Kidney Exchange: The Impact of Large Exchanges, Altruistic Donation, and Center Incentives

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2007-2008 Barcelona JOCS Seminar on
Game Theory and its Applications
October 29th, 2007

Kidney Transplants

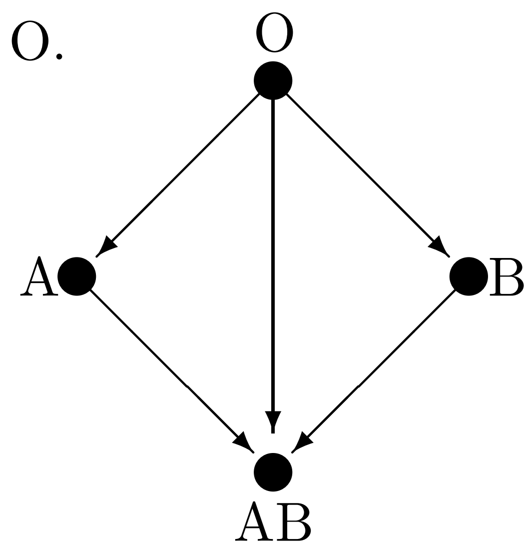
- There are over 70,000 patients on the waiting list for cadaver kidneys in the U.S.
- In 2003 there were over 8,600 transplants of cadaver kidneys performed in the U.S.
- In the same year, about 3,400 patients died while on the waiting list.
- In 2003 there were also over 6,400 transplants of kidneys from *living* donors, a number that has been increasing steadily from year to year.

Institutional Constraint: No Money

- The shortage of kidney increases by about 3,500 kidneys each year in the U.S.
- In U.S., National Organ Transplant Act (NOTA) of 1984 prohibits "any person to knowingly acquire, receive, or otherwise transfer any human organ for valuable consideration for use in human transplantation."
- There is a rich literature on whether the ban on buying and selling of kidneys be repealed (ex: Becker & Elias 2007).

Medical Constraint: Blood Type Compatibility

- There are four blood types: A, B, AB and O.
- In the absence of other complications:



  * Type O kidneys can be transplanted into any patient;
  * type A kidneys can be transplanted into type A or type AB patients;
  * type B kidneys can be transplanted into type B or type AB patients; and
  * type AB kidneys can only be transplanted into type AB patients.

Medical Constraint: Tissue Type Compatibility

- Tissue type or HLA type: Combination of six proteins.
- Prior to transplantation, the potential recipient is tested for the presence of preformed antibodies against donor HLA.
  If present (*positive crossmatch*), the transplantation cannot be carried out.

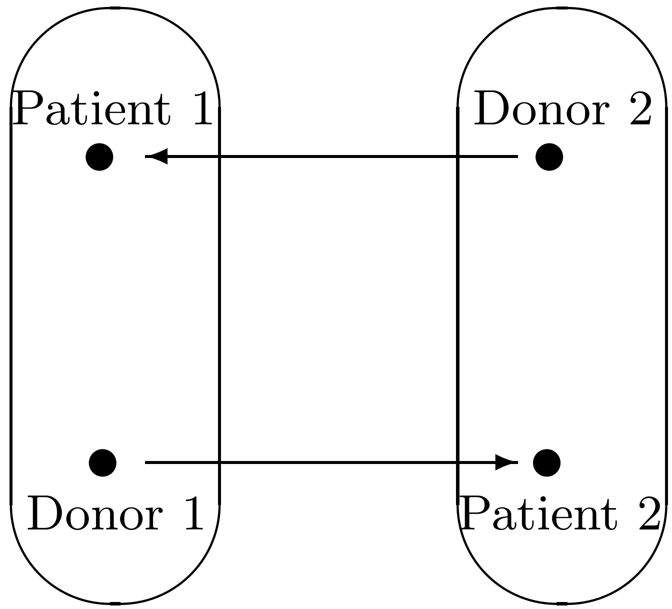
Deceased Donor Kidneys

- U.S. Congress views deceased donor kidneys offered for transplantation as a national resource, and the National Organ Transplant Act of 1984 established the Organ Procurement and Transplantation Network (OPTN).
- Run by the United Network for Organ Sharing (UNOS), it has developed a centralized priority mechanism for the allocation of deceased donor kidneys.

Live Donor Transplants:
Until Recently Much Less Organized

- A patient identifies a willing donor and, if the transplant is feasible, it is carried out.
- Otherwise, the patient remains on the queue for a deceased donor kidney, while the donor returns home.
- In the period 2000-2004, additional possibilities have been utilized in a small number of cases through exchanges between two incompatible couples.

Paired Exchange



- Until recently very rare: In the period 2000-2004 five paired exchanges in New England.
- In 2000 the transplantation community issued a *consensus statement* indicating it as "ethically acceptable."
- *Incentives Constraint*: All four operations shall be carried out simultaneously!

Recent Developments

- Roth, Sönmez, Ünver (*QJE* 2004, *JET* 2005): Analyses kidney exchange as a market/mechanism design problem.
- Our collaboration with Drs Frank Delmonico and Susan Saidman lead to the establishment of New England Program for Kidney Exchange (NEPKE) in 2005.
- Other kidney exchange programs (Allience for Paired Donation, Johns Hopkins University Kidney Paired Donation Program) are launched and following the example of NEPKE they adopted an "optimal allocation" methodology.
- In 2007 a bill that amends NOTA to clarify that "kidney exchange shall not be considered to involve the transfer of a human organ for valuable consideration has passed both in the House and in the Senate. Currently there is a movement through national exchange in the U.S.

Altruistic Kidney Exchange

Tayfun Sönmez, Utku Ünver

I: the set of incompatible pairs

C: the set of compatible pairs

M^{μ}: those who are matched under matching μ

T^{μ}: pairs who receive a transplant under matching μ

Feasible Exchanges & Preferences

1. An **ordinary two-way exchange**: This is an exchange between two incompatible pairs that are mutually compatible.
2. An **altruistically unbalanced two-way exchange**: This is an exchange between an incompatible and a compatible pair that are mutually compatible.

Assumption: Patients are indifferent between all compatible kidneys.

Maximality and Pareto Efficiency

The following result extends a well-known result in discrete optimization literature.

Proposition 1: For any two Pareto-efficient matchings μ, ν, we have $|T^{\mu}| = |T^{\nu}|$.

Moreover, the choice of incompatible pairs and compatible pairs can be completely separated.

Proposition 2: Let μ, ν be two Pareto-efficient matchings such that $M^{\mu} \cap C = A$ and $M^{\nu} \cap I = J$. Then there exists a Pareto-efficient matching η such that $M^{\eta} = A \cup J$.

Corollary: Let μ be a Pareto efficient matching such that $M^{\mu} \cap I = J$. Then there exists a Pareto efficient matching η be such that $M^{\eta} \cap I = J$ and $|M^{\eta} \cap C| \leq |M^{\nu} \cap C|$ for any Pareto efficient matching ν.

Efficient Kidney Exchange: Coincidence of Wants in Markets with Compatibility-Based Preferences

Alvin Roth, Tayfun Sönmez, Utku Ünver, *AER* 2007

Simulations

Assuming blood-unrelated pairs:

# Pairs	2-way	2&3-way	2&3&4-way	Unrestricted
25	8.86	11.272	11.824	11.992
50	21.792	27.266	27.986	28.09
100	49.708	59.714	60.354	60.39

Example 1. Consider a population of 7 incompatible patient-donor pairs consisting of:

* O-A, O-B (difficult to match O patients)
* A-B, A-B, B-A (more A-B than B-A pairs)
* A-A (an odd number of A-A pairs)
* B-O (scarce O donor)

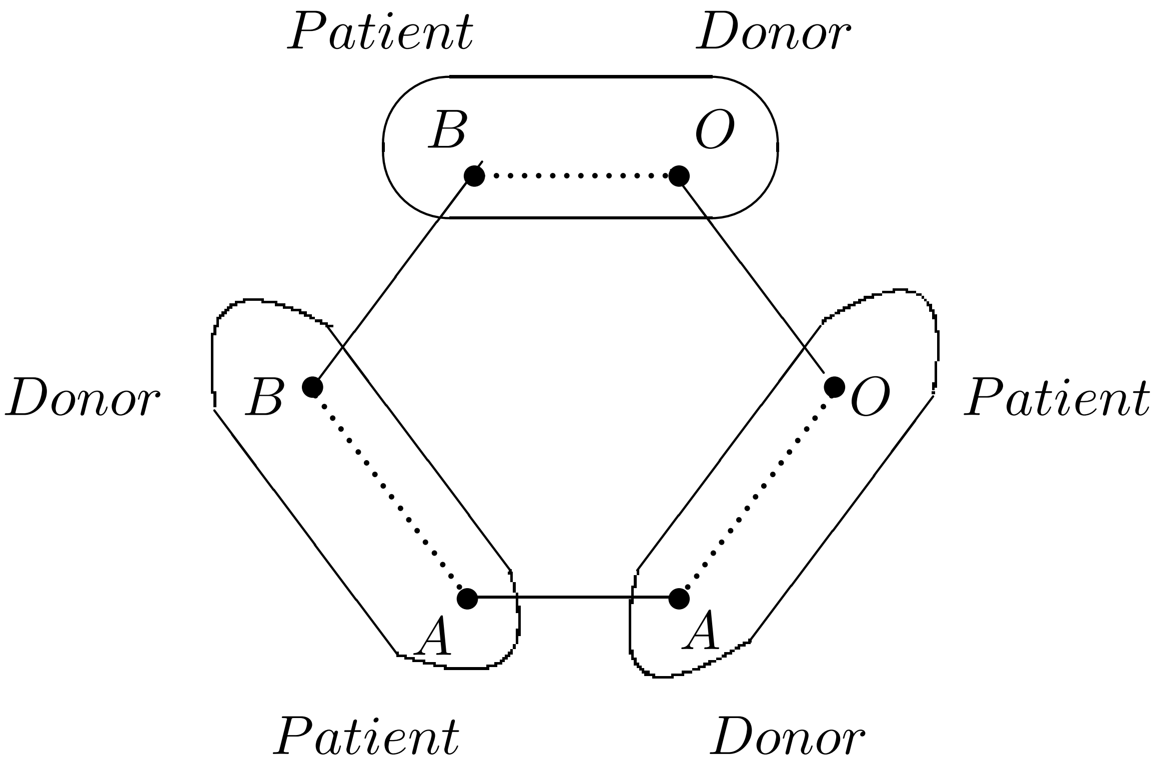
Suppose patients are indifferent between all compatible donors and they are not tissue-type incompatible with other donors.

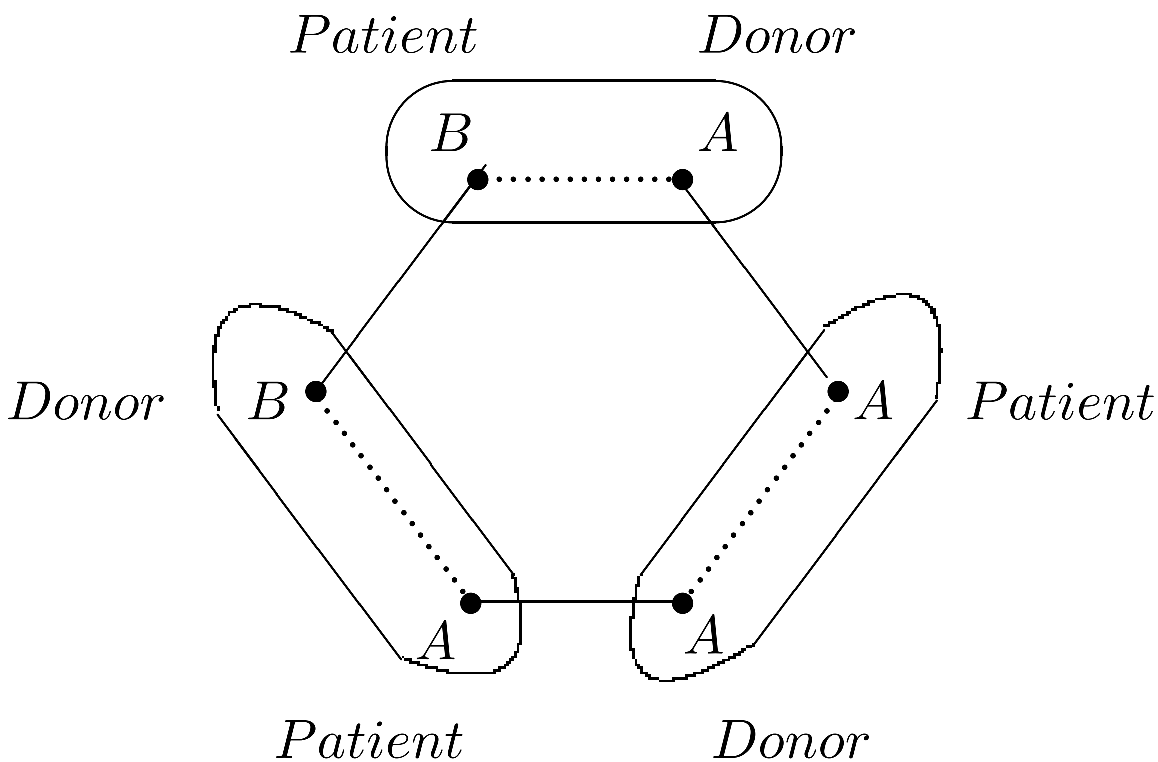
- If only two-way exchanges are possible: 4 transplants

$$(B-O, O-B); (B-A, A-B)$$

- If three-way exchanges are feasible as well: 6 transplants

$$(B-O, O-A, A-B); (B-A, A-A, A-B)$$


Patient
Donor
B
O
Donor
B
O
Patient
A
A
Patient
Donor


Patient
Donor
B
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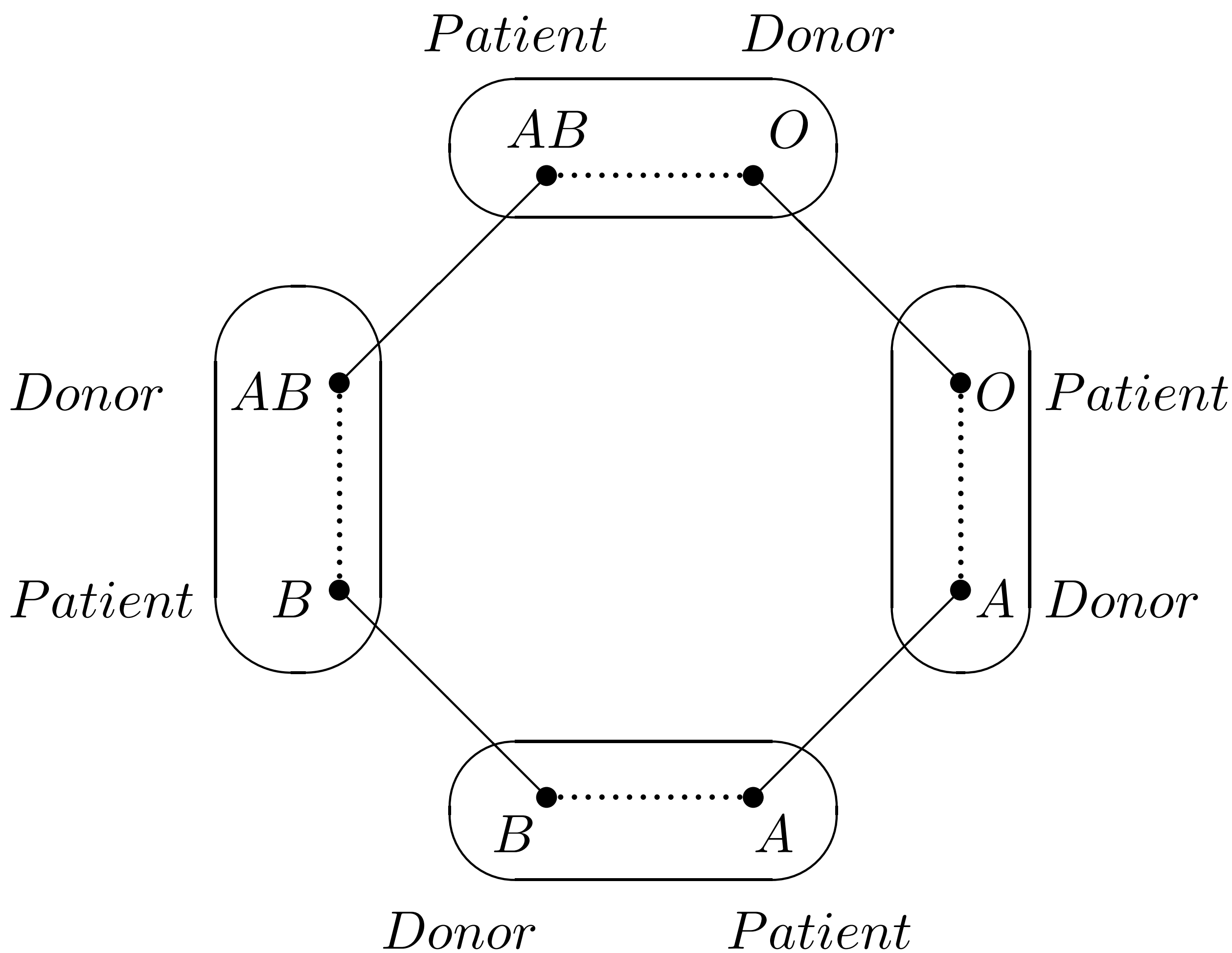
4-Way Exchanges Add Much Less



Assumptions

***Assumption 1* (Upper bound assumption).** No patient is tissue-type incompatible with another patient's donor.

***Assumption 2* (Large population assumption).** Regardless of the maximum number of pairs allowed in each exchange, pairs of types O-A, O-B, O-AB, A-AB, and B-AB are on the "long side" of the exchange in the sense that at least one pair of each type remains unmatched in each feasible set of exchanges.

***Assumption 3*.** #(A-B) > #(B-A).

***Assumption 4*.** There is either no type A-A pair or there are at least two of them. The same is also true for each of the types B-B, AB-AB, and O-O.

Results

Proposition 3: For any patient population obeying Assumptions 1 and 2, the maximum number of patients who can be matched with only two-way exchanges is:

$$
\begin{aligned}
&2\,(\#(\text{A-O}) + \#(\text{B-O}) + \#(\text{AB-O}) + \#(\text{AB-A}) + \#(\text{AB-B})) \\
&\quad + (\#(\text{A-B}) + \#(\text{B-A}) - |\#(\text{A-B}) - \#(\text{B-A})|) \\
&\quad +2\left(\left\lfloor \frac{\#(\text{A-A})}{2} \right\rfloor + \left\lfloor \frac{\#(\text{B-B})}{2} \right\rfloor + \left\lfloor \frac{\#(\text{O-O})}{2} \right\rfloor + \left\lfloor \frac{\#(\text{AB-AB})}{2} \right\rfloor \right)
\end{aligned}
$$

Proposition 4: For any patient population for which Assumptions 1-4 hold, the maximum number of patients who can be matched with two-way and three-way exchanges is:

$$
\begin{aligned}
&2\left(\#(\text{A-O}) + \#(\text{B-O}) + \#(\text{AB-O}) + \#(\text{AB-A}) + \#(\text{AB-B})\right)\\
&\quad + \left(\#(\text{A-B}) + \#(\text{B-A}) - |\#(\text{A-B}) - \#(\text{B-A})|\right)\\
&\quad + \left(\#(\text{A-A}) + \#(\text{B-B}) + \#(\text{O-O}) + \#(\text{AB-AB})\right)\\
&\quad + \min\{(\#(\text{A-B}) - \#(\text{B-A})), (\#(\text{B-O}) + \#(\text{AB-A}))\}\\
&\quad + \#(\text{AB-O})
\end{aligned}
$$

To summarize, the marginal effect of three-way kidney exchanges is:

$$
\begin{aligned}
&\#(\text{A-A}) + \#(\text{B-B}) + \#(\text{O-O}) + \#(\text{AB-AB})\\
&\quad -2\left(\left\lfloor\frac{\#(\text{A-A})}{2}\right\rfloor + \left\lfloor\frac{\#(\text{B-B})}{2}\right\rfloor + \left\lfloor\frac{\#(\text{O-O})}{2}\right\rfloor + \left\lfloor\frac{\#(\text{AB-AB})}{2}\right\rfloor\right)\\
&\quad + \min\{(\#(\text{A-B}) - \#(\text{B-A})), (\#(\text{B-O}) + \#(\text{AB-A}))\} + \#(\text{AB-O})
\end{aligned}
$$

Proposition 5: For any patient population in which Assumptions 1-4 hold, the maximum number of patients who can be matched with two-way, three-way and four-way exchanges is:

$$
\begin{aligned}
2\,&(\#(\text{A-O}) + \#(\text{B-O}) + \#(\text{AB-O}) + \#(\text{AB-A}) + \#(\text{AB-B})) \\
&+ (\#(\text{A-B}) + \#(\text{B-A}) - |\#(\text{A-B}) - \#(\text{B-A})|) \\
&+ (\#(\text{A-A}) + \#(\text{B-B}) + \#(\text{O-O}) + \#(\text{AB-AB})) \\
&+ \min\{(\#(\text{A-B}) - \#(\text{B-A})), (\#(\text{B-O}) + \#(\text{AB-A}) + \#(\text{AB-O}))\} \\
&+ \#(\text{AB-O})
\end{aligned}
$$

Remark: In the absence of tissue-type incompatibilities between patients and other patients' donors, the marginal effect of four-way kidney exchanges is bounded above by the rate of the very rare AB-O type.

A general model of type-specific exchange

There are n types of agents and each pair i consists of a receiving agent P_i and a donating agent D_i.

$\mathcal{B}$: The set of agent types

Compatibility: Given two agent types X,Y $\in \mathcal{B}$,

$$\text{X} \succeq \text{Y}$$

is read as X is *compatible* with Y and it means a donating agent of type X can be matched with a receiving agent of type Y.

Assumption: Compatibility relation $\succeq$ satisfies:

1. $\text{X} \succeq \text{X}$ for any X $\in \mathcal{B}$ (**reflexivity**)
2. $\text{X} \succeq \text{Y}$ and $\text{X} \neq \text{Y} \Rightarrow \text{Y} \not\succeq \text{X}$ for any X,Y $\in \mathcal{B}$ (**asymmetry**)
3. $\text{X} \succeq \text{Y}$ and $\text{Y} \succeq \text{Z} \Rightarrow \text{X} \succeq \text{Z}$ (**transitivity**)

Generalization of Earlier Assumptions

Assumption 2'. Let X,Y $\in \mathcal{B}$ be such that X $\succeq$ Y and X $\neq$ Y. Regardless of the maximum number of pairs allowed in each exchange, pairs of type X-Y are on the "long side" of the exchange in the sense that at least one pair of type X-Y remains unmatched in each matching.

Assumption 4'. For any X $\in \mathcal{B}$, there is either no type X-X pair or there are at least two of them.

Theorem 1 (n-way exchange suffices): Suppose the compatibility relation satisfies reflexivity, asymmetry, and transitivity. Consider a population for which Assumptions 2' and 4' hold and let μ be any maximal matching (when there is no restriction on the size of the exchanges that can be included in a matching). Then there exists a maximal matching ν which consists only of exchanges involving at most n pairs, under which the same set of pairs are matched as in μ.

Corollary (4-way exchange suffices for kidney exchange): Consider a patient population for which Assumptions 1, 2, 4 hold and let μ be any maximal matching (when there is no restriction on the size of the exchanges that can be included in a matching). Then there exists a maximal matching ν which consists only of two-way, three-way, and four-way exchanges, under which the same set of patients benefit from exchange as in matching μ.

Center Incentives in Kidney Exchange

Alvin Roth, Tayfun Sönmez, Utku Ünver

t: A generic transplant center

I_t: the set of incompatible pairs at center t

Each center can submit it's pairs to a centralized clearinghouse (such as a regional kidney exchange program) or can match them within the center. The latter will result in efficiency loss.

Assumption: Each center wants to maximize the number of own patients matched.

Individual Rationality

- A mechanism is **individually rational** if each center always receives as many transplants as it would receive by itself.
- A **priority mechanism**: For a given priority ordering of pairs, match the first pair (if you can), subject to that constraint match the second pair (if you can), and so on so forth.

  A priority mechanism is not individually rational but it can be easily modified to satisfy this mild requirement through endogenizing the priority ordering.
- π: a priority ordering π of all incompatible pairs

  π_t: the restriction of π to incompatible pairs in center t
- $\tilde{I}_t$: the set of incompatible pairs who are matched via the priority mechanism induced by π_t when only pairs in I_t participate in exchange

  $\tilde{I} = \cup_{t \in T} \tilde{I}_t$

- Modified priority ordering $\tilde{\pi}$:
  1. Any incompatible pair in $\tilde{I}$ has higher priority under $\tilde{\pi}$ than any incompatible pair in $I \setminus \tilde{I}$,
  2. the relative priority ordering among incompatible pairs in $\tilde{I}$ under $\tilde{\pi}$ is the same as their relative priority ordering under π, and
  3. the relative priority ordering among incompatible pairs in $I \setminus \tilde{I}$ under $\tilde{\pi}$ is the same as their relative priority ordering under π,

  This uniquely determines a priority ordering.

Proposition 6: The priority mechanism induced by the endogenous priority ordering $\tilde{\pi}$ is individually rational.

Can Full Participation be Guaranteed?

Unfortunately No!

Proposition 7: There exists no Pareto efficient mechanism where full participation is always a dominant strategy for each transplant center.

***Example*:** There are two transplant centers A, B, three incompatible pairs $a_1, a_2, a_3 \in I_A$ in center A, and four incompatible pairs $b_1, b_2, b_3, b_4 \in I_B$ in center B.

List of feasible exchanges: (a_1, a_2), (a_1, b_1), (a_2, b_1), (a_3, b_3), (b_1, b_2), (b_2, b_3).

There are four Pareto efficient matchings. Whichever one is chosen, one of the centers has an incentive to keep some of its pairs to be matched within its center:

1. $\mu_1 = \{(a_1, a_2), (b_1, b_2), (a_3, b_3)\}$: If B keeps pairs b_2, b_3 to be matched within the center, the only Pareto efficient matching of the remaining economy is $\nu = \{(a_1, b_1)(a_2, b_4)\}$.
2. $\mu_2 = \{(a_1, b_1), (a_2, b_4), (a_3, b_3)\}$: If B keeps pairs b_2, b_3 to be matched within the center, the only Pareto efficient matching of the remaining economy is $\nu = \{(a_1, b_1)(a_2, b_4)\}$.
3. $\mu_3 = \{(a_1, b_1), (a_2, b_4), (b_2, b_3)\}$: If A keeps pairs a_1, a_2 to be matched within the center, the only Pareto efficient matching of the remaining economy is $\eta = \{(a_3, b_3)(b_1, b_2)\}$.
4. $\mu_4 = \{(a_2, b_4), (a_3, b_3), (b_1, b_2)\}$: If A keeps pairs a_1, a_2 to be matched within the center, the only Pareto efficient matching of the remaining economy is $\eta = \{(a_3, b_3)(b_1, b_2)\}$.

Conclusion

- Opportunity to use tools of market/mechanism design in the health sector.
- In the present context not only economic theory provides guidence to solve an important real-life problem, but also the real-life problem helps advancing economic theory.