Komplex Analysis Weeklong-School and Workshop KAWA6

Pisa, March 23–28, 2015

Titles and Abstracts — School

Laura DeMarco

(Northwestern University)

Dynamical moduli spaces and elliptic curves

I will introduce the moduli space of rational functions on $\mathbb{P}^1$, of a given degree $d > 1$, as a space of dynamical systems. I will discuss dynamical questions (such as structure of the bifurcation locus, features of degenerating families) and connections with arithmetic geometry.

Joaquim Ortega-Cerdà

(Universitat de Barcelona)

Fekete points, an overview

I will present an overview of the distribution of Fekete points in several contexts. These are points that minimize a certain energy and that appear naturally in problems ranging from electrostatics to approximation theory. We will overview the results in different contexts, from the classical weighted Fekete points in the plane to more elaborated versions in complex Riemannian manifolds. We will explore how the connection to other sets of points can be enlightening in their study and will provide some applications.

Valentino Tosatti

(Northwestern University)

The Kähler-Ricci flow

The Ricci flow is an evolution equation which deforms a Riemannian metric in the direction of its Ricci tensor, with the goal of making it more "round". If the underlying manifold is complex and the initial metric is Kähler, then so are the evolved metrics, and the flow is called the Kähler-Ricci flow. When the manifold is also compact, the flow becomes intimately related to the complex structure of the manifold. If the manifold is algebraic, Song and Tian have recently initiated a program relating the convergence properties of the flow to the minimal model program in birational geometry.

In these lectures I will give an introduction to the Kähler-Ricci flow, and present some results which fit in this framework. Possible topics to be covered are: the characterization of the maximal existence time of the flow, the long time behavior on some minimal Kähler manifolds, the formation of singularities in finite time.

Special lecture by Jean-Pierre Demailly

On Thursday at 18:00 there will be a special lecture:

Jean-Pierre Demailly

(Université de Grenoble I)

Proof of the Kobayashi conjecture on the hyperbolicity of very general hypersurfaces

The Green-Griffiths-Lang conjecture stipulates that for every projective variety X of general type over $\mathbb{C}$, there exists a proper algebraic subvariety of X containing all non constant entire curves $f \colon \mathbb{C} \to X$. Using the formalism of directed varieties, we prove here that this assertion holds true in case X satisfies a strong general type condition that is related to a certain jet-semistability property of the tangent bundle T_X. We then use this fact to confirm a long-standing conjecture of Kobayashi (1970), according to which a very general algebraic hypersurface of dimension n and degree at least $2n+2$ in the complex projective space $\mathbb{P}^{n+1}$ is hyperbolic.

Titles and Abstracts — Workshop

Matthieu Astorg

(Université de Toulouse III Paul Sabatier)

A polynomial endomorphism of $\mathbb{C}^2$ with a wandering Fatou component

For a holomorphic map $f\colon M \to M$, where M is a complex manifold, the Fatou set is the largest open set on which the iterates of f form a normal family, and a Fatou component is a connected component of the Fatou set. A celebrated theorem of Sullivan (70's) assert that all Fatou component are eventually periodic when f is a rational map and $M = \mathbb{P}^1(\mathbb{C})$. Various classes of counter-examples have been found and studied when $M = \mathbb{C}$ and f is an entire transcendental function. We give a construction for a counterexample in dimension 2, with $M = \mathbb{P}^2(\mathbb{C})$, and f a polynomial skew-product, using techniques of parabolic implosion. (Joint work with X. Buff, R. Dujardin, H. Peters and J. Raissy.)

Fabrizio Bianchi

(Université de Toulouse III Paul Sabatier & Università di Pisa)

Holomorphic motion for the Julia sets of holomorphic families
of endomorphisms of $\mathbb{CP}(k)$

We build measurable holomorphic motions for Julia sets of holomorphic families of endomorphisms of $\mathbb{CP}(k)$ under various equivalent notions of stability. This generalizes the well-known result obtained by Mane-Sad-Sullivan and Lyubich in dimension 1 and leads to a coherent definition of the bifurcation locus in this setting. Since the usual 1-dimensional techniques no longer apply in higher dimension, our approach is based on ergodic and pluripotential methods. (Joint work with François Berteloot.)

Sandrine Daurat

(Université de Paris Est Marne la Vallée)

Dynamics in the basin of attraction

We are interested in the description of the dynamics in the basin of attraction of an attracting set A. For this, we will rely on works of J. Diller, R. Dujardin and V. Guedj on "small" topological degree rational mappings of a complex projective surface, works themselves based on those of E. Bedford, M. Lyubich et J. Smillie. We establish the laminarity of the Green current T for a large family of endomorphisms. We also build weakly hyperbolic measure of saddle type which represent equidistribution of saddle periodic points included in A, together with the distribution of the images of almost every point in the basin of attraction in the sense of the trace measure of T.

Eleonora Di Nezza

(Imperial College London)

Regularizing properties and uniqueness of the Kæhler-Ricci flow

Let X be a compact Kæhler manifold. I will show that the Kæhler-Ricci flow, as well as its twisted version, can be run from an arbitrary positive closed current, and that it is immediately smooth in a Zariski open subset of X. Moreover, if the initial data has positive Lelong number we indeed have propagation of singularities for short time. Finally, I will prove a uniqueness result in the case of zero Lelong numbers. (This is a joint work with Chinh Lu.)

Anna Fino

(Università di Torino)

Solvmanifolds and Hermitian Geometry

I will present general results on compact solvmanifolds admitting an invariant complex structure. We will consider in particular the case of compact solvmanifolds with holomorphically trivial canonical bundle, studying the existence of special Hermitian metrics.

Vincent Guedj

(Université de Toulouse III Paul Sabatier)

Weak complex Monge-Ampère flows

Studying the (long-term) behavior of the Kæhler-Ricci flow on mildly singular varieties, one is naturally lead to study weak solutions of degenerate parabolic complex Monge-Ampère equations. In this talk we develop a viscosity theory for such flows and study the (normalized) Kæhler-Ricci flow on varieties with canonical singularities, generalizing results of Song and Tian. (This is joint work with P. Eyssidieux and A. Zeriahi.)

Sione Ma'u

(University of Auckland)

Transfinite diameter on varieties

Transfinite diameter is a geometric notion that plays a central role in classical potential theory and complex analysis. In several complex variables and pluripotential theory, the Fekete-Leja transfinite diameter is a natural generalization. An important study of it was Zaharjuta's 1975 paper. His methods for studying transfinite diameter combined algebra and analysis, i.e. manipulating polynomials, and taking limits. The algebraic part in this case was relatively trivial. I am interested in transfinite diameter on an algebraic variety. Recently with David Cox (Amherst, MA), we adapted Zaharjuta's techniques to study transfinite diameter on algebraic varieties in $\mathbb{C}^n$ that behave "nicely" at infinity. Almost all the additional work is in the algebra. I will describe our work and illustrate it with concrete examples.

Francine Meylan

(Université de Fribourg)

Chern-Moser operators and polynomial models in CR geometry

We consider the fundamental invariant of a real hypersurface in $\mathbb{C}^N$ -its holomorphic symmetry group- and analyze its structure at a point of degenerate Levi form. Generalizing the Chern-Moser operator to hypersurfaces of finite multitype, we compute the Lie algebra of infinitesimal symmetries of the model and provide explicit description for each graded component. Compared with a hyperquadric, it may contain additional components consisting of nonlinear vector fields defined in terms of complex tangential variables. As a consequence, we obtain exact results on jet determination for hypersurfaces with such models. The results generalize directly the fundamental result of Chern and Moser from quadratic models to polynomials of higher degree.

Samuele Mongodi

(Università di Roma "Tor Vergata")

Weakly complete surfaces

It is well known that a complex space which admits a strictly plurisubharmonic exhaustion function is Stein. What can we say if we only know that there exists an exhaustive plurisubharmonic function, whose Levi form may degenerate somewhere? We try to answer to this question for 2-dimensional complex manifolds, with the further assumption that such an exhaustion psh function is real analytic. I will give some relevant examples and state a classification result for weakly complete surfaces. I will also sketch the main steps of the proof and, time permitting, add some details about the methods used. (Joint work with G. Tomassini and Z. Slodkowski.)

Yusuke Okuyama

(Kyoto Institute of Technology)

Equidistribution of rational functions having a superattracting periodic point

We will talk about an approximation of the activity current T_c in the parameter space of a holomorphic family f of rational functions having a marked critical point c by parameters for which c is periodic under f, i.e., is a superattracting periodic point. This partly generalizes a Dujardin-Favre theorem for rational functions having preperiodic points, and refines a Bassanelli-Berteloot theorem on a similar approximation of the bifurcation current T_f of the holomorphic family f.

Evgeny Poletsky

(Syracuse University)

Extensions of Riemann domains and functions on them

In our talk we will discuss new developments in the problem of extensions of Riemann domains and functions on them using analytic disks. This usage was originated by B. Jöricke to construct envelopes of holomorphy and then used by F. Larusson and the speaker to subextend plurisubharmonic functions to holes in the domain. The method itself relies on homotopies of analytic disks and we will discuss the latest advancements in the holomorphic homotopy theory of analytic disks on general manifolds. Finally, we will show how an h-analog for the fundamental group can be introduced.

Martin Sera

(University of Wuppertal)

A Generalization of Takegoshi's Relative Vanishing Theorem

I would like to present a generalization of Takegoshi's relative version of the Grauert-Riemenschneider vanishing theorem. Under some natural assumptions, we extend Takegoshi's vanishing theorem to the case of Nakano semi-positive coherent analytic sheaves on singular complex spaces. We also obtain some new results about proper modifications of torsion-free coherent analytic sheaves.

Poster Session

Posters will be displayed in the conference room, on friday and saturday, and poster sessions will take place during the coffee breaks of the workshop.

- Ines Adouani (Université Pierre et Marie Curie Paris 6) *Lower bound of admissible functions on the Grassmannian $G_{m;nm}(\mathbb{C})$*
- Hassan Jolany (Université Lille 1) *Song-Tian theory in Kähler Ricci flow*
- Lucas Kaufmann Sacchetto (Université Pierre et Marie Curie Paris 6) *Monge-Ampère measures with Hölder continuous potential and integrability of p.s.h. functions*
- Zoryana Mozhyrovska (Lviv Commercial Academy) *Universality of composition operators on spaces of analytic functions*