2007. I thank Christian Broda, Jonathan Eaton, and Tim Kehoe for comments and useful suggestions, as well as participants in presentations at NYU, New York Federal Reserve Bank, 2007 Midwest Trade and Theory Spring Meeting, and 2008 North American Econometric Society Winter Meeting. This paper was written while visiting the Economics Department at NYU whose hospitality is very much appreciated. Financial support was provided by the Spanish Ministry of Education and Science (PNICDI co-financed by the FEDER), project SEJ2005-07200, and by the Fundación Séneca de la Región de Murcia, project 05710/PHCS/07 and Ayudas para estancias externas de investigadores. Contact: falcala@um.es.

1. INTRODUCTION

Recent empirical research has documented the importance of country specialization across both the horizontal and the vertical (quality) dimensions of goods to characterize the current patterns of trade. Existing models tend to concentrate on only one of these two dimensions and neglect their possible connections. However, there is evidence suggesting that these connections may be important. This paper provides a simple integrated model where both the horizontal and the vertical dimensions of trade are present and their interactions are investigated. The paper shows that the factors that create absolute and comparative advantages across goods also play an important role in the vertical specialization within goods.

Schott (2004), Hummels and Klenow (2005), and Khandelwal (2007), among others, have shown strong evidence of the significance of the quality dimension in characterizing current international trade. Horizontal specialization (specialization across goods) seems to be fading, while vertical specialization (specialization within goods along the quality dimension) is becoming increasingly important.[1] Still, the importance of the horizontal dimension of specialization cannot be underestimated. For example, Hummels and Klenow (2005) show that this dimension plays a key role in the expansion of exports as countries become richer. They found that a 10-percent increase in per capita income brings about, on average, an 8.5-percent increase in the range of goods being exported (the extensive margin of exports) and a 0.9-percent increase in the unit price of exports (the quality margin, with unit price interpreted as a proxy for quality). Along the same lines, Kehoe and Ruhl (2002) showed that the extensive margin accounts for the bulk of trade growth after trade liberalizations. Ideally, trade models should be able to incorporate both the horizontal and the vertical dimensions of specialization.

There are several general equilibrium models analyzing the patterns of country specialization along the quality dimension (Flam and Helpman 1987, Falvey and Kierzkowski 1987, Grossman and Helpman 1991, Stokey 1991, and Murphy and Shleifer 1997, among others). They predict that richer countries specialize in producing higher quality, which conforms to the general evidence in Schott (2004), and Hummels and Klenow (2005). The main underlying argument is that richer countries have a larger endowment in human or physical capital which provides a comparative advantage in producing higher quality. A limitation of these models is that they assume either only one vertically differentiated good in the economy (together with some non-differentiated good) or only one quality level per good at each point of time. As a consequence, they cannot account for the simultaneous existence of the extensive and the quality margins of exports, and the possible interactions between horizontal and vertical specialization. Moreover, some evidence suggests that richer countries do not always produce higher

[1] For example, Schott (2004) has shown that 62% of industrial goods imported by the US in 1994 sourced from both low- and high-wage countries, and that this figure rose steadily from a 30-percent in 1972.

quality than poorer countries. For example, the highest quality varieties of tropical and semi-tropical crops like tobacco or coffee do not originate from the richest producers. Schott (2004) found that about 50-percent of the industrial goods exported by low-wage as well as high-wage countries showed a significant (at the 10 percent level) positive correlation between unit values and exporter per capita GDP. However, this leaves the other 50-percent of selected industrial goods without a significant correlation between income and the proxy for quality. This evidence calls for digging deeper into the determinants of the relative quality of exports across countries.

Likely candidates to contribute to producing higher quality of a given good are absolute and comparative advantage in that good. Going back to the example of tropical crops, a country may produce the best cigars because its soil and climate are the best to grow tobacco. It is then likely that the country also has a horizontal specialization in the production of cigars. The quality of exports of a given good and the absolute or the comparative advantage in that good are likely to be related. As a consequence, less developed countries may export higher quality than richer countries in goods where they have an (absolute or a comparative) advantage. Section 2 provides two examples. I use coffee as a primary good that may have significant quality differences across producers, and men's cotton shirts as an industrial good that has a wide set of low- as well as high-income exporters. It is shown for these goods that exporter *revealed comparative advantage* is positively and significantly correlated with the unit value of exports. Moreover, the significance of revealed comparative advantage increases when income per capita is also included in the regression. Thus, these illustrative cases show that, at least for some goods, specialization across goods is related to vertical specialization within each good and that this relationship is independent of a potential link through income per capita.

A second limitation of available trade models on country specialization along the vertical dimension is that they assume homogeneous producers within each country. However, heterogeneity of firms' efficiency is a prominent phenomenon whose consideration has proven to be very fruitful in capturing important features of international trade.[2] It is shown in this paper that accounting for this heterogeneity is also important for describing the patterns of vertical specialization. Moreover, firm heterogeneity helps explaining why each country's range of exported goods, as well as exported qualities within each good, often overlaps with other countries' ranges (even if wages and efficiency across industries differ).[3]

[2] See Bernard and Jensen (1995) for pioneering empirical work; Bernard, Eaton, Jensen, and Kortum (2003) and Melitz (2003) for path breaking general equilibrium trade models; and Eaton, Kortum, and Kramarz (2005) for how the heterogeneous-firm framework coupled with Cournot equilibrium fits regularities on the distribution of firms and market shares across output destinations. Tybout (2003) and Bernard et al. (2007) review the literature. Bernard, Redding and Schott (2007) analyze firm heterogeneity and comparative advantage in a two-country two-factor two-industry model with no quality differentiation.

[3] The intuitive argument is that firms' choices on output quality are likely to be correlated with their efficiency. As a consequence, the distribution of firm efficiency within each industry, in each country, is likely to be an important

This paper builds a simple two-country Ricardian model with a continuum of goods that can be produced along a continuum of quality levels. Furthermore, each good can be produced by a set of firms that are heterogeneous in terms of their efficiencies. Thus, the model introduces the quality dimension as well as firm heterogeneity into the Dornbusch, Fischer and Samuelson (1977) model (DFS). This model provides the basis to investigate the interactions between horizontal specialization across goods and vertical specialization within goods.

The paper focuses on the implications for international trade of technological differences across countries, industries, and firms in markets with many vertically differentiated goods. In contrast with the more complex supply side, it greatly simplifies the demand side. In particular, it assumes the same homothetic demand across goods and quality varieties in both countries. To be sure, non-homotheticies are important in shaping the patterns of trade along the quality dimension (see Hallak 2006, Choi, Hummels and Xiang 2006, and Fieler 2008). However, homotheticity may prove to be a useful simplification enabling the derivation of sharp predictions that are consistent with the empirical evidence. Moreover, there is no reason to expect that the predictions in this paper would be reversed by introducing non-homotheticies.

In the model's equilibrium, more-efficient firms produce higher quality.[4] Therefore firm heterogeneity implies that each good is produced in every country in many different qualities. The spectrum of qualities produced by each country in each industry (good) is likely to overlap with the other country's spectrum. This gives rise to non-trivial patterns of country market shares across quality varieties where the richer country does not necessarily produce the highest quality. The main implications of the model are as follows. Country market shares across goods are a continuous decreasing function of comparative costs. Richer countries export a wider set of goods (the extensive margin). Within each industry, (*i*) the highest quality is produced by the country with the absolute advantage in the industry; (*ii*) the lowest quality is produced by the country with the lowest wage; (*iii*) country average quality is decreasing in comparative costs and increasing in the wage level. Thus, the model integrates the analysis of horizontal specialization across goods with the analysis of quality specialization within each good. In so doing, it reveals important connections between these two dimensions. Results are consistent with the empirical literature cited above and with the specific empirical motivation provided in the paper.

The paper is organized as follows. Section 2 provides some empirical evidence on the relationship between export unit values and exporter revealed comparative advantage. Section 3 lays out the model.

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determinant of the distribution of output qualities. Then, overlapping distributions of firm efficiencies across countries will imply overlapping distributions of output qualities.

[4] See Alcalá and Hernández (2006), Baldwin and Harrigan (2007), Johnson (2007), Verhoogen (2008), and Kugler and Verhoogen (2008) for other models with this feature analyzing different empirical implications.

Section 4 analyzes specialization across goods. Section 5 analyzes quality specialization within goods. Section 6 concludes.

2. QUALITY AND COMPARATIVE ADVANTAGE ACROSS GOODS: SOME EMPIRICAL EVIDENCE

This section provides illustrative evidence that exporter comparative advantage (jointly with exporter income per capita) is positively correlated with export unit value (used as a proxy for average quality). I consider two cases: a primary good (coffee) and an industrial good (men's cotton shirts). These products were selected since previous studies showed that they are exported by a wide array of countries with different income levels and that their unit values may widely differ across producers.

As the basic observable measure of comparative advantage I use the index of *revealed comparative advantage*, *RCA* (Balassa 1965). This index is a measure of relative export performance (or specialization) by industry and country. The index for country *i* and good *j* can be defined as $RCA_i(j)=100*(EXP_i(j)/EXP_i)/(EXP_W(j)/EXP_W)$; where $EXP_i(j)$ is country *i*'s exports of good *j* to the world, EXP_i is its total exports, $EXP_W(j)$ is total international trade of good *j*, and EXP_W is total world trade (all variables in value terms). I also consider a *quantity variation* of this concept which may be labeled as *quantity revealed comparative advantage* (*QRCA*). This measure has the same definition as $RCA_i(j)$ except that $EXP_i(j)$ is replaced by the number of units of good *j* exported by country *i*. Hence $QRCA_i(j)=RCA_i(j)/unit\ value_i(j)$. Although the empirical measure that is directly connected to the theoretical model below is *RCA*, using the *QRCA* measure serves as a robustness check for the empirical relationship.

The basic relationship to be estimated is:

$$(1) \qquad \text{Log } unit\ value_i(j) = a_0 + a_1 \text{ Log } PCGDP_i + a_2 \text{ Log } RCA_i(j) + u_i\,;$$

where $unit\ value_i(j)$ is the ratio of the value of country *i*'s exports of good *j* over the quantity exported (in kilograms or in number of items, depending on the commodity), $PCGDP_i$ is country *i*'s PPP per capita GDP, and u_i is the error term. Data used to estimate this equation are from United Nations Commodity Trade Statistics Database (SITC, rev.3, available online at http://comtrade.un.org/) except for *PCGDP* which is from the World Development Indicators, World Bank, 2007. All data correspond to 2005.

Figures 1-3 summarize the main point in this section. Figure 1 depicts the scatter plot of the log of revealed comparative advantage against the log of unit value of exports for the case of coffee exports to the world market.[5] Figure 2 depicts a partial scatter plot drawn using the results from estimating equation

[5] Since the UN *comtrade* statistics for coffee exports reflect re-exports by many non-producers (the problem remains after using the UN *comtrade* data on re-exports), the sample was restricted to countries included in the International Coffee Organization list of main exporters of coffee (see http://www.ico.org/about_statistics.asp). Also note the

(1) for men's cotton shirts exports to the US. The vertical axis measures Log*unit value*$_i$(*j*)–(a_0+a_1Log*PCGDP*$_i$) using the coefficient estimates of $\mathbf{a_0}$ and $\mathbf{a_1}$, whereas the horizontal axis measures Log*RCA*$_i$(*j*). Figure 3 does the same using Log*QRCA*$_i$(*j*) instead of Log*RCA*$_i$(*j*). In all cases, a positive relationship between comparative advantage and export unit values becomes apparent.

The details of regressions are shown in Tables 1-3. Table 1 shows OLS estimates of equation (1) for coffee exports to the world market. Per capita GDP is not significant by itself, whereas comparative advantage is positive and significant by itself at 5-percent level. When both variables are included in the regression, both coefficients and significances increase. Columns 4 and 5 repeat the regressions using *QRCA*. The size of the coefficient for *QRCA* is somewhat reduced but is still significant at the 5-percent level in the joint regression with *PCGDP*.

Tables 2 and 3 show that these results are not exclusive to primary goods but may also hold for industrial goods. As already noted, I consider men's cotton shirts exports to the US (comparative advantage is computed using world exports). The sample includes all the countries exporting at least 500 items to the US in 2005, as reported by the US. All columns report OLS regressions except column 5 of each of these two tables which report 2SLS regressions. Table 2 uses the *RCA* measure of comparative advantage. *RCA* is not significant by itself but is positive and significant at the 1-percent level when *PCDGP* is included in the regression (column 3). Figure 2 was drawn using these last results.

The number of units of *j* exported by a country *i*, $n_i(j)$, may also have some impact on the unit value of its exports.[6] I therefore consider $n_i(j)$ as an additional control in the regressions (column 4). The coefficient on *RCA* increases when *quantity exported* is included. The coefficient of *quantity exported* is also significant and has the expected negative sign. Since endogeneity may be a problem with $n_i(j)$ (recall that it is used to compute unit values of exports) I instrument it using the logs of country *i*'s population and openness (exports+imports of goods and services over GDP).[7] Results are in column 5. Comparison of columns 4 and 5 reveals that coefficients are almost identical in the OLS and the 2SLS estimations. Table 3 performs exactly the same empirical analysis using the *QRCA* measure instead of *RCA*. All the results are qualitatively the same with both measures, even if the estimated coefficients for *QRCA* are somewhat lower than those for *RCA*. Figure 3 was drawn using the results in column 4 of this table.

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outlier with a log(*unit value*) about 0.5 and log(*RCA*) about –13 which corresponds to Philippines. This country has a surprising high unit value for 2005 in comparison to unit values in 2004 and 2006. Results would significantly improve if this outlier were dropped.

[6] For example, if two countries have the same per capita GDP and the same *RCA* but different sizes, the larger country will tend to export a larger quantity of the good. Then, if there is some horizontal differentiation related to each specific exporting country (the Armington hypothesis) besides vertical differentiation, we should expect a negative relationship between $n_i(j)$ and *unit value*$_i$(*j*).

[7] Table 4 reports the first stage regressions for these instruments.

In sum, country per capita income and comparative advantage show positive and jointly significant correlations with export unit values. Furthermore, either *PCGDP* or *RCA* may not be significant when including only one of them in the regression.[8] In the following sections I build a trade model of horizontal and vertical specialization that predicts these conditional correlations. The model is also able to predict other previously documented facts already cited such as the richer-country extensive margin of exports.

3. THE MODEL

Consider a two-country economy. Home and foreign countries are denoted *H* and *F*, respectively. Subscript *W* indicates world aggregates. There is a measure-one continuum of goods indexed by *j*. Each good defines an *industry*. Every good can be produced along a continuum of qualities. For each good, there is an infinite set of efficiency-heterogeneous potential producers in each country. In equilibrium, only a finite measure of firms will be active. Each firm produces only one good and chooses which quality and how many units to produce taking as given the other firms' quality and quantity choices (*Cournot* equilibrium). Firm *k* from country *i* in industry *j* produces $x_{ki}(j)$ units of good *j* with quality $q_{ki}(j)$. Firms choosing zero output are said to be inactive. There are no transportation costs so that the production of firm $k_i(j)$ has the same price $P_{ki}(j)$ in both countries.

3.1. Demand

Denote by $c_{ki}^h(j)$ country-*h* representative agent's consumption of firm $k_i(j)$'s output. Consumers from both countries maximize the same utility function

$$(2) \qquad \int_0^1 \ln\left(\sum_{i=H,F}\sum_k q_{ki}(j)c_{ki}^h(j)\right)dj$$

with respect to $c_{ki}^h(j)$, for every $k_i(j)$, subject to $Y_h = \int_0^1\left[\sum_{i=H,F}\sum_k P_{ki}(j)c_{ki}^h(j)\right]dj$; where Y_h is country-*h* representative consumer's income. $\sum_{i=H,F}\sum_k q_{ki}(j)c_{ki}^h(j)$ may be referred to as the number of *quality units* of good *j* consumed by *h*. The first-order conditions of maximizing (2) are straightforward:

$$(3) \qquad \frac{P_{ki}(j)}{P_{k'i'}(j)} = \frac{q_{ki}(j)}{q_{k'i'}(j)};$$

$$(4) \qquad \sum_{i=H,F}\sum_k P_{ki}(j)c_{ki}^h(j) = Y_h.$$

[8] This may be more likely to happen for products where per capita income and comparative advantage are negatively correlated. In the sample used, correlation between (the logs of) these two variables are –0.24 in the case of coffee and –0.25 in the case of men's cotton shirts.

Condition (3) states that the relative price between any two varieties of the same good *j* produced by firms $k_i(j)$ and $k'_{i'}(j)$, is given by their relative quality (e.g., the products of two firms producing the same quality variety of the same good are perfect substitutes and the marginal rate of substitution between any two quality varieties of the same good is constant). Condition (4) states that expenditure is the same across all goods, as with any symmetric Cobb-Douglas utility function. Denote by *P*(*j*) the price of a unit of good *j* with quality equal to 1. Then, from expression (3) we have:

$$P_{ki}(j) = P(j) \cdot q_{ki}(j) ,$$

I will refer to *P*(*j*) as the *price level in industry j*. Using this expression to substitute in (4) and assuming market clearing for each firm's output (e.g., $x_{ki}(j) = \sum_{h=H,F} c_{ki}^{h}(j)$), yields the price level in industry *j* as a function of firms' output and quality choices:

$$(5) \qquad P(j) = \frac{Y_H + Y_F}{\sum_{i=H,F} \sum_{k} q_{ki}(j) x_{ki}(j)} .$$

This is the inverse demand function to be used in solving for the Cournot equilibrium of each industry.

The general equilibrium of the economy with this demand setting yields a determinate composition for each firm's output in terms of both quantity and quality, and therefore a determinate composition of world consumption. However, if no further considerations are made, each representative consumer is indifferent in this equilibrium between consuming any quality variety of each good, as long as relative prices between quality varieties satisfy (3). In other words, the composition of each individual's consumption basket in terms of quality varieties within each good is indeterminate even if the composition of aggregate world consumption is fully determinate. Since a characterization of exports requires a determinate composition of each country representative agent's consumption basket, I will informally consider a common slight variation of utility function (2) that renders this composition fully determinate. I will assume an infinitesimal preference for variety over the varieties produced by the set of firms. As a result, each individual's consumption basket is a scaled down version of the world's consumption basket. Moreover, the composition of each country's exports is exactly the same as the composition of its production. Formally, if υ_i is country *i*'s share in world income ($\upsilon_H = Y_H/(Y_H+Y_F) = 1-\upsilon_F$), then country *i* consumes a portion υ_i of every firm's output, and exports a portion $1-\upsilon_i$ of every domestic firm's output. Under this assumption, the vertical and horizontal characterization of each country's output in Sections 4 and 5 should also be interpreted as a characterization of its exports.

3.2. Technology

Labor is the only production factor. Increasing output quality comes at the cost of lower output per worker. Efficiency of firm k from country i in industry j is given by the product of three positive parameters: T_i, $a_i(j)$, and z_k; where T_i is a country-specific aggregate efficiency parameter, $a_i(j)$ is a country-industry-specific efficiency parameter, and z_k is a firm-specific efficiency parameter. Firm $k_i(j)$'s production function is given by:

$$(6) \qquad x_{ki}(j) = \left[T_i a_i(j) z_k\right]^{1-\sigma} \frac{l_{ki}(j)}{e^{q_{ki}(j)/[T_i a_i(j) z_k]^{\sigma}}}, \quad 0 < \sigma \leq 1;$$

where $l_{ki}(j)$ is its input of labor. The parameter σ measures the extent to which more-efficient firms have a relative advantage in producing higher quality goods (σ=0 would imply that higher efficiency is neutral with respect to quality). The parameter T_i captures differences across countries in general sources of productivity (e.g., generic human capital, good institutions, and public infrastructures). Parameter $a_i(j)$ captures country-industry-specific asymmetries, which may be due to differences in specialized knowledge, skills, and natural resource endowments. z_k captures firm-specific components such as entrepreneur's skills and the myriad of physical and procedural elements that characterize a firm and cannot be easily imitated.

Units of goods are normalized so that $T_F=a_F(j)=1$ for all j. Thus, we can drop subscripts for home technology parameters; i.e., $T_H=T$ and $a_H(j)=a(j)$. The function $a(j):[0,1]\rightarrow \mathrm{R}_{++}$ is assumed to be continuous, differentiable, and strictly decreasing. There are an infinite number of potential firms in each country and sector, indexed k=0,...,∞, which are ordered inversely with respect to efficiency. Hence firm 0 is the most efficient one. Its efficiency is normalized z_0=1. In equilibrium, only a finite measure of firms will be active. The distribution of firm efficiencies z_k is the same in all industries and countries.[9]

Country i's wage is denoted by w_i. The foreign wage is used as the *numeraire*: w_F=1. Let $c_{ki}(q, j)$ be the (constant) marginal cost of firm k from country i producing good j with quality q:

$$(7) \qquad c_{ki}(q,j) = \frac{w_i}{\left[T_i a_i(j) z_k\right]^{1-\sigma}} e^{q/[T_i a_i(j) z_k]^{\sigma}}.$$

The use of the terms absolute and comparative advantage may lead to some confusion when both horizontal and vertical specialization are considered and become intertwined. I reserve these terms for comparison of advantage across goods and avoid their use in the analysis of trade along the quality dimension. Country H (respectively, F) is said to have an *absolute advantage* over country F (resp. H) in

[9] The assumption that the distribution of firm-specific parameters z_k is the same in every industry and country is not necessary for the results in the paper but it greatly simplifies the exposition. The same results can be obtained by just assuming the following first-order stochastic dominance of firm productivities across countries: if $T_i a_i(j)>T_h a_h(j)$ then $T_i a_i(j) z_{ki}(j)>T_h a_h(j) z_{kh}(j)$; where $z_{ki}(j)$ and $z_{kh}(j)$ are efficiency of the k^{th} most efficient firm in industry j in countries i and h, respectively.

industry j if $Ta(j)>1$ (resp. $Ta(j)<1$). This implies that for each k, efficiency of firm k from country H, $Ta(j)z_k$, is higher than efficiency of firm k from country F. At any rate, even if the home country has an absolute advantage in the industry, some foreign firms may be more efficient than some home-country firms in that industry. Additionally, I will refer to the ratio $[w_i/T_ia_i(j)]/[w_h/T_ha_h(j)]$ as country i's *comparative cost* in industry j with respect to country h. Note that, due to normalizations, $w_F/T_Fa_F(j)=1$ for all j. Hence, home country's comparative cost in industry j is simply $w/Ta(j)$. Country H (respectively, F) is said to have a *cost advantage* over country F (resp. H) in industry j if $w/Ta(j)<1$ (resp. $w/Ta(j)>1$).

3.3. Equilibrium

Each firm maximizes profits $\pi_{ki}(j)=x_{ki}(j)[q_{ki}(j)P(j)-c_{ki}(q,j)]$ with respect to its output $x_{ki}(j)$ and quality $q_{ki}(j)$, taking as given the inverse industry demand function (5) and other firms' output and quality choices (Cournot equilibrium). From each firm's first order conditions of maximization we have:

$$(8) \qquad s_{ki}(j)=\begin{cases} 1-e\dfrac{w_i}{P(j)T_ia_i(j)z_k} & \text{if} \quad 1-e\dfrac{w_i}{P(j)T_ia_i(j)z_k}\geq 0; \\ 0 & \text{otherwise.} \end{cases}$$

$$(9) \qquad q_{ki}(j)=\left[T_ia_i(j)z_k\right]^{\sigma};$$

where $s_{ki}(j)$ is firm $k_i(j)$'s world market share in value terms, $s_{ki}(j)\equiv q_{ki}x_{ki}(j)/\sum_{h=H,F}\sum_g q_{gh}(j)x_{gh}(j)$.[10] The least efficient active firm from i in industry j will be denoted by $\bar{k}_i(j)$. Firm $\bar{k}_i(j)$ satisfies:

$$(10) \qquad z_{\bar{k}_i(j)}=\frac{e\,w_i}{P(j)T_ia_i(j)}.$$

Expressions (8) and (9) imply that market share and output quality are increasing in the firm's efficiency. The positive link between efficiency and market shares is common to *Cournot* models with heterogeneous firms. On the other hand, the positive link between efficiency and quality is the consequence of the relative advantage of more efficient firms in producing higher quality (e.g., $\sigma>0$ in expression (6)).

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[10] See Appendix. This approach brings about exactly the same result than a Cournot equilibrium where each firm first chooses how many *quality units* of the good to produce (i.e., it chooses the product $x_{ki}(j)\cdot q_{ki}(j)$), given the other firms' production of quality units; and second, it chooses which combination of quantity and quality minimizes the cost of producing this optimal number of quality units. It may then be noted that, for optimal quality choices, firm $k_i(j)$'s cost per unit of quality $c_{ki}(q_{ki}(j),j)/q_{ki}(j)$ is equal to $w_i/[T_ia_i(j)z_k]$. Hence the parameter σ does not play a role when comparing costs across firms and countries in equilibrium.

Some discussion of this second link will clarify some differences and similarities between this and other trade models with quality differentiation. The link between efficiency and quality tends to be present in all models of trade with quality-differentiated goods. Notwithstanding, models differ in the level at which this link is established: it may be established at the aggregate level, at the industry level, or at the firm level, depending on the source of efficiency differences. It is useful in this respect to recall the three components efficiency differences in this paper: a country aggregate component; a country-industry-specific component; and a firm-specific component. The general equilibrium models of international specialization with only one quality differentiated good such as Flam and Helpman (1987) (see the rest of references in the Introduction) only consider the first component of efficiency differences. As a consequence, they obtain a positive aggregate-economy link between efficiency and quality: richer countries specialize in producing the higher quality goods. Heterogeneous firm models of international trade with quality differentiation (see references in footnote 4) only consider the third source of efficiency differences. Therefore, the equilibrium is not characterized in terms of country characteristics.[11] The model in this paper considers the industry-country-specific component of firms' efficiency (which may be termed the *Dornbusch-Fischer-Samuelson component*), in addition to the aggregate and the firm specific components. This component is what brings about a link between the country's absolute advantage in a particular industry and average output quality. A country's absolute advantage in a given industry implies that, on average, its firms will tend to be more efficient relative to the world, and therefore will tend to produce higher quality. Still, the link is not completely straightforward but conditional on other circumstances. If the country also has a low wage, inefficient firms will also be able to stay active in the market. This will tend to reduce average quality of the country's output, even if it has an absolute advantage in the good. These are the issues analyzed in detail in Section 5.

In addition to the equilibrium conditions (8)-(9) obtained from firm maximization, industry equilibrium requires that market shares add up to 1. Let ψ_{ij} be the sum of country-i firms' market shares in industry j: $\psi_{ij} \equiv \sum_{k=0}^{\bar{k}_i(j)} s_{ki}(j)$. Note from (8) that ψ_{ij} is a function of the ratio $P(j)T_i a_i(j)/w_i$. Industry j equilibrium condition is:

$$(11) \qquad \psi_{Fj}\big(P(j)\big)+\psi_{Dj}\left(P(j)\frac{Ta(j)}{w}\right)=1\,.$$

For any wage and industry technology parameters –more specifically, for any ratio $Ta(j)/w$– there is always an equilibrium industry price level $P(j)^*$ such that (11) is satisfied. Intuitively, given technology and wages, all firm market shares given by (8) would go to zero for an industry price level $P(j)$ sufficiently low (note that even producing output of zero quality is costly). On the other hand, for an

[11] These models tend to focus on other issues and stylized facts such as the correlations between destination-market remoteness, export quality, and firms' inputs quality and wages.

industry price level high enough we would have $\psi_{Fj}(P(j))+\psi_{Dj}(P(j)Ta(j)/w)>1$. Continuity of market shares on $P(j)$ ensures the existence of an industry equilibrium price level.

> **Proposition 3.1:** *For any $T>0$ and $w>0$, there exists a price level $P_j^*>0$ such that industry j is in equilibrium (i.e., expression (11) is satisfied). Moreover, P_j^* is a continuous and decreasing function of the ratio $Ta(j)/w$, $P_j^* = P^*(Ta(j)/w)$.*
> Proof: See Appendix.

In addition to equilibrium in every industry, the general equilibrium of the world economy requires that labor demand matches aggregate labor supply in each country. Labor supply is assumed to be equal to one in each country. Using expressions (6), (8), and (9) we get the following labor market equilibrium condition for each country:

$$(12) \qquad \int_0^1 l_i(j)dj = \frac{1}{w_i} Y_W \int_0^1 \left(\sum_{k=0}^{\infty} s_{ki}(j)\left[1-s_{ki}(j)\right]\right)dj = 1; \quad i = H, F.$$

These two equilibrium conditions determine the relative wage w and the scale of world income Y_W. The relative wage w may be seen as determining how the sum of industry market shares is distributed between the two countries in a way that is consistent with their relative labor supplies (and productivities). In turn, world income Y_W adjusts to the scale that is consistent with the absolute size of world labor supply.[12]

> **Proposition 3.2:** *For any $T>0$, there exists a wage w^* and a world income Y_W^* satisfying all the equilibrium conditions. Moreover, w^* is continuous and strictly increasing in T.*
> Proof: See Appendix.

Given w^* and Y_W^*, we can solve for the rest of variables. Given w^*, the function $P_j^*=P^*(Ta(j)/w)$ determines prices. Then, expressions $\psi_{ij}(P(j)T_ia_i(j)/w_i)$ determine country market shares in each industry. In turn, production levels are obtained using market shares, Y_W^*, and prices.

4. Specialization Across Goods

This section characterizes country specialization across industries. Implications of the model are more easily derived by letting the number of firms vary in a continuous way. In what follows I assume that there is a continuous number of potential firms in each industry and country. In every industry and

[12] Some intuition on how the world economy equilibrium is reached, is as follows. For a home wage w low enough, home country market shares would be equal to one in all industries. For w high enough the opposite is true: foreign-country firms would get all the market in all industries. Since industry equilibrium prices and country market shares are continuous in w, there is an intermediate w^* such that the distribution of market shares across countries is consistent with their relative labor supplies. Then, the scale of the world economy, Y_W, adjusts to the absolute size of the labor supply.

country, the firm-specific efficiency parameters z_k are given by a continuous and differentiable function $g(k)$, g:[0,∞)→ (0,1], such that $g(0)=1$ and $dg/dk<0$. The analysis in the previous section carries over exactly the same by substituting sums across the set of firms with integrals. In equilibrium, only a finite measure $\bar{k}_i(j)$ of firms from each country is active in each industry.[13]

4.1. Aggregate efficiency, wages, and income

In this subsection I obtain some intermediate results that may not be so interesting by themselves but that will prove to be very useful in the following. An increase in a country's aggregate efficiency lowers its firms' costs and raises their incentives to hire more labor and increase production. This increases wages and lowers the prices in industries where the country is a producer. However, the wage increase and the price reductions do not completely offset the positive impact of the aggregate efficiency increase on firms' profits (except in industries where the country is the unique producer). The following proposition summarizes these and other related facts.

Proposition 4.1: *An increase in the home-country aggregate efficiency T brings about: (i) a less than proportional increase in the home wage w; (ii) a less than proportional reduction in the price level* $P^*(j)$ *and an increase in the* $P^*(j)T/w$ *ratio in those industries where both countries are producers; (iii) an increase in the home-country relative income* υ.

Proof: See Appendix.

Results (*i*) and (*iii*) imply that country aggregate efficiency, wage, and relative income are positively connected. Throughout the rest of the paper I use the expressions *country with higher aggregate efficiency*, *country with the higher wage*, and *richer country* as roughly equivalent.[14]

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[13] The use of the continuum in the Cournot setting is not infrequent in the literature (in particular, in the comparative static analysis of the Cournot equilibrium with respect to the number of firms). It is of great mathematical convenience even if somewhat counterintuitive. Note that when each firm takes as given the distribution of competitors' output and qualities, it is irrelevant whether the set of competitors is measured in discrete or in continuous units. Then, given competitors' choices, the firm's optimal output and quality is computed as if it were a *full measure-one firm* (thus, having a non-negligible impact on the industry equilibrium). This implies the same first order conditions as in the model with a discrete number of firms (i.e., expressions (A.1) and (A.2) in the Appendix). Finally, each firm's output computed in this way is integrated with the rest of firms' output to constitute industry output as in any other model using the convention of a continuum of agents.

[14] The equivalence may not be exact if the share of profits in national income is not the same in both countries. A sufficient condition for this equivalence to be precise is that the schedule $a(j)$ is symmetric; where symmetry is defined as $a(j)=1/a(1-j)$ for every j. To see this, consider an economy where this condition holds and assume $T=1$. It can then be shown that both countries would have the same wage and income (moreover, $P(1-j)=a(j)P(j)$). Now, consider the case $T\neq1$. Since according to Proposition 4.1 we have $dw^*/dT>0$ and $d\upsilon/dT>0$, we conclude that $w^*>1$ and $\upsilon>0.5$ if and only if $T>1$. However, even if the schedule $a(j)$ is not symmetric, the richer country will also have a higher wage as long as the difference between the countries' aggregate efficiencies is large enough.

4.2. The Extensive Margin of Exports

How does horizontal specialization relate to income and comparative cost? The model's basic implications on horizontal specialization can be presented graphically. Consider Figure 4 which is drawn for a given set of technology parameters and the corresponding relative equilibrium wage w^*. Define country i's *marginal firm* in industry j as the firm that would just be in the margin of being active should firms from the other country have zero share in market j (this could be the consequence of the other country having very low productivity in this industry). Subscript M will denote marginal firm variables. Figure 4 draws firm costs $w_i/T_i a_i(j) z_k$ as a function of the industry,[15] for four types of firms: home-country most efficient firms (k=0) and marginal firms (k=M); and, similarly, foreign-country most efficient firms and marginal firms. Solid lines correspond to home-country firms whereas dotted lines correspond to foreign-country firms. For each country, the lower line corresponds to the most efficient firms' costs (recall that, due to normalizations, $w_F/T_F a_F(j) z_0=1$). Upper lines correspond to marginal firms' costs. Since $a(j)$ is decreasing, domestic costs are increasing as we move towards higher j.

Expression (8) implies that two firms $k_H(j)$ and $k_F'(j)$ in a given industry, one from each country, have the same market share if they have the same cost ratio. That is, if $w/Ta_j z(k_H(j)) = z(k_F'(j))$. Now note that for industry $j=\bar{j}_H(T)$, the most efficient home firm has the same cost ratio as the foreign marginal firm. Hence both firms have zero market share since, by definition, marginal firms have zero market share. Then, for $j>\bar{j}_H(T)$ the most efficient home firm has higher costs than the marginal foreign firm. Hence home-country output for $j\geq\bar{j}_H(T)$ is zero. Symmetrically, for $j\leq\bar{j}_F(T)$ foreign output is zero. Now, any increase in T shifts downwards the domestic schedules since the ensuing increase in the equilibrium wage w^* is less than proportional (Proposition 4.1). It therefore moves both cutoffs $\bar{j}_H$ and $\bar{j}_F$ to the right. Therefore,

> **Proposition 4.2:** *An increase in a country's aggregate efficiency expands the range of industries where the country is a producer.*

As long as higher income is linked to aggregate efficiency, this proposition implies that richer countries export a wider set of goods. As pointed out in the Introduction, Hummels and Klenow (2005) have shown that this richer countries' extensive margin of exports has a large quantitative importance.

The formal argument for this result is as follows. Denote by $\bar{P}$ the value of $P(j)$ that solves $\psi_{Fj}(P(j))=1$.[16] This is the equilibrium price at the cutoff industry $\bar{j}_H$. Moreover, the most efficient domestic firm in

[15] The ratio $w_i/T_i a_i(j) z_k$ is firm k's cost *per unit of quality* (or, equivalently, per unit of output value); see footnote 10.

[16] $\bar{P}$ is the same for all j; see the proof of Proposition 2.1 for details. It can also be shown that marginal firms' efficiency is given by $z_M=e/\bar{P}$.

industry $\bar{j}_H$ must be exactly on the edge of being active. Hence, $\bar{j}_H(T)$ is the industry satisfying $s_{0H}(\bar{j}_H) = 1 - ew^*(T)/\left[\bar{P}Ta(\bar{j}_H)z_0\right] = 0$. That is,

$$(13) \qquad a(\bar{j}_H) = e\frac{w^*(T)}{\bar{P}T}.$$

Differentiating with respect to *T* and recalling Proposition 4.1, yields the result in Proposition 4.2:

$$\frac{d\bar{j}_H}{dT} = \frac{1}{\partial a(\bar{j}_H)/\partial \bar{j}_H}\frac{a(\bar{j}_H)}{T}\left[\frac{dw^*}{dT}\frac{T}{w^*} - 1\right] > 0.$$

4.3. Comparative Costs and Country Market Shares

The cutoff between industries where the home country has a cost advantage and those where the foreign country does corresponds to the crossing of the two upper lines in Figure 4. This cutoff is denoted by $\bar{j}^{CA}$ and satisfies

$$(14) \qquad Ta(\bar{j}_H^{CA})/w^*(T) = 1.$$

This is the single cutoff determining country specialization in the DFS model without transportation costs. In this model, country *H* is the only producer and exporter for $j < \bar{j}^{CA}$, and *F* is the only producer and exporter for $j > \bar{j}^{CA}$. In the model in this paper, continuity of firms' market shares on wages and efficiency parameters leads to the following

> **Proposition 4.3:** *Home county's market share in industry j is a continuous and decreasing function of its comparative cost w/Ta(j).*

Figure 5 illustrates the pattern of market shares implied by this proposition. It is straightforward to check this proposition taking into account that country *i*'s market share is the sum of its firms' markets shares. Expression (8) implies that each domestic firm's market share is decreasing in $w/Ta(j)$. Moreover, expression (10) for $\bar{k}_H(j)$ implies that the set of domestic active firms is decreasing in $w/Ta(j)$. Thus, lower comparative cost implies larger home-country market share both because the number of its active firms is larger and because each firm has larger market share. Note that lower ratio $w/Ta(j)$ also implies a lower industry price level $P(j)^*$ (Proposition 3.1), which reduces foreign output and markets shares.

5. COUNTRIES' OUTPUT QUALITY WITHIN EACH INDUSTRY

This section characterizes countries' specialization within each industry along the quality dimension. Subsection 5.1 investigates which countries produce each quality within each industry, and how this relates to wages and absolute advantage. In turn, Subsection 5.2 focuses on average quality. Results are compared at the end of the section with the evidence in Section 2.

5.1. Who Produces Which Qualities Within each Industry?

For obvious reasons, the analysis in this section focuses on industries in the interval $(\bar{j}_F, \bar{j}_H)$ where both countries have positive production. Consider an industry in this interval. Active firms from country *i* span the interval of efficiencies $(T_i a_i(j) z_{\bar{k}i(j)}, T_i a_i(j)]$ which determines the interval of qualities produced by the country. From expression (9), it is clear that the country with the highest value of $T_i a_i(j)$ (i.e., the country with the absolute advantage in this industry) produces the highest quality. This expression also implies that the least efficient active firm in each country produces the lowest quality in that country. Now, who does produce the lowest quality in the world market? Consider the least efficient firms from *H* and *F*: $\bar{k}_H(j)$, $\bar{k}_F(j)$. From (9) and (10), we have:

$$1 = \frac{e\, w_H}{P(j)\left[q_{\bar{k}H}(j)\right]^{1/\sigma}} = \frac{e\, w_F}{P(j)\left[q_{\bar{k}F}(j)\right]^{1/\sigma}}.$$

This yields

$$(15) \qquad \frac{q_{\bar{k}H}(j)}{q_{\bar{k}F}(j)} = \left(\frac{w_H}{w_F}\right)^{\sigma}.$$

Therefore, the lowest quality is produced in the country with the lowest wage. The reason is that lower wages allow lower-efficiency firms to be competitive. In turn, lower efficiency implies lower quality. Now, consider the range of quality varieties produced by country *i*. The span of this range is measured by the ratio between the highest and the lowest quality so that it is independent of units:

$$(16) \qquad \frac{q_{0i}(j)}{q_{\bar{k}i}(j)} = \left(\frac{T_i a_i(j)}{T_i a_i(j) z_{\bar{k}i(j)}}\right)^{\sigma} = \frac{1}{\left(z_{\bar{k}i(j)}\right)^{\sigma}}.$$

Since for any $P(j)$, $z_{\bar{k}i(j)}$ is lower for the country with lower cost $w_i/T_i a_i(j)$ (see expression (10)), we conclude that the country with the cost advantage in industry *j* produces a wider spectrum of qualities. The reason is that the range of active firms' efficiencies is wider in the county with the cost advantage.[17] Summarizing,

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[17] See Bernard, Redding and Schott (2007) for a similar result in a heterogeneous firm model with no quality differentiation. Also note that this last result implies an *extensive margin of exports within the quality dimension*: an

Proposition 5.1: *Consider an industry where both countries have positive market shares.*

(i) The highest quality is produced in the country with the absolute advantage.

(ii) The lowest quality is produced in the country with the lowest wage.

(iii) The country with the cost advantage produces a wider spectrum of qualities.

Figures 6 and 7 illustrate this proposition assuming that the home country has higher wage level. Figure 6 considers an industry where the home country has an absolute advantage. Hence it is the unique producer of the highest qualities $q\in(q_{0F}(j),\ q_{0H}(j)]$. On the other hand, the foreign country is the unique producer of the lowest qualities $q\in(q_{\bar{k}F}(j),\ q_{\bar{k}H}(j))$ since it has lower wage. Figure 7 considers an industry where the foreign country has an absolute advantage. In this case, the foreign country is the unique producer of the highest qualities $q\in(q_{0H}(j),\ q_{0F}(j)]$ as well as the lowest qualities. The richer (home) country only produces some intermediate qualities.

Proposition 5.1 highlights some of the connections between vertical and horizontal specializations. Comparative costs $w/Ta(j)$ determine market shares across goods as well as the relative width of the range of qualities produced by each country. Furthermore, each of the two components of this ratio plays a specific role: a high denominator (high absolute advantage) involves producing the high qualities; whereas a low numerator (low wage) involves producing the low qualities because it allows less-efficient firms to be active. Of course, both circumstances are simultaneously possible as in Figure 7.

Figure 8 provides an overall picture of the countries' horizontal and vertical specializations. In comparison to Figure 4, it includes a new dotted line that depicts the inverse of country *H*'s efficiency across industries; i.e., $1/Ta(j)$. As in the rest of figures, this new line is drawn assuming that country *H* has a higher wage than *F* (i.e., $w>1$), so that it is below the $w/Ta(j)$ line. Industry $\bar{j}^{AA}$ is the cutoff between industries where the home country has an absolute advantage (to the left of $\bar{j}^{AA}$) and industries where the foreign country has it (to the right). According to Proposition 5.1, country *H* produces the highest-quality varieties to the left of $\bar{j}^{AA}$ and country *F* does so to the right. Moreover, to the left of $\bar{j}^{CA}$ country *H* also has a cost advantage and produces the widest spectrum of quality varieties (symmetrically for *F* to the right of this cutoff). As explained for Figure 4, both countries have positive production for industries between $\bar{j}_F$ and $\bar{j}_H$. Between $\bar{j}^{AA}$ and $\bar{j}_H$, the poorer country is the only producer of the highest quality varieties in spite of the richer country also being a producer of these goods. To the right of $\bar{j}_H$ the

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increase in a country's income brings about an increase in the range of exported qualities within each industry. The reason is that an increase in a country's aggregate efficiency raises its comparative cost advantage in all industries where it is an exporter (since, according to Proposition 4.1, the ensuing wage increase is less than proportional).

poorer country is also the unique producer of the highest qualities, but in a trivial sense since the richer country does not produce these goods at all. The lowest qualities of all the goods are produced in country F except to the left of $\bar{j}_F$, which corresponds to the interval of goods not produced in this poorer country.

Note that $\bar{j}^{AA}$ in Figure 8 could be to the right of $\bar{j}_H$. This would imply that the richer country does not produce some of the goods for which it has an absolute advantage.[18] Another particular case occurs when $Ta(1)>1$ (the richer H country has an absolute advantage in all goods) and $Ta(1)/w>z_M$ (it produces some varieties of every good). Country specialization in this case roughly corresponds to the one described in trade models with only one quality differentiated good: the richer country is the only producer and exporter of the higher-quality goods.

Proposition 5.1 points out that the production of the highest qualities is not directly linked to country income but to absolute advantage in each specific industry. Still, it is likely that the richer countries produce the higher qualities of most goods because those countries are richer just because they have an absolute advantage (i.e., they are more efficient) in more or more important industries. Some sources of absolute advantage may have an imperfect, low, or even null correlation with per capita income (e.g., specific natural resources or histories of specialization that created some local knowledge and other positive externalities in particular industries). However, the general sources of absolute advantage (e.g., high average education, easy and cheap access to financial resources, good institutions, or public infrastructures) tend to be positively correlated with income. The fact that richer countries tend to produce the highest qualities for a larger set of goods than poorer countries is reflected in the model as follows:

> **Proposition 5.4:** *The richer country produces the highest qualities for a set of industries that is larger than the set of industries for which it has a cost advantage.*

This can be checked using Figure 8. If H is richer, then $1/Ta(\bar{j}^{CA}) < w/Ta(\bar{j}^{CA}) = 1 = 1/Ta(\bar{j}^{AA})$. Therefore, since $a(j)$ is decreasing, $\bar{j}^{CA}$ is always to the left of $\bar{j}^{AA}$.

5.2. Average Quality

5.2.1. Average quality and income level

Average quality of world's and country i's output in industry j are, respectively:

[18] The country with the absolute advantage in a given industry may gradually abandon it as wages increase. For example, a state like Florida might have the highest absolute advantage for producing oranges, but it would loose market share as its wage level rises (Proposition 4.3). The market share reduction will concentrate on the cheapest (lower-quality) varieties which are produced by the least efficient firms. Eventually, for sufficiently high wages, even the most efficient firms would be unable to survive in spite of the absolute advantage. Clearly, this may be different with labor mobility across countries.

$$Q_W(j) \equiv \sum_{i=H,F} \int_0^{\bar{k}_i} r_{ki}(j) q_{ki}(j) dk; \quad Q_i(j) \equiv \frac{\int_0^{\bar{k}_i} r_{ki}(j) q_{ki}(j) dk}{\int_0^{\bar{k}_i} r_{ki}(j) dk};$$

where $r_{ki}(j)$ is firm $k_i(j)$'s share in the world market of j, in physical units of output. Since

$$r_{ki}(j) \equiv \frac{x_{ki}(j)}{\sum_{i=H,F} \int_0^{\bar{k}_i} x_{ki}(j) dk} = s_{ki}(j) \frac{Q_W(j)}{q_{ki}},$$

we have:

$$\text{(17)} \qquad Q_i(j) = \frac{\int_0^{\bar{k}_i} s_{ki}(j) dk}{\int_0^{\bar{k}_i} s_{ki}(j) / q_{ki}(j) dk} = w_i^{\sigma} \left(\frac{T_i a_i(j)}{w_i} \right)^{\sigma} \frac{\int_0^{\bar{k}_i} s_{ki}(j) dk}{\int_0^{\bar{k}_i} s_{ki}(j) / (z_{ki}(j))^{\sigma} dk}.$$

Note that differences across countries in market shares $s_{ki}(j)$ and in the measure of active firms $\bar{k}_i(j)$ can only arise as a consequence of differences in comparative costs $w_i/T_i a_i(j)$ (see expressions (8) and (10)). It is then immediate the following

> **Proposition 5.2:** *Consider an industry where both countries have positive production. Conditional on comparative costs, higher country wage implies higher average quality.*

The intuitive argument for this result is as follows. Two countries with the same cost level $w_i/T_i a_i(j)$ in a given industry will have the same measure of active firms; and for every k, firm k in one country will have the same market share than the corresponding firm k in the other country. However, if the wage in one of the countries is higher, then it must be the case that this country has higher absolute efficiency in the industry. Thus, for every pair of firms with the same cost and market share, one from each country, the firm from the country with higher wage is more efficient and produces higher quality.

This proposition suggests that regressions between export average quality and country per capita income should be run conditional on some measure of comparative costs. This motivates the empirical approach in Section 2, as we discuss at the end of this section. Still, unconditional regressions between average quality of exports and country per capita income have delivered significant positive coefficients for a large set of industrial goods, as well as at the aggregate level (see Schott 2004, and Hummels and Klenow 2005). Proposition 5.2 is consistent with these empirical findings as long as, for a large set of industries, cross-country differences in costs are moderate or are not positively correlated with country per capita income (since, as Proposition 5.3 states below, larger comparative costs bring about lower quality).

5.2.2. Average quality and comparative advantage

Consider now the average quality consequences of differences in comparative costs. Given wages, lower comparative costs in an industry imply higher absolute advantage. In an economy with homogeneous firms and the same wages, average output quality would unquestionably be higher in the country with higher absolute advantage in the industry. However, the relationship may be uncertain if firms are heterogeneous. The reason is that higher absolute advantage involves a larger set of active firms in the country and a reallocation of market shares across firms producing different qualities. Market shares of less-efficient firms (which produce lower quality) could increase relative to the market shares of more-efficient firms when the country's efficiency increases. In fact, it is straightforward from (8) that this will be the case. For some distribution of firm efficiencies, this could give rise to a negative composition effect such that higher efficiency involves lower average output quality. Nevertheless, reasonable assumptions on the distribution of efficiency across firms may rule out this possibility.

Let us consider the following distribution pattern of firms' efficiency, which satisfies the general characteristics assumed on $g(k)$ at the beginning of Section 4:

(18) $$z_k = e^{-\theta k}\ , \quad \theta > 0\ , \quad k \in [0, \infty).$$

Larger θ involves wider heterogeneity across firms. This distribution is flexible enough to approximate a wide array of possible industry configurations. Using (18) to substitute in (17) we have:

(19) $$Q_i(j) = \frac{\frac{1}{\theta}\left[\ln b + \frac{1}{b} - 1\right]}{\left(\frac{1}{T_i a_i(j)}\right)^{\sigma} \frac{1}{\theta}\frac{1}{\sigma}\left[\frac{1}{1+\sigma} b^{\sigma} + \frac{\sigma}{1+\sigma}\frac{1}{b} - 1\right]}; \quad \text{where } b \equiv \frac{P(j) T_i a_i(j)}{w_i e}.$$

The derivative of the log of $Q_i(j)$ with respect to country's efficiency in industry j, $T_i a_i(j)$, yields:

(20) $$\frac{\partial Q_i(j)}{\partial (T_i a_i(j))}\frac{T_i a_i(j)}{Q_i(j)} = \frac{1}{\theta}\frac{s_{0i}(j)}{\psi_i(j)}\left[1 - \frac{Q_i(j)}{q_{0i}(j)}\right] > 0.$$

Thus, higher efficiency in industry j implies higher average quality. Note that differences in $Q_i(j)$ across countries only depend on differences in the wage level and efficiency in the industry. Moreover, conditional on wages, higher efficiency in the industry implies lower comparative cost. Therefore, we have the following

> **Proposition 5.3:** *Consider an industry where both countries have positive production. Conditional on wages, lower country comparative cost implies higher average output quality.*

The empirical exercise in Section 2 is directly related to Propositions 5.2 and 5.3. Section 2 shows for two illustrative goods that *PCGDP* and *RCA* are jointly and significantly correlated with export unit values (whereas unconditional correlations are not significant in some cases). Still, to make more transparent the connection between that empirical section and the propositions, I now check the relationship between the observable *RCA* index used in the regressions and the comparative cost concept used in the propositions.

Let us restate the definition of *RCA* using the notation developed so far (recall that country *i* exports a fraction (1-υ_i) of each good):

$$(21) \qquad RCA_i(j) = 100 \cdot \frac{(1-\upsilon_i)\psi_{ij} Y_W}{(1-\upsilon_i) Y_i} / \frac{\sum_{h=H,F}(1-\upsilon_h)\psi_{hj} Y_W}{\sum_{h=H,F}(1-\upsilon_h) Y_h} = 100 \cdot \frac{\psi_{ij}(T_i a_i(j)/w_i)}{\upsilon_i} \cdot \zeta(j)\,;$$

where $\zeta(j) \equiv \sum_{h=H,F}(1-\upsilon_h)\upsilon_h / \sum_{h=H,F}(1-\upsilon_h)\psi_{hj}$. Note that $\zeta(j)$ only depends on the industry and therefore enters the expression for $RCA_i(j)$ in the same way for all countries. Hence, conditional on the country's income share $\upsilon_i = Y_i/Y_W$, there is an increasing one-to-one mapping between the country's *RCA* in a given industry and its comparative cost ratio $T_i a_i(j)/w_i$. Therefore, we can use *RCA* to test Propositions 5.2 and 5.3 by including exporter's *GDP* in the estimating equation, in addition to *RCA* and *PCGDP* (this last variable being used as a proxy for the wage level). When the log of exporter's *GDP* is added to the estimated equations in Section 2 it turns out to be not significant, whereas the significance of *PCGDP* and *RCA* stays the same. Therefore, the evidence in Section 2 is entirely consistent with the model and, in particular, with Propositions 5.2 and 5.3.

6. Concluding Comments

Recent empirical research has documented the importance of country specialization across both the horizontal and the vertical dimensions of goods to characterize the current patterns of trade. Existing models tend to concentrate on only one of these two dimensions of specialization and neglect their possible connections. However, the evidence provided at the beginning of this paper suggests that these connections may be important. This paper provides an integrated model where both the horizontal and the vertical dimensions of trade are present and their interactions are investigated. The paper shows that the factors that create absolute and comparative advantages across goods also play an important role in the vertical specialization within goods. The model is consistent with the specific empirical motivation in the paper as well as with previously documented facts. Some of the most important simplifications of the model relate to demand. Generalizing this component of the analysis seems an especially interesting direction for further research.

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APPENDIX

Firms' Profit Maximization First Order Conditions

Each firm maximizes profits $\pi_{ki}(j)=x_{ki}(j)[q_{ki}(j)P(j)-c_{ki}(q,j)]$ with respect to $x_{ki}(j)$ and $q_{ki}(j)$, taking as given industry inverse-demand function (5) and the other firms' output and quality choices. Thus, from firm $k_i(j)$'s perspective $\frac{\partial P(j)}{\partial x_{ki}(j)}=-\frac{P(j)q_{ki}}{\sum_{g=H,F}\sum_h q_{hg}(j)x_{hg}(j)}$ and $\frac{\partial P(j)}{\partial q_{ki}(j)}=-\frac{P(j)x_{ki}}{\sum_{g=H,F}\sum_h q_{hg}(j)x_{hg}(j)}$.

Hence profit maximization yields the following FOC:

$$\text{(A.1)}\qquad \frac{\partial \pi_{ki}(j)}{\partial x_{ki}(j)}=q_{ki}(j)P(j)+x_{ki}(j)q_{ki}(j)\frac{\partial P(j)}{\partial x_{ki}(j)}-c_{ki}(q,j)=P(j)-s_{ki}(j)P(j)-\frac{c_{ki}(q,j)}{q_{ki}(j)}=0;$$

$$\text{(A.2)}\qquad \frac{\partial \pi_{ki}(j)}{\partial q_{ki}(j)}=P(j)+q_{ki}(j)\frac{\partial P(j)}{\partial q_{ki}(j)}-\frac{\partial c_{ki}(q,j)}{\partial q_{ki}(j)}=P(j)-s_{ki}(j)P(j)-\frac{\partial c_{ki}(q,j)}{\partial q_{ki}(j)}=0.$$

Then, (A.1) and (A.2) yield (8) and (9) in the main text.

Proof of Proposition 3.1

First, let us characterize the functions $\psi_{Fj}(P(j))$ and $\psi_{Hj}(P(j)Ta(j)/w)$. Note that (10) implies that the number of active firms $\bar{k}_i(j)$ is increasing in $P(j)$. Moreover, taking into account expression (8) we have that ψ_{Fj} is continuous in $P(j)$ and satisfies:

(i) if $P(j) \le e/z_0 = e$, then $\psi_{Fj}(P(j)) = 0$;

(ii) if $P(j) > e$, then $\psi_{Fj}(P(j)) > 0$ and strictly increasing;

(iii) since $\lim_{P(j)\to\infty}\psi_{Fj}(P(j))>1$, there exists $\bar{P}$, $\bar{P}>e$, such that $\psi_{Fj}(\bar{P})=1$.

Note that $\bar{P}$ does not depend on any parameter specific to industry j due to the normalization of foreign industry parameters. Similarly, it is easy to check that ψ_{Hj} is also a continuous and increasing function of the ratio $P(j)Ta(j)/w$. Moreover,

$$\psi_{Hj}(P(j)Ta(j)/w)=\begin{cases}0 & \text{if and only if} \quad P(j)Ta(j)/w\le e;\\ 1 & \text{if and only if} \quad P(j)Ta(j)/w=\bar{P}.\end{cases}$$

Now, the following claim completes de proof of Proposition 3.1 by showing how the equilibrium price P_j^* is determined as a function of any possible value of the ratio $Ta(j)/w$.

Claim 3.1.A: *For any T, w, and the industry parameter a(j), there is an industry equilibrium price* P_j^**. Moreover,* P_j^* *is a continuous and decreasing function of the ratio Ta(j)/w, the same for all j, satisfying:*

$$\begin{cases} P_j^* = \overline{P} & \text{if } Ta(j)/w \in (0\,, e/\overline{P}], \quad \text{impying } \psi_{Hj} = 0\,, \psi_{Fj} = 1; \\ \overline{P} > P_j^* > e & \text{if } Ta(j)/w \in (e/\overline{P}\,, \overline{P}/e), \quad \text{impying } \psi_{Hj} > 0\,, \psi_{Fj} > 0; \\ P_j^* = \overline{P}w/Ta(j) \le e & \text{if } Ta(j)/w \in [\overline{P}/e\,, \infty); \quad \text{impying } \psi_{Hj} = 1\,, \psi_{Fj} = 0. \end{cases}$$

Moreover, if $Ta(j)/w \in (e/\overline{P}\,, \overline{P}/e)$ *then* $0 > \frac{\Delta P_j^*}{P_j^*} / \frac{\Delta(Ta(j)/w)}{Ta(j)/w} > -1$.

The Claim is proven as follows:

(*i*) For $0 < Ta(j)/w \le e/\overline{P}$ we must have $P_j^* = \overline{P}$ since for any ratio $Ta(j)/w$ in that interval, $P(j) < \overline{P}$ would imply $\psi_{Fj}<1$ and $\psi_{Hj}=0$ (and $P(j) > \overline{P}$ implies $\psi_{Fj}>1$). Hence $\psi_{Hj}=0$ and $\psi_{Fj}=1$.

(*ii*) Now consider $Ta(j)/w \ge \overline{P}/e$. We then have $P_j^* = \overline{P}w/[Ta(j)] \le e$, since: (1) for a price lower than this expression we would have $\psi_{Hj}<1$ and $\psi_{Fj}=0$; (2) for a price higher than this expression we would have $\psi_{Hj}>1$. Hence $\psi_{Hj}=1$ and $\psi_{Fj}=0$. Note that at the initial point of the interval defining this case, $Ta(j)/w = \overline{P}/e$, this result implies $P_j^* = e$.

(*iii*) Consider now the intermediate values $Ta(j)/w \in (e/\overline{P}, \overline{P}/e)$. Starting from $Ta(j)/w = e/\overline{P}$ and $P_j^* = \overline{P}$, any increase in $Ta(j)/w$ raises home-country market share above 0. This must be compensated by a reduction in P_j^* so that (11) can be satisfied by means of a reduction in the foreign market share. Moreover, the relative increase $\Delta(Ta(j)/w)/(Ta(j)/w)$ must be larger (in absolute terms) than the relative reduction in the equilibrium price $\Delta P_j^*/P_j^*$ (otherwise the home country share would be either unaffected or reduced while the foreign share is reduced). This will be the case until ψ_{Fj} turns out equal to 0 as $Ta(j)/w$ reaches $e/\overline{P}$. □

Proof of Proposition 3.2

Consider equation (12) in the text:

$$\int_0^1 l_i(j)dj = \int_0^1 \sum_{k=0}^{\infty} e \frac{x_{ki}(j)}{[T_i a_i(j) z_k]^{1-\sigma}} dj = \frac{1}{w_i} \int_0^1 \sum_{k=0}^{\infty} P_j^* q_{ki}(j) x_{ki}(j) e \frac{w_i}{P_j^* T_i a_i(j) z_k} dj$$
$$= \frac{1}{w_i} Y_W \int_0^1 \sum_{k=0}^{\infty} s_{ki}(j)[1 - s_{ki}(j)] dj = 1\,; \quad i = H, F.$$

Recall that market shares $s_{ki}(j)$ are functions of the ratio T/w and prices P_j^*; and that, in turn, prices are also functions of the ratio T/w. Hence we define: $\Psi_i(T/w) \equiv \int_0^1 \sum_{k=0}^{\infty} s_{ki}(j)[1 - s_{ki}(j)] dj$. Dividing the last two terms above for $i=F$ by the same expression for $i=H$ yields:

(A.3) $$w \frac{\Psi_F(T/w)}{\Psi_H(T/w)} = 1\,.$$

This condition can be used to substitute for one of the labor-market equilibrium conditions (12). Wages satisfying (A.3) guarantee that country shares in production match relative labor supplies, whereas (12) for either H or F can be used to insure that world income Y_W adjusts to the scale that is consistent with the absolute size of world labor supply.

Foreign market shares $s_{kF}(j)$ are increasing in P_j^* and therefore decreasing in T/w. To the contrary, domestic shares are increasing in T/w (since they positively depend on this ratio and since the elasticity of P_j^* with respect to T/w is lower than 1; see the last statement in Claim 3.1.A in the proof of Proposition 3.1). Then, note that $a(1)\equiv a_1=\min(a(j))$ and $a(0)\equiv a_0=\max(a(j))$. Hence from Claim 3.1.A (in the proof of Proposition 3.1) we have that $T/w=e/(\overline{P}a_0)$ implies $\psi_{Hj}=0$ all j; and $T/w=\overline{P}/(a_1 e)$ implies $\psi_{Fj}=0$ all j.

Now, assume the distribution of efficiencies is such that no firm has more than half the market share in any industry (e.g., $s_{0i}(j)<0.5$, all j; this assumption is not necessary but greatly simplifies the proof). It is then easy to check that:

$$\Psi_H(T/w)\equiv\int_0^1\sum_{k=0}^{\infty}s_{kH}(j)\left[1-s_{kH}(j)\right]dj:\begin{cases}T/w=e/(\overline{P}a_0) & \Rightarrow \Psi_H(T/w)=0;\\ e/(\overline{P}a_0)<T/w\le \overline{P}/(a_1e) & \Rightarrow \Psi_H(T/w)>0 \text{ and increasing.}\end{cases}$$

$$\Psi_F(T/w)\equiv\int_0^1\sum_{k=0}^{\infty}s_{kF}(j)\left[1-s_{kF}(j)\right]dj:\begin{cases}e/(\overline{P}a_0)\le T/w<\overline{P}/(a_1e) & \Rightarrow \Psi_F(T/w)>0 \text{ and decreasing}\\ T/w=\overline{P}/(a_1e) & \Rightarrow \Psi_F(T/w)=0.\end{cases}$$

Note that, as integrals of the continuous functions s_{ij} in T/w, $\Psi_H(T/w)$ and $\Psi_F(T/w)$ are also continuous in T/w. Define the function $\Phi(T/w):(e/(\overline{P}a_0)\,,\overline{P}/(a_1e)]\rightarrow[0,\infty)$ as $\Phi(T/w)\equiv\Psi_F(T/w)/\Psi_H(T/w)$. This function is characterized as follows:

$$\begin{cases}\lim_{T/w\rightarrow e/(\overline{P}a_0)}\Phi(T/w)=\infty\\ \Phi(T/w) \text{ is strictly decreasing in } T/w & \text{if } e/(\overline{P}a_0)<T/w<\overline{P}/(a_1e);\\ \Phi(T/w)=0 & \text{if } T/w=\overline{P}/(a_1e)\end{cases}$$

Now, for any given $T\in(0,\infty)$, the product $w\cdot\Phi(T/w)$ can be seen as a function of w in the interval $[a_1e/\overline{P}T\,,\overline{P}a_0T/e)$, which is again continuous in w and satisfies:

$$w\cdot\Phi(T/w)\quad\begin{cases}\lim_{w\rightarrow\overline{P}a_0T/e}w\cdot\Phi(T/w)=\infty.\\ \text{strictly increasing in } w & \text{if } \overline{P}a_0T/e>w>a_1e/\overline{P}T\,;\\ =0 & \text{if } w=a_1e/\overline{P}T\end{cases}$$

Therefore for any $T>0$ there exists w^*, $a_1e/\overline{P}T<w^*<\overline{P}a_0T/e$, satisfying the general equilibrium condition $w^*\cdot\Phi(T/w^*)=1$. Moreover, since $\Phi(T/w)$ is decreasing in T/w, $w^*=w^*(T)$ is increasing in T. □

Proof of Proposition 4.1

To prove this proposition, first consider the following intermediate result:

Claim 4.1.A *(differentiability of country market shares and prices)***:** *With a continuous number of firms,* $\psi_{ij}(P(j)T_i a_i(j)/w_i)$ *is differentiable. Furthermore,* $\frac{\partial\Psi_H(T/w)}{\partial(T/w)}>0$ *and* $\frac{\partial\Psi_F(T/w)}{\partial(T/w)}<0$.

To prove this Claim, define $m\equiv P(j)T_i a_i(j)/w_i$ and consider firm and country market shares as functions of m. Note from expression (8) in the main text that each firm's market share $s_{ki}(j)$ is continuous and differentiable at all points m>0, except at the point where the firm switches from being inactive to being active. At this point, the firm's market share is continuous but not differentiable:

$$\lim_{m_h\to m^-}\frac{\partial s_{\bar{k}i}(j)}{\partial m}=0\neq\lim_{m_h\to m^+}\frac{\partial s_{\bar{k}i}(j)}{\partial m}=\frac{1}{m}.$$

However, the country's market share ψ_{ij} is differentiable at all points:

$$\lim_{m_h\to m^-}\frac{\partial}{\partial m}\psi_{ij}(m_h)=\lim_{m_h\to m^-}\frac{\partial}{\partial m}\int_0^{\bar{k}ij}\left(1-\frac{e}{m_h g(k)}\right)dk$$
$$=\lim_{m_h\to m^-}\left[\int_0^{\bar{k}ij}\frac{\partial}{\partial m}\left(1-\frac{e}{mg(k)}\right)dk+\left(1-\frac{e}{mg(\bar{k}_i(j))}\right)\frac{\partial\bar{k}_i(j)}{\partial m}\right]=\int_0^{\bar{k}ij}\frac{e}{m^2 g(k)}dk=\lim_{m_h\to m^+}\frac{\partial}{\partial m}\psi_{ij}(m_h).$$

Therefore, $\psi_{ij}(m)$ is differentiable with respect to m, for all m>0. Moreover, $\partial\psi_{ij}(m)/\partial m>0$.

Now, assuming to simplify the proof that the distribution of efficiencies is such that no firm controls half or more of the market in any industry (i.e., $s_{0i}(j)<0.5$, all j), we have:

$$\frac{\partial\Psi_F(T/w)}{\partial(T/w)}=\int_0^1\left(\frac{d\sum_{k=0}^{\infty}s_{kF}(j)[1-s_{kF}(j)]}{d(T/w)}\right)dj=\int_0^1\left(\frac{\partial\sum_{k=0}^{\infty}s_{kF}(j)[1-s_{kF}(j)]}{\partial P_j^*}\frac{dP_j^*}{d(T/w)}\right)dj<0.$$
$$\frac{\partial\Psi_H(T/w)}{\partial(T/w)}=\int_0^1\left(\frac{d\sum_{k=0}^{\infty}s_{kH}(j)[1-s_{kH}(j)]}{d(T/w)}\right)dj=\int_0^1\left(\frac{\partial\sum_{k=0}^{\infty}s_{kH}(j)[1-s_{kH}(j)]}{\partial(P_j^*T/w)}P_j^*\left[1+\frac{dP_j^*}{d(T/w)}\frac{T/w}{P_j^*}\right]\right)dj>0.$$

Where the last inequality takes into account that $0\geq\frac{\Delta P_j^*}{P_j^*}/\frac{\Delta(T/w)}{T/w}\geq-1$ for all j, with strict inequalities for an open interval of industries (see Claim 3.1.A). This completes the proof of Claim 4.1.A.

Now, to check (i) in Proposition 4.1 use Claim 4.1.A to differentiate expression (A.3) with respect to T, which yields:

(A.4) $$0<\frac{dw^*}{dT}\frac{T}{w^*}=\frac{\varepsilon_\Phi}{\varepsilon_\Phi-1}<1;\text{ where }\varepsilon_\Phi\equiv\frac{\partial\left[\Psi_F(T/w)/\Psi_H(T/w)\right]}{\partial(T/w)}\frac{T/w}{\Psi_F(T/w)/\Psi_H(T/w)}<0.$$

To check (*ii*), note first that both countries have strictly positive market shares if $Ta(j)/w\in(e/\overline{P}\,,\overline{P}/e)$ (see Claim 3.1.A). Differentiating $P_j^*(Ta(j)/w)$ with respect to T and using Claim 3.1.A and (A.4) yields:

(A.5) $$-1<\frac{dP_j^*}{dT}\frac{T}{P_j^*}=\frac{\partial P_j^*}{\partial(T/w)}\frac{T/w}{P_j^*}\left[1-\frac{dw}{dT}\frac{T}{w}\right]<0\text{ for } j \text{ such that } a(j)\in(\frac{ew}{\overline{P}T},\frac{\overline{P}w}{eT}).$$

Then, for industries where the home country is not a producer or it is the unique producer we have:

$$\begin{cases}\dfrac{dP_j^*}{dT}\dfrac{T}{P_j^*}=0 & \text{for } j \text{ such that } \quad a(j)\in(0\,,\dfrac{ew}{\overline{P}T}];\\[2ex] \dfrac{dw}{w}-\dfrac{dP_j^*}{P_j^*}=\dfrac{dT}{T} & \text{for } j \text{ such that } \quad a(j)\in[\dfrac{\overline{P}w}{eT}\,,\infty).\end{cases}$$

Finally, to check (*iii*) consider country *i*'s income:

$$Y_i=\int_0^1\left[\int_0^{\bar{k}}P(j)q_{ki}(j)x_{ki}(j)dk\right]dj=\mathrm{Y}_W\int_0^1\psi_{ij}dj.$$

The elasticity of relative income with respect to home aggregate efficiency is given by:

$$\frac{dY_H}{dT}\frac{T}{Y_H}-\frac{dY_F}{dT}\frac{T}{Y_F}=\int_0^1\left[\frac{d\psi_{Hj}}{dT}\frac{T}{\psi_{Hj}}\right]dj-\int_0^1\left[\frac{d\psi_{Fj}}{dT}\frac{T}{\psi_{Fj}}\right]dj>0.$$

The sign follows from the arguments above in this Proposition implying $d\psi_{Hj}/dT\geq 0$ and $d\psi_{Fj}/dT\leq 0$; with strict inequalities for industries where both countries have positive production. Thus, country *i*'s share in world income υ_i is increasing in its aggregate efficiency. □

TABLE 1
Unit Values and Exporter Characteristics: Coffee

	Dependent variable is log unit value				
	(1)	(2)	(3)	(4)	(5)
PCGDP	0.042		0.059**		0.059**
	(0.029)		(0.024)		(0.028)
Log *RCA*		0.085**	0.095***		
		(0.032)	(0.032)		
Log *QRCA*				0.060	0.075**
				(0.037)	(0.037)
Number of observations	34	34	34	34	34
Adj. R^2	0.05	0.20	0.28	0.08	0.14

This table reports OLS results of regressing the log of unit values of coffee exports to the world market on exporter PPP per capita income (*PCGDP*) and the log of exporter revealed comparative advantage (*RCA*) or the log of exporter quantity revealed comparative advantage (*QRCA*). All regressions include a constant. Data are from United Nations Commodity Trade Statistics Database, except *PCGDP* which is from WDI World Bank. Data correspond to 2005. The sample of exporters was restricted to countries included in the International Coffee Organization list of the main exporters of coffee. Heteroskedasticity-consistent standard errors in parenthesis. *** means significant at 1%; ** 5%; * 10%.

TABLE 2
Unit Values and Exporter Characteristics: Men's Cotton Shirts

	Dependent variable is log unit value				
	(1)	(2)	(3)	(4)	(5)
Log *PCGDP*	0.553***		0.620***	0.560***	0.560***
	(0.080)		(0.082)	(0.076)	(0.083)
Log *RCA*		0.054	0.122***	0.162***	0.163***
		(0.047)	(0.038)	(0.039)	(0.040)
Log *quantity exported*				–0.090***	–0.090**
				(0.021)	(0.035)
Number of observations	70	70	70	70	70
Adj. R^2	0.38	0.01	0.47	0.56	0.58

Results of regressing unit values of men's cotton shirts exports to the US on exporter PPP per capita income (*PCGDP*), exporter revealed comparative advantage (*RCA*) and number of units exported to the US (*quantity exported*). Columns 1-4 use OLS. Column 5 uses two-stage least squares using exporter population and openness as instruments for quantity exported. The sample includes all countries exporting at least 500 items to the US market. All variables are in logs and all regressions include a constant. Data correspond to 2005 and are from United Nations Commodity Trade Statistics Database, except *PCGDP* and population which are from WDI World Bank. Heteroskedasticity-consistent standard errors in parenthesis. *** means significant at 1%; ** 5%; * 10%.

TABLE 3
Unit Values and Exporter Characteristics: Men's Cotton Shirts

	Dependent variable is log unit value				
	(1)	(2)	(3)	(4)	(5)
Log *PCGDP*	0.553***		0.620***	0.576***	0.573***
	(0.080)		(0.086)	(0.081)	(0.086)
Log *QRCA*		–0.008	0.084**	0.130***	0.133***
		(0.045)	(0.036)	(0.039)	(0.043)
Log *quantity exported*				–0.089***	–0.094**
				(0.024)	(0.041)
Number of observations	70	70	70	70	70
Adj. R^2	0.38	–0.01	0.42	0.50	0.52

Results of regressing unit values of men's cotton shirts exports to the US on exporter PPP per capita income (*PCGDP*), exporter quantity revealed comparative advantage (*QRCA*) and number of units exported to the US (*quantity exported*). Columns 1-4 use OLS. Column 5 uses two-stage least squares using exporter population and openness as instruments for quantity exported. The sample includes all countries exporting at least 500 items to the US market. All variables are in logs and all regressions include a constant. Data correspond to 2005 and are from United Nations Commodity Trade Statistics Database, except *PCGDP* and population which are from WDI World Bank. Heteroskedasticity-consistent standard errors in parenthesis. *** means significant at 1%; ** 5%; * 10%.

TABLE 4
First Stage Regressions: Men's Cotton Shirts

	Dependent variable is Log of quantity exported	
	(1)	(2)
Log *PCGDP*	–0.681*	–0.536
	(0.361)	(0.405)
Log *RCA*	0.385*	
	(0.206)	
Log *QRCA*		0.437**
		(0.213)
Log *Population*	1.26***	1.21***
	(0.258)	(0.257)
Log *Openness*	1.85***	1.68**
	(0.686)	(0.701)
Number of observations	70	70
F statistic	9.52	10.21
Adj. R^2	0.37	0.39

This table reports OLS results of regressing the log of the number of men's cotton shirts exported to the US (*quantity exported*) on the log of exporter PPP per capita income (*PCGDP*), the log of exporter revealed comparative advantage (*RCA*) (or the log of exporter quantity revealed comparative advantage, *QRCA*, in column 2), the log of exporter population, and the log of exporter openness (exports+imports of goods and services over GDP). This last two variables are used as instruments for *quantity exported* in the 2SLS regressions reported in columns 5 of Tables 2 and 3. The sample includes all countries exporting at least 500 items to the US market. All regressions include a constant. Data correspond to 2005 and are from United Nations Commodity Trade Statistics Database, except *PCGDP*, population, and openness which are from WDI World Bank. Heteroskedasticity-consistent standard errors in parenthesis. *** means significant at 1%; ** 5%; * 10%.

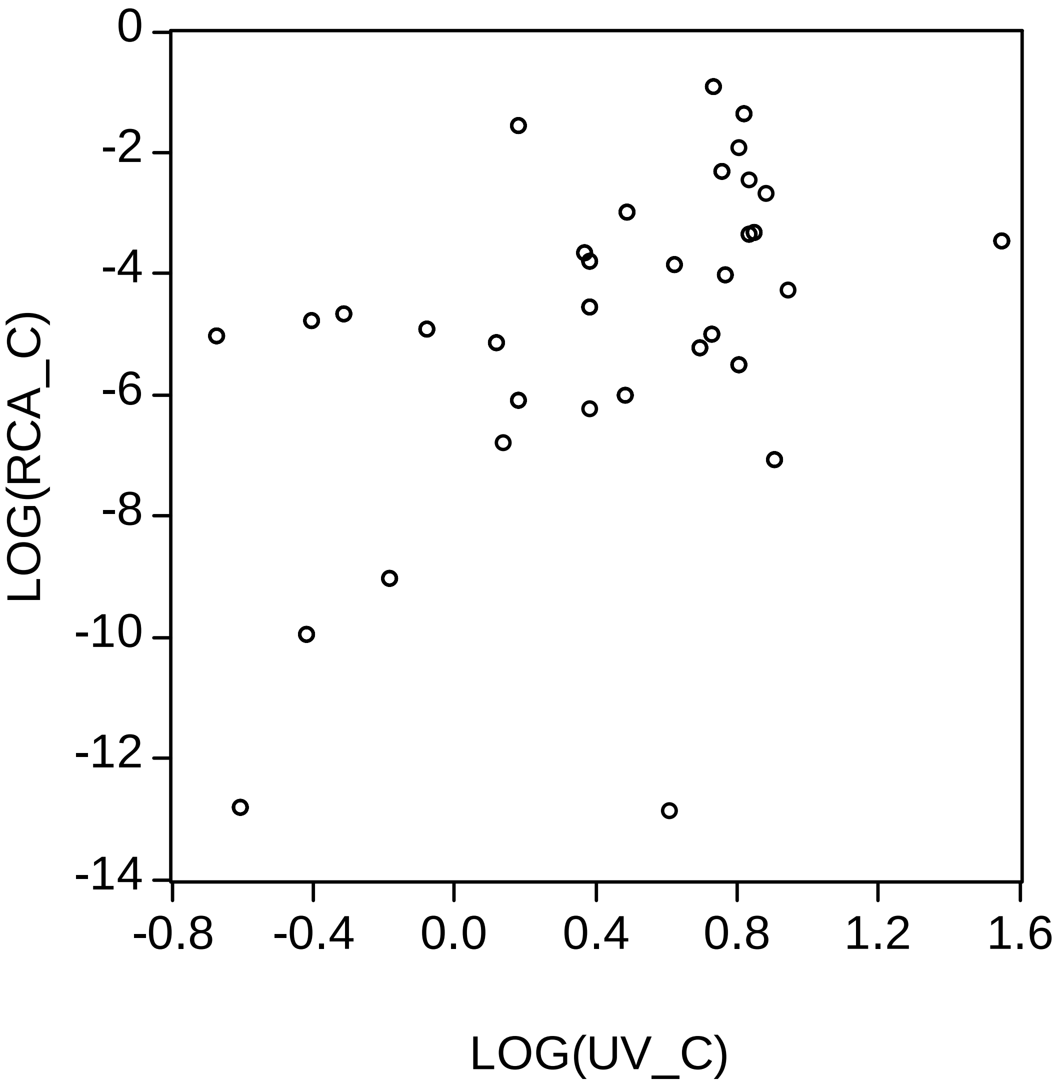

Figure 1: Scatter plot: revealed comparative advantage–unit value. Coffee

The horizontal axis measures the log of exporter revealed comparative advantage in coffee. The vertical axis measures the log of unit value of coffee exports (units are kilograms).

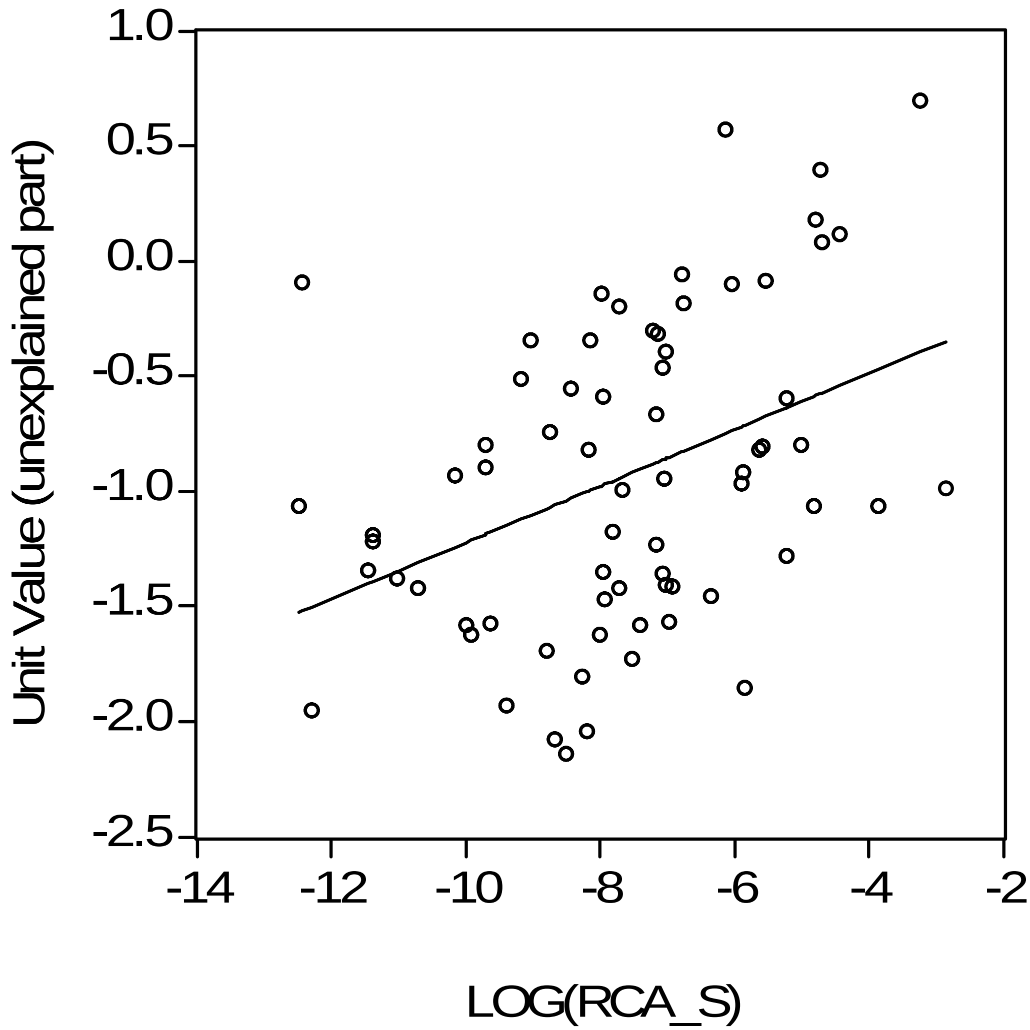

Figure 2: Partial scatter plot: revealed comparative advantage–unit value. Men's cotton shirts

The vertical axis measures Log *unit value of Men's Cotton Shirts (US market)* – ($\hat{a}_0$ + $\hat{a}_1$ Log *PCGDP*) with the coefficient estimates $\hat{a}_0$ and $\hat{a}_1$ taken from column (3) in Table 2. The horizontal axis measures the log of exporter revealed comparative advantage in Men's Cotton Shirts.

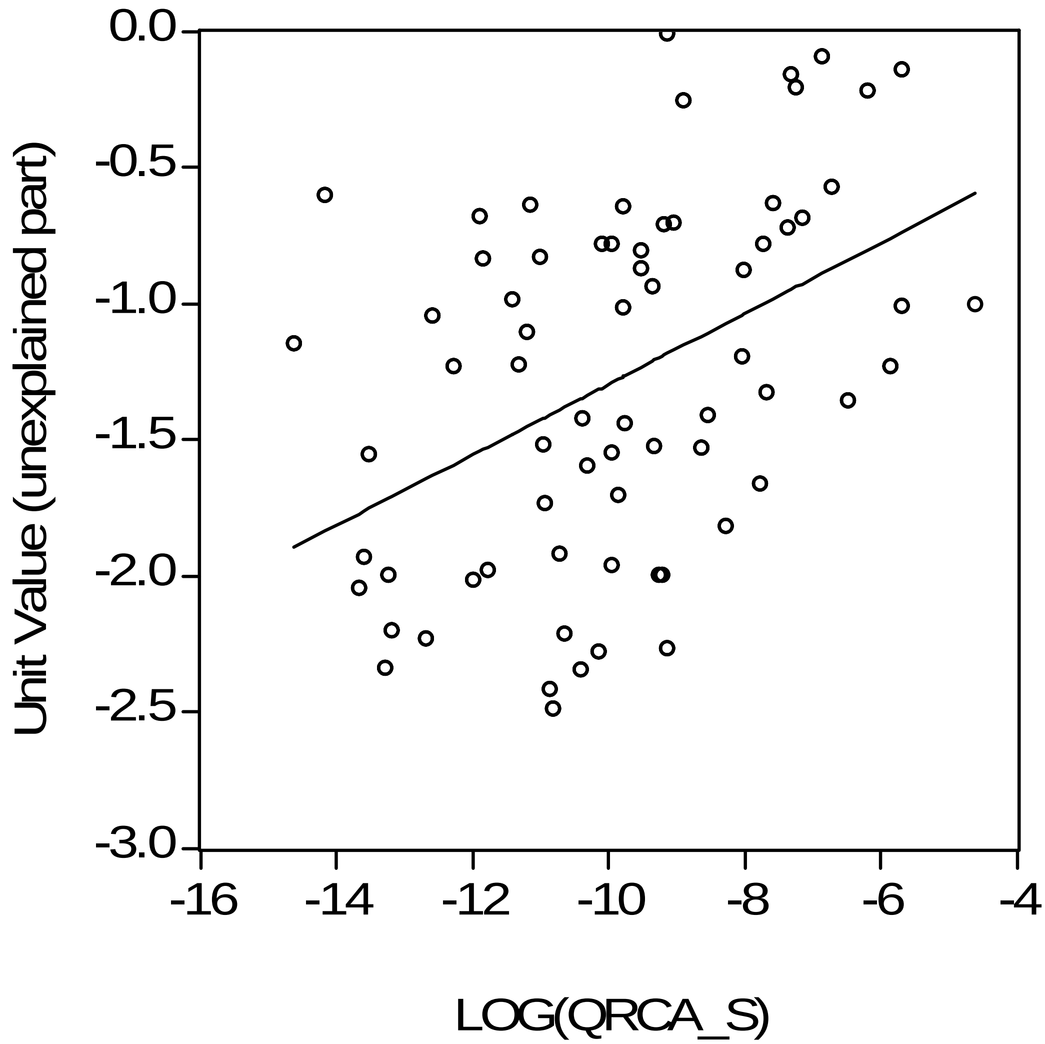

Figure 3: Partial scatter plot: quantity revealed comparative advantage–unit value. Men's cotton shirts

The vertical axis measures Log *unit value of Men's Cotton Shirts* – ($\hat{a}_0$ + $\hat{a}_1$ Log *PCGDP* + $\hat{a}_2$ Log *quantity exported*) with the coefficient estimates $\hat{a}_\mathbf{0}$, $\hat{a}_\mathbf{1}$, and $\hat{a}_2$ taken from column (4) in Table 3. The horizontal axis measures the log of quantity exporter revealed comparative advantage in Men's Cotton Shirts.

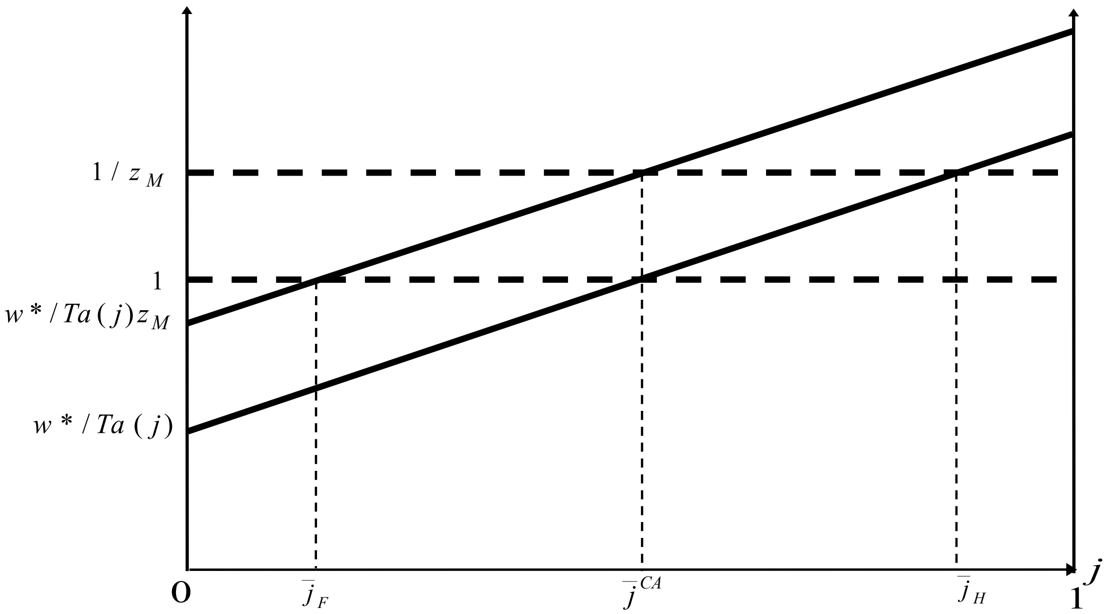

Figure 4: Specialization across goods

Each line shows costs across the different industries for four types of firms: the most efficient firm (firm 0; e.g., z_0=1) and the marginal firm (firm *M*) in the home country (solid lines) and the foreign country (dashed lines).

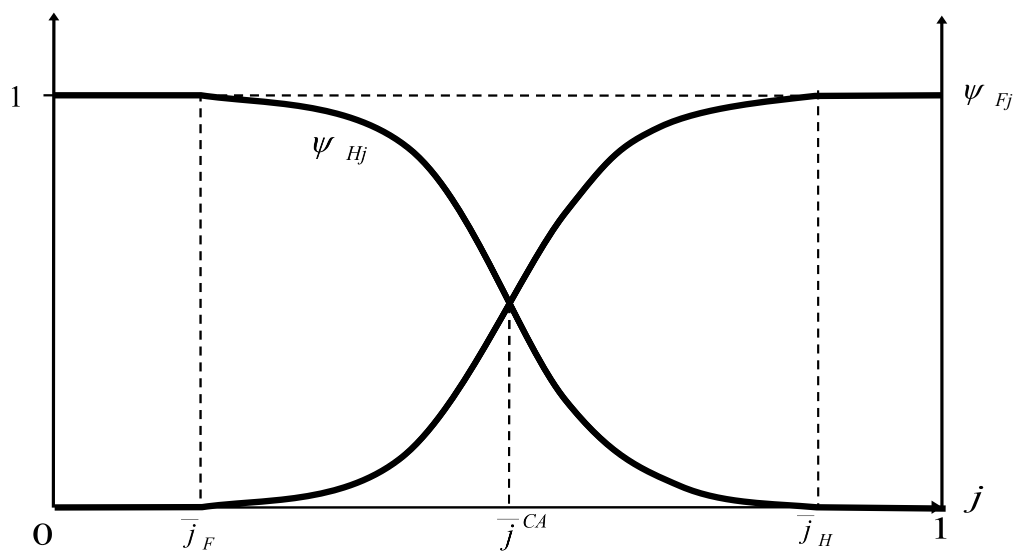

Figure 5: Country market shares across industries

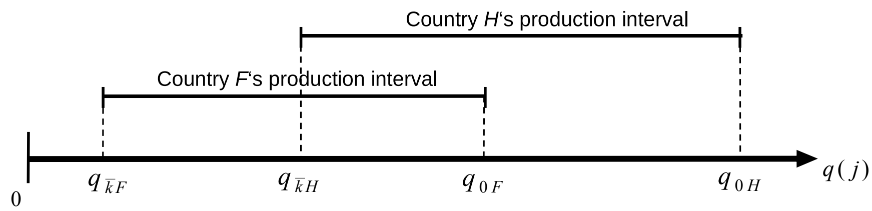

Figure 6: Interval of qualities produced by each country: The case of an industry where the richer country (*H*) has an absolute advantage

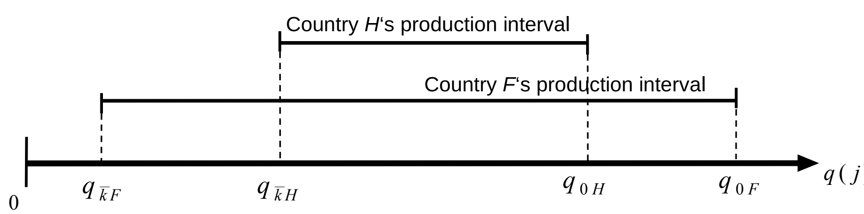

Figure 7: Interval of qualities produced by each country: The case of an industry where the poorer country (*F*) has an absolute advantage.

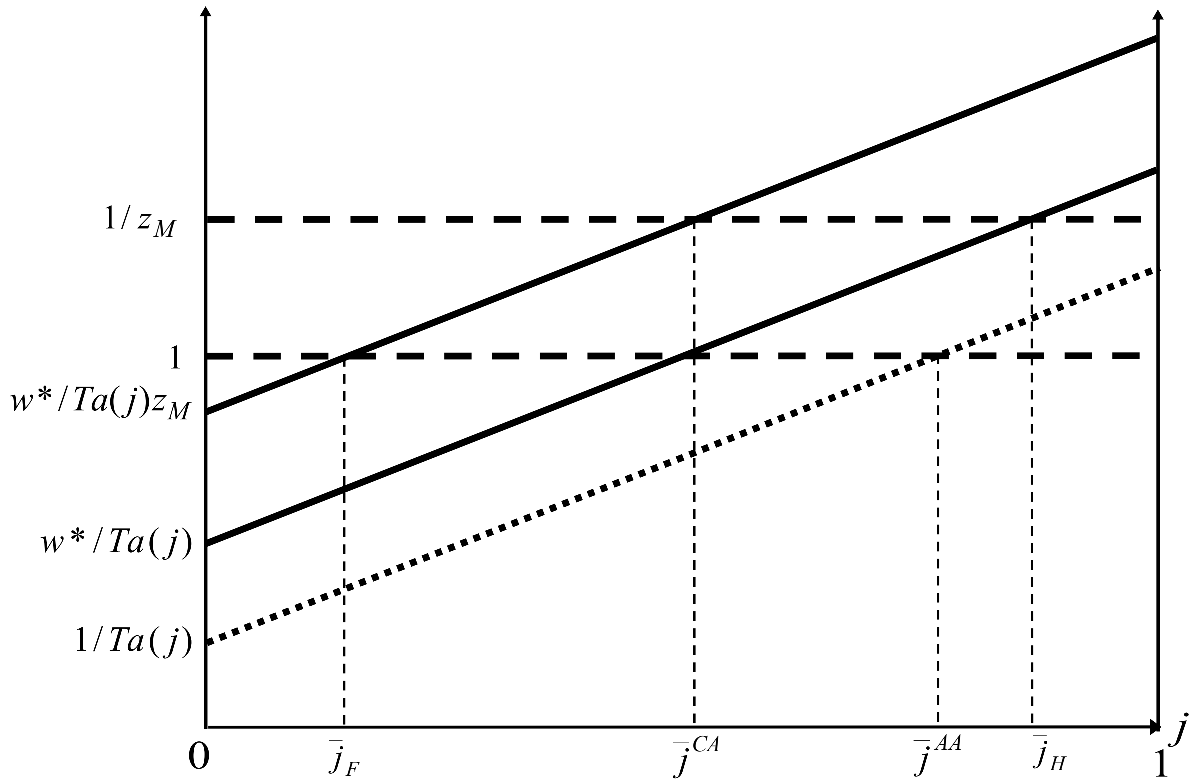

Figure 8: Specialization across goods and quality specialization within goods

This figure adds the inverse of country *H*'s efficiency across industries $1/Ta(j)$ (the dotted line) to the lines already in Figure 4. Country *H* is assumed to have a higher wage level than country *F* (i.e., $w^*>1$) For goods between $\bar{j}_F$ and $\bar{j}_H$ both countries have positive production. To the left (respectively, right) of $\bar{j}^{AA}$ country *H* (resp. country *F*) has an absolute advantage and produces the highest qualities. To the left (resp. right) of $\bar{j}^{CA}$ it has a cost advantage and produces the widest set of qualities. To the right of $\bar{j}_F$ country *F* produces the lowest qualities.