

FORMAL CRITERIA OF NON-TRUTH-FUNCTIONALITY

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1. *Truth-Functional Meaning*

The distinction between truth-functional and non-truth-functional logical and linguistic contexts is vital to a complete philosophical semantics. Even if we believe, as Wittgenstein maintains in the *Tractatus Logico-Philosophicus*, that there are no non-truth-functional proposition-building operations, we still stand in need of a theoretical criterion by which truth-functional contexts and operators can be distinguished from the non-truth-functional components of a formal or colloquial language.

Standardly, the categories of non-truth-functional meaning in logic and language have included especially:

Stock Categories of Non-Truth-Functional Meaning

1. Propositional attitude contexts (belief, doubt, fear, hope, etc.).
2. *De dicto* quotation or indirect reference contexts.
3. *De dicto* numerability contexts (numbering syntax items in distinct coreferential terms or logically equivalent sentences).
4. Modal contexts (according to Quine's *argumentum salva non veritate*).

Of course, there are many other non-truth-functional aspects of meaning if we consider the problem in its most general terms. The fact that a given term refers to a particular object is not truth-functional, and equally the assignment of truth values to the

ultimate propositional subcomponents of a truth-functionally complex well-formed formula, sometimes also called atomic wffs, is not itself a matter of truth-functional meaning. If we know that proposition p is true and that q is true, then we know truth-functionally that $\neg p$ is false, $\neg q$ is false, and $p \wedge q, p \vee q, p \rightarrow q$ are true, whereas $p \rightarrow \neg q, \neg p \wedge q, \neg p \vee \neg q$, and other truth-functionally constructed wffs are false. What is not truth-functionally determined is whether and why p is true and whether and why q is true. When we advance from propositional to predicate-quantificational logic, the situation is exactly the same. There too we can proceed truth-functionally only when we have non-truth-functionally determined truth value assignments for such basic predications as Fa, Gb , and the like, and for quantifications of the form $\forall xFx, \exists xGx$, among others, none of which is itself a truth-functional aspect of meaning. At least, these are non-truth-functional, if we agree, as most logicians have argued, that the meaning of $\forall xFx$ is not simply reducible to $Fa \wedge Fb \wedge Fc \wedge \dots$ (in lieu of the further assertion that a, b, c are *all* of the objects in the respective quantificational domain), and that $\exists xGx$ is not simply reducible to $Ga \vee Gb \vee Gc \vee \dots$ (etc.).

If we consider the possibilities for devising a formal criterion of non-truth-functionality, we soon run into surprising limitations. Such limitations take the form of distinctive counterexamples to efforts to define the concept of non-truth-functionality that appear to exhaust the range of logical analyses by which non-truth-functional contexts and operators might be defined.

2. *Universal Criterion of Non-Truth-Functionality*

Intuitively, we may think of a non-truth-functional context as intensional. The relation between non-truth-functionality and intensionality is nevertheless not as straightforward as one of logical equivalence.

∀-Criterion

$$\text{NTF}(\varphi^{\ulcorner \urcorner}) \leftrightarrow \forall S \diamond ([V(S) = T \wedge V(\varphi^{\ulcorner S \urcorner}) = F] \vee [V(S) = F \wedge V(\varphi^{\ulcorner S \urcorner}) = T])$$

This criterion initially appears very promising. It establishes a context as non-truth-functional just in case it is possible for the truth value of any sentence to be irrelevant to the truth value of the completed context when the sentence is introduced. We can see the criterion at work in collaborating applications, including standard intensional, and in this case also intentional contexts. Let $\varphi^{\ulcorner \urcorner} = \text{It is believed that } \ulcorner \urcorner$; and let $S =$ (supposedly) any sentence whatsoever, for example, ‘It is raining (at a certain place P and time t)’. Then, clearly, in this case at least, the context gratifyingly looks to be non-truth-functional according to the criterion. This, indeed, is precisely as we should hope and expect, because it is possible for the sentence to be true but for no one to believe that it is raining, or for it to be believed that it is raining when in point of fact the sentence is false.

3. *Counterexamples to the Universal Criterion*

So far so good. Unfortunately, the context is vulnerable to a family of counterexamples. Here is one suggestive, logically interesting case that in itself is not yet decisive, but that points toward a deeper difficulty in the criterion. Let $\varphi^{\ulcorner \urcorner} = \text{It is believed that } \ulcorner \urcorner$, as before. Now, however, consider as a substitution instance for

sentence S , the sentence: ‘At least one sentence is believed to be true’. Now, when we introduce the sentence into the context, we still have an intuitively intensional and hence non-truth-functional context, since nothing has changed about the context itself. This makes the left-hand side of the criterion true (that is, it is true that the context is non-truth-functional), while the right-most disjunct in the right-hand side of the criterion is false. The reason is that it is not possible for the context completed with the inserted sentence S as stipulated to be false when S is true, nor for the completed context to be true when S is false. For then we have:

It is believed that \ulcorner At least one sentence is believed to be true \urcorner .

If the sentence S to which the operator ϕ is applied is false, in this instance, then so is $\phi \ulcorner S \urcorner$. The example is not immediately conclusive, because it only rules out one of the disjuncts in the criterion’s *analysans*. When we consider the other disjunct, the situation is different, since it does not follow from the fact that at least one sentence is believed to be true that the sentence ‘At least one sentence is believed to be true’ is itself believed to be true.

However, there are further interesting consequences to consider already in connection with this first effort to identify a counterexample to the above criterion. Since the left-most disjunct (for short, $\diamond T \ulcorner F \urcorner$) within the criterion, at least under universal quantification, is rendered inoperative, as the (partial) counterexample proves, we must concentrate on the right-most disjunct ($\diamond F \ulcorner T \urcorner$), and expect it to bear all the weight of explicating a truth functional analysis of the concept of non-truth-functionality where the (partial) counterexample holds. Thus, if the criterion, relying on the right-most disjunct

is correct, then, as a matter of alethic logical necessity, it *must* logically be the case that the sentence ‘At least one sentence is believed’ is false, that is, that universal doubt or global skepticism is *logically* impossible. This, needless to say, is highly implausible. Less problematically, perhaps, but also implausibly, it must also be logically possible in that case for the sentence ‘At least one sentence is believed’ itself *not* to be (actually) believed.

If the difficulties attending the first of these requirements does not sustain the counterexample to the proposed universal criterion of non-truth-functionality, we nevertheless encounter a far more damaging criticism in the following application. What, if anything, prevents the first counterexample from completely undermining the criterion is the slim possibility that even though it is true that at least one sentence is believed, no one actually believes it; that is, that it is logically possible that no one actually believes the sentence ‘At least one sentence is believed’, or believes that the sentence is true. This possibility, however, seems in turn to presuppose that no one reflectively and doxastically responsibly considers the sentence; for, if they do, then it is impossible to imagine that as a result they would not at least dispositionally come to believe that the sentence is true.

Taking a cue from this consideration, without yet elevating it to the status of a completely decisive counterexample, let us now modify the original counterexample to produce a logically strengthened alternative. To proceed, we begin with a more precise definition of the concept of reflective doxastically-epistemically responsible judgment. It might have this content:

Definition:

For any judgment x and any proposition p , x is a *reflective doxastically-epistemically responsible considered judgment* $\leftrightarrow p$ is consciously entertained in thought, and is recognized as a result from the standpoint of a proper application of reasonable doxastic or epistemic principles that p is true, false (or has some variant truth value status).

Consider, next, a modification in which we revise the intensional (and hence intuitively non-truth-functional) operator ϕ to make it instead, ‘It is reflectively and doxastically-epistemically responsibly believed that $\lceil S \rceil$ ’. In this first application, furthermore, let us give as sentence S , the tautology (any will do), ‘ $p \rightarrow p$ ’. Then we have the completed context:

It is reflectively and doxastically-epistemically responsibly believed that $\lceil p \rightarrow p \rceil$.

In this application, we expect that since S cannot be false, it is logically impossible for it not to be reflectively and doxastically-epistemically responsibly believed. Of course, if the example has any force, it is clearly due to what we are prepared to pack into the meaning of the phrase ‘reflectively and doxastically-epistemically responsibly believed’. Building into the phrase the meaning needed to guarantee that $\phi \lceil S \rceil$ for the proposed values nevertheless does nothing whatsoever to change the fact that the context is intensional, and hence intuitively non-truth-functional. On the logically *per impossibile* assumption that the tautology $S [p \rightarrow p]$ is false, then,

classically, at least, anything whatsoever deductively follows, from which in particular, as a result, it also follows that it is false that $\phi \ulcorner S \urcorner$, that it is false that ‘It is reflectively and doxastically-epistemically responsibly believed that $\ulcorner p \rightarrow p \urcorner$ ’. Classically, of course, it *also* follows that the completed context is true; but since on that assumption we have a (classical) truth-functional paradox, we no longer have a clearcut situation in which the satisfaction of the criterion in this application guarantees the non-truth-functionality of the evidently intensional and hence intuitively non-truth-functional context.

Since the above application indicates difficulties when the criterion is put to use in testing the truth-functionality or non-truth-functionality of doxastic contexts satisfied by tautologies (and similar counterpart cases can obviously be devised involving contradictions or syntactical logical inconsistencies), let us turn to another refined type of counterexample that does not involve logical truths but contingencies, and that, as a bonus, works simultaneously to discredit both disjuncts in the original proposed truth functional criterion of non-truth-functionality. As such, it provides a model for the kind of counterexamples that defeat the universally quantified formalization of the criterion. We now have, substituting the equally intensional and thus putative non-truth-functionality of *doubt* rather than *belief*:

It is reflectively doxastically-epistemically responsibly and upon consideration doubted that \ulcorner At least one sentence has at some time been doubted \urcorner .

The above sentence represents a completed intuitively intensional, non-truth-functional linguistic context that on examination does not satisfy the requirements

stipulated in the universally quantified formulation of the truth functional criterion of non-truth-functionality under consideration.

Here is the dilemma. Suppose in the first place that it is true that at least one sentence has at some time been doubted. Then it is logically impossible, as we have weighted these concepts, for anyone to reflectively and doxastically-epistemically responsibly doubt that at least one sentence has at some time been doubted, and hence that the context, according to the criterion, counterintuitively, is not non-truth-functional. Suppose, then, secondly, that it is false that at least one sentence has at some time been doubted. It follows implausibly, then, once again, that universal doubt or global skepticism is logically excluded. Moreover, given the specifics of this revised construction, it further follows logically that it cannot be, as these concepts have been defined, reflectively and doxastically-epistemically responsibly doubted that at least one sentence has at some time been doubted. For doubting the sentence S taken as argument for the context $\phi^{\uparrow \downarrow}$, will itself constitute a sentence that according to the assumption stands under doubt. The implication, once again, in this case, is that the proposed universal truth functional criterion of non-truth-functionality fails.

Thus, the universal form of the truth functional criterion of non-truth-functionality is too strong; the *analysans* insufficiently characterizes non-truth-functionality in at least some applications, as the counterexamples show. Nor can we justifiably tinker with the statement of the criterion so that just this family of counterexamples is excluded as a threat to its integrity. Such a tactic would be objectionably *ad hoc*, because the contexts in question are patently intensional, and as such undoubtedly non-truth-functional. It is

just that the universal truth functional criterion does not rightly identify these contexts as non-truth-functional.

4. *Existential Criterion of Non-Truth-Functionality*

The next natural choice, then, is to consider a logically weaker, existentially quantified counterpart of the previous formula. We do this in the following statement:

∃-Criterion

$$\text{NTF}(\varphi^{\ulcorner \urcorner}) \leftrightarrow \exists S \diamond [[V(S) = T \wedge V(\varphi^{\ulcorner S \urcorner}) = F] \vee [V(S) = F \wedge V(\varphi^{\ulcorner S \urcorner}) = T]]$$

Here, rather than trying to satisfy a demand for universal satisfaction of truth functional independence of completed contexts and the sentences by which they are completed, we need only find a single sentence for which it is either logically possibly true under the same circumstances in which the putatively non-truth-functional context completed by the insertion of the sentence is false, or logically possibly false under the same circumstances in which the putatively non-truth-functional context completed by the insertion of the sentence is true. The mere fact that there is a category of intensional context counterexamples does nothing to gainsay the availability of there being at least one such qualifying sentence for any non-truth-functional context.

5. *Counterexamples to the Existential Criterion*

The trouble with the existentially revised form of the criterion is even more glaring and relatively easier to demonstrate. Where the context to be tested as before is $\varphi^{\ulcorner \urcorner}$, now, for *any* sentence S , we encounter a fatal difficulty if only we make the operator φ within the context propositional negation, \neg , so that $\forall S[\varphi^{\ulcorner S \urcorner} \leftrightarrow \neg S]$. Clearly, here

context $\varphi^{\ulcorner \urcorner}$ is truth functional rather than non-truth-functional. A range of similar counterexamples can also be adduced for other propositional operators. Since propositional operations are one and all truth functional, they defeat not only the existential but also the original universal formalization of the criterion.

Again, we cannot simply exclude propositional connectives as appropriate substituends for φ in $\varphi^{\ulcorner \urcorner}$. To do so would not only be objectionably *ad hoc*, but, more importantly, it would amount to stipulating that only (if not all and only) a certain filter of contexts is non-truth-functional provided it does not contain any truth functional contexts. This in turn is tantamount to saying that a context is non-truth-functional just in case it is non-truth-functional. To legislate such an exclusionary prohibition on the attempt to advance a truth functional criterion of non-truth-functionality as a consequence in addition to being objectionably *ad hoc* would therefore be viciously circular.

As a final question, let us consider the problem of whether there is any point to offering the above detailed discussion of counterexamples to the universal formalization of the truth functional criterion of non-truth-functionality when the propositional operator counterexamples to the existential formalization also serve as counterexamples to the universal formalization. The main reason is that we need to be suspicious from the outset of the logically weaker merely existential formalization as an adequate criterion of non-truth-functionality. For it does not seem sufficiently general to consider a context as non-truth-functional if there is only a limited number of sentences, possibly no more than one, whose truth value is logically independent of the truth value of the completed contexts prescribed by the criterion completed by inserting the sentence. Thus, we see that for an intuitively non-truth-functional context that is decidedly not dependent on the

interpretation of ϕ as a propositional operator, we get a negative result when applying the criterion to a particular highly limited choice of sentences.

Here in the process we discover another interesting family of counterexamples to both the universal and existential formalizations of the truth functional criterion of non-truth-functionality. For this class of counterexamples, let $\phi =$ 'The number designated in the attached sentence $\lceil \cdot \rceil$ is π '. Then we consider the following pool of sentences S to insert within and thereby complete the context: (S1) ' π is a rational number'; (S2) '2 is a rational number'; (S3) ' π is an irrational number'; (S4) '2 is an irrational number'; etc. Now we observe that although the operator ϕ is not intuitively non-truth-functional, (S1) is necessarily false, whereas the completed context $\phi \lceil (S1) \rceil$ is true; (S2) is necessarily true, but $\phi \lceil (S2) \rceil$ is false; (S3) is necessarily true and $\phi \lceil (S3) \rceil$ is true; (S4) is necessarily false, and $\phi \lceil (S4) \rceil$ is false. Additionally, for all the indefinitely many sentences that do not contain *any* number, whether π or another, the application of the criterion gives no useful result whatsoever.

What we learn from this set of cases is that the mere fact that there are *some* sentences that satisfy the truth value independence test for contexts completed by those sentences is insufficient to properly support the judgment that the context in question is non-truth-functional, so that the existential formalization of the criterion is an unreliable measure of whether or not the context is non-truth-functional.

Consider, then, an intensional and hence again intuitively presumably non-truth-functional context that is satisfied by some but not all sentences, again discrediting the merely existential formalization of the criterion while providing another interesting

category of counterexamples to both existentially and universally quantified versions. The following intuitively intensional and presumably non-truth-functional context (although for purposes of the present argument it does not matter whether the context is actually truth functional), is now adduced. We let $\phi =$ If reflectively, doxastically-epistemically responsibly considered at all, it is believed that the number designated in the attached sentence $\lceil \cdot \rceil$ is a number. As sentence substituends, consider these representative possibilities: (S5) ‘3 is an even number’; (S6) ‘3 is an odd number’. Now we discover that (S5) is necessarily false, but $\phi \lceil (S5) \rceil$ is necessarily true, while (S6) is necessarily true, and the completed context $\phi \lceil (S6) \rceil$ is still necessarily true.

The immediate moral of the story is that trying to implement a truth functional criterion of non-truth-functionality by reference to only some rather than all sentence substituends does not give reliable results. The context is either truth functional or non-truth-functional. The truth functional criterion of non-truth-functionality existentially formulated and implemented by means of inserting the above choice of sentences supports both evaluations that the context is (sometimes) truth functional and that it is (sometimes) non-truth-functional, depending on the sentence that is inserted. Thus, we arrive at the conclusion from another direction that we have yet to identify an adequate truth functional criterion of non-truth-functionality, universally or existentially formalized.

6. Implications and Limitations of the Counterexamples

The inference to be drawn from all of these challenges to truth functional criteria of non-truth-functionality needs to be carefully articulated. We must take special pains to

avoid exaggeration and misinterpretation. We have not yet arrived at the position where we can conclude that there absolutely can be no truth functional characterization of non-truth-functionality.

We have, after all, offered no closure argument to show that we have exhausted all possible formalizations of truth functional criteria. The counterexamples we have presented to the most obvious truth functional formulations nevertheless establish the difficulty of identifying an adequate truth functional criterion, and casts doubt where previously it might have been thought painless and theoretically unproblematic to characterize non-truth-functionality truth functionally. The challenge that emerges from this consideration of counterexamples is to devise a truth functional criterion that avoids all variations of the clusters of counterexamples we have seen arise with respect to the most likely truth functional formalizations in several variations, with the prediction that a satisfactory solution to the problems posed by these and kindred counterexamples may finally be impossible to achieve.

If that is true, in turn, moreover, then, with the same cautionary qualification, another important conclusion follows. For not only is there a strong presumption in that event that it will be impossible even in principle for there to be an adequate truth functional criterion of non-truth-functionality. It will additionally, and perhaps more surprisingly, follow by complementarity of conceptual analysis that it is impossible, in that case, even in principle, for there to be an adequate truth functional criterion of truth functionality.