

BARCELONA MODNET CONFERENCE  
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ABSTRACTS OF TALKS

HANS ADLER

Strict orders prohibit elimination of imaginaries

**Abstract:** Shelah defined a hierarchy of strong order properties that are in strength between the strict order property and the tree property. Theories with the strict order property cannot eliminate a certain infinitary hyperimaginary. On the other hand, a certain amalgamation condition that is consistent with the strong order property implies elimination of hyperimaginaries.

ITAÏ BEN YAACOV

On a general notion of lovely pairs

**Abstract:** The notion of *beautiful pairs* of models of a stable theory was defined by Poizat. It was extended to simple theories, yielding the notion of *lovely pairs*, by Vassiliev in the rank 1 case, and then by Pillay, Vassiliev and the speaker in the general case. An analogous notion of lovely pairs with respect to algebraic independence makes sense for general geometric structures, and turns out to coincide with *dense pairs* for o-minimal theories (van den Dries, followed by Dolich, Berenstein, Onshuus, Gunaydin, Vassiliev and possibly others).

In this talk I shall describe a general definition of lovely pairs with respect to any notion of independence which satisfies sufficiently many axioms, e.g., non forking in stable or simple theories, non thorn forking in rosy theories, algebraic independence in geometric structures. As such it encompasses all familiar notions (at least to the speaker) of lovely pairs. Somewhat surprisingly, the notion of independence does *not* have to be symmetric, so one can speak, for example, of lovely pairs of models of a dependent theory with respect to non forking.

I shall describe what is known, and ask many questions.

FRANCK BENOIST

Differential schemes for rings with a Hasse derivation

**Abstract:** Because of the quantifier elimination for existentially closed Hasse fields, type-definable sets are given, up to boolean combination, as zero sets of differential polynomials. As for ACF, it may be fruitful to consider geometric objects given by schemes rather than by naive algebraic sets. I will focus on some difficulties arising

when we try to develop a theory of schemes for Hasse rings and give some partial results to avoid them.

### ALEXANDRE BOROVIK

#### Generic automorphisms of simple groups of finite Morley rank

**Abstract:** I will discuss an approach – suggested some time ago by Udi Hrushovski – to a possible classification of simple groups of finite Morley rank via a study of groups of fixed points of their generic automorphisms. I will outline some results in that direction (they belong mostly to Pinar Ugurlu).

### ZOE CHATZIDAKIS

#### To be announced

### RAF CLUCKERS

#### Lipschitz continuity properties for $p$ -adic definable functions

**Abstract:** In general, a function  $f$  between normed spaces is said to be Lipschitz continuous with constant  $C$  when for every  $x, y$  in the domain one has

$$|f(x) - f(y)| \leq C|x - y|.$$

On the reals, K. Kurdyka proved that a real subanalytic function with bounded partial derivatives is piecewise Lipschitz continuous with some constant  $C$ . Most of the techniques used by Kurdyka break down in the  $p$ -adics, like paths, path integrals, the idea of the length of a path, the description of the length as an integral, the mean value theorem, and so on, in part by the total discontinuity of the  $p$ -adics. On the other hand, definable functions in the  $p$ -adics are of course defined using only a finite amount of data, hence there can be no behaviour of oscillation or of infinitely many “jumps”, and thus, intuitively, Kurdyka’s result must hold for the  $p$ -adics too. We will discuss the  $p$ -adic situation.

### ADRIEN DELORO

#### Small groups of odd type

(Joint work with Eric Jaligot)

**Abstract:** According to the Cherlin-Zilber conjecture, infinite simple groups of finite Morley rank should be algebraic. As bad groups of Morley rank 3 - the worst kind of possible counter-examples - are not known not to exist, one sees that a proof of the algebraicity conjecture cannot rely on a naive induction on the Morley rank: the case  $rk = 3$  is still open! Hence the concept of smallness has to be revised, using another notion than the Morley rank. In the algebraic category, the Lie rank is a good indication of the size. It turns out that the only simple algebraic group of Lie rank 1 is  $PSL_2$ . Several identification theorems for  $PSL_2$  among groups of finite Morley rank have been proved since the beginning of the story; we shall recast the main ideas and expose the last avatar of a recognition argument first used by Nesin. As the Lie rank does not generalize to abstract groups of finite Morley rank, our notion of smallness is local solvability, in the following sense. A

group is locally solvable if normalizers of non-trivial, definable, connected, solvable subgroups are still solvable. As many good things, this concept is borrowed from finite group theory. Theorem. Let  $G$  be a locally solvable non-solvable connected group of finite Morley. Assume that the Sylow 2-subgroups of  $G$  are toral-by-finite, and that the centralizers of involutions are solvable. Then  $G$  is isomorphic to  $PSL_2$  over an algebraically closed field of characteristic not 2, or to something else - but we have a clear idea.

### LOU VAN DEN DRIES

#### O-minimal triangulation respecting a standard part map

(report on joint work with J. Maříková)

**Abstract:** Let  $R$  be an o-minimal field and  $V$  a proper convex subring of  $R$  with ordered residue field  $\mathbf{k}$  and standard part map (residue map)  $st : V \rightarrow \mathbf{k}$ . Let  $\mathbf{k}_{\text{ind}}$  be the expansion of  $\mathbf{k}$  by the relations  $st(X) \subseteq \mathbf{k}^n$  where  $X \subseteq R^n$  is definable in  $R$ . When is  $\mathbf{k}_{\text{ind}}$  o-minimal? In her thesis Jana Maříková answers this question positively if  $(R, V)$  satisfies a certain first order scheme  $\Sigma$ . More precisely: *if  $(R, V) \models \Sigma$ , then the definable relations of  $\mathbf{k}_{\text{ind}}$  are exactly the boolean combinations of the basic relations  $st(X)$ .*

In subsequent joint work with Maříková we proved the converse of the italicized statement. The main tool is a triangulation result respecting the standard part map that seems to be new even in the semialgebraic case.

### PANTELIS E. ELEFThERIOU

#### A semi-linear group which is not affine

**Abstract:** We present an example of a group  $G$  definable in an ordered divisible abelian group  $\mathcal{M} = \langle M, <, +, 0 \rangle$  which does not admit an affine embedding; in other words, there is no  $\mathcal{M}$ -definable isomorphism between  $G$  and any other  $\mathcal{M}$ -definable group  $K$  as topological groups, such that the topology on  $K$  is the subspace one.

### DAVID EVANS

#### Combinatorial geometries of the Hrushovski constructions

**Abstract:** This talk is about the strongly minimal sets constructed by Hrushovski in his 1993 paper 'A new strongly minimal set'. In particular, I will discuss the PhD work of Marco Ferreira on the combinatorial geometry given by algebraic closure in these structures.

### OLIVIER FRÉCON

#### Algebraic groups and abstract isomorphisms

**Abstract:** We consider two connected affine algebraic groups  $G$  and  $H$  over an algebraically closed field  $K$ . We discuss the consequences of the existence of an abstract isomorphism from  $G$  to  $H$ .

This talk will be in French with slides in English.

### GRIGORY GARKUSHA

#### Classifying finite localizations of quasi-coherent sheaves

**Abstract:** Given a quasi-compact, quasi-separated scheme  $X$ , we classify the tensor localizing subcategories of finite type of quasi-coherent sheaves  $Qcoh(X)$  in terms of topology which is dual to the Zariski topology on  $X$ . This topology is closely related to the Ziegler topology on the injective spectrum. As an application, we reconstruct  $X$  from  $Qcoh(X)$ . Also, a bijective correspondence between the tensor thick subcategories of perfect complexes on  $X$  and the tensor localizing subcategories of finite type in  $Qcoh(X)$  is established.

### IMMANUEL HALUPCZOK

#### Trees of definable sets in the $p$ -adic integers

**Abstract:** Let  $X \subset \mathbb{Z}_p^n$  be a definable set in the  $p$ -adic integers, and write  $X_\lambda := \{a + p^\lambda \mathbb{Z}_p^n \mid a \in X\}$  for the set of balls of “radius”  $\lambda$  intersecting  $X$ . There is a natural tree structure on  $\bigcup_\lambda X_\lambda$  given by the inclusion of balls. (The root of the tree is the whole ball  $\mathbb{Z}_p^n$ .)

The trees one obtains in this way are far from arbitrary. For example, there is an old result of Denef relating the number  $\#X_\lambda$  of nodes at different depths. The goal of this talk is to present a conjecture which gives a precise combinatorial description of the class of trees which one can obtain from a definable set.

One reason these trees are interesting is that they exactly recall the definable set up to isometry: isometries between definable sets induce isomorphisms of trees. From the conjectural description of the trees one can obtain a decomposition of the original definable set  $X$  into simpler sets, which is well adapted to the  $p$ -adic measure. This might be useful, for example, for  $p$ -adic (and motivic) integration.

Another reason that the trees are interesting is the following: Suppose  $X$  is a variety. Then close to any non-singular  $x \in X$ , the tree will just be isomorphic to the tree of  $\mathbb{A}^{\dim X}$ . On the other hand, if  $x$  is a singular point, then the tree close to  $x$  can get much more complicated. Thus the trees might provide a new understanding of singularities.

### GARETH JONES

#### Model completeness for certain Pfaffian structures

**Abstract:** I show that the expansion of the real field by a total Pfaffian chain is model complete in a language with symbols for the functions in the chain, the exponential and all real constants. In particular, the expansion of the reals by all total Pfaffian functions is model complete.

**JONATHAN KIRBY****Raising to generic powers**

(Joint work with A.J.Wilkie and Martin Bays)

**Abstract:** We prove unconditionally a Schanuel property for raising to a generic real power, leading to the hope that the real field with a generic power function can be proved to be decidable.

**SONIA L'INNOCENTE****Possible exponentiations over the universal enveloping algebra of  $\mathfrak{sl}_2(\mathbb{C})$** 

(Joint work with Angus Macintyre and Françoise Point [1])

**Abstract:** This report aims at defining some possible exponentiations over the universal enveloping algebra, denoted by  $U$ , of the Lie algebra  $\mathfrak{sl}_2(\mathbb{C})$  of the traceless  $2 \times 2$  matrices with the coefficients in the field of complex numbers  $\mathbb{C}$ . By using the universal property of  $U$  and the characterization of any simple representation  $V_\lambda$  of  $\mathfrak{sl}_2(\mathbb{C})$  of dimension  $\lambda + 1$  (with  $\lambda \in \mathbb{N} - \{0\}$ ), we investigate *exponential maps* indexed by  $\lambda$  from  $U$  to the special linear group  $GL_{\lambda+1}(\mathbb{C})$ . We describe some of the properties of these maps, for instance their kernels. Then, we define an exponential map  $\text{EXP}$  from  $U$  to any non-principal ultraproduct  $\prod_{\mathcal{V}} GL_{\lambda+1}(\mathbb{C})$  of the groups  $GL_{\lambda+1}(\mathbb{C})$ , where  $\mathcal{V}$  is a non principal ultrafilter on  $\mathbb{N}$  and we will investigate some of its properties, for instance its kernel. Then, by a suitable norm, we show that this exponential map  $\text{EXP}$  is continuous and that the subgroup generated by  $\text{EXP}(U)$ ,  $\langle \text{EXP}(U) \rangle$ , is a topological group. Then, we construct an asymptotic cone, a metric space with a distance, into which we embed  $U$  and we show that  $U$  has a faithful action on that space.

## REFERENCES

- [1] S. L'Innocente, A. Macintyre, F. Point, Possible exponentiations over the universal enveloping algebra of  $\mathfrak{sl}_2\mathbb{C}$ , *Preprint*.

**FRANÇOIS LOESER****Topological properties of spaces of stably dominated types in ACVF**

**Abstract:** We shall present some recent results with E. Hrushovski on the topology of spaces of stably dominated types in ACVF. The spaces are closely connected to Berkovich spaces and we shall put emphasis on their tameness.

**ANGUS MACINTYRE****Algebraic properties of Zilber's exponential**

**Abstract:** I will survey what is known on the algebraic properties of Zilber's exponential, in particular outlining the proof (jointly with Paola D'Aquino and Giuseppina Terzo) that it satisfies the Schanuel Nullstellensatz. For the complex field, this property was proved thirty years ago by Henson and Rubel using deep

complex analysis, a method not available in Zilber's fields.

**GUILLAUME MALOD**  
**Complete families for circuit classes**

**Abstract:** We construct a complete polynomial family for VP, a complexity class characterized by arithmetic circuits of polynomial size and degree. Although very simple, this construction yields a first explicit family. We also study the completeness of other polynomial families for the arithmetic classes.

**AMADOR MARTIN PIZARRO**  
**On relative CM-triviality**

**Abstract:** Hrushovski's ab initio construction exhibited a strongly minimal set whose geometry is non-locally modular but it does not arise from the presence of a field. The proof that no field was interpretable was by using a combinatorial notion, called CM-triviality, related to a specific point-line-plane configuration. In his fusion of two strongly minimal sets, Hrushovski claims that the fusion is CM-trivial over the base data. We will introduce a notion of relative CM-triviality and we will show the relation of this notion to the presence of a field or a bad group.

**ALEXEY MURANOV**  
**Independence property and hyperbolic groups**  
(Joint work with Eric Jaligot and Azadeh Neman)

**Abstract:** A *CSA-group* is a group  $G$  in which every maximal abelian subgroup  $A$  is *malnormal*, i.e. satisfies  $A \cap A^g = \{1\}$  for each  $g \in G \setminus A$ . If  $f$  is a function from the set of primes to  $\mathbb{N} \sqcup \{\infty\}$ , we call a *CSA<sub>f</sub>-group* any *CSA-group* which contains no elementary abelian  $p$ -subgroup of rank  $f(p) + 1$  for any prime  $p$  for which  $f(p)$  is finite. For every such  $f$ , the class of *CSA<sub>f</sub>-groups* is universally axiomatized. We show that if  $f(2) = 0$  (no involutions), then existentially closed *CSA<sub>f</sub>-groups* have the independence property. We proceed as follows.

If  $\phi(\bar{x}, \bar{y})$  is a formula in a given language  $\mathcal{L}$ , and  $\mathcal{C}$  is any (not necessarily elementary) class of  $\mathcal{L}$ -structures, we say that  $\phi$  has the *independence property relative to  $\mathcal{C}$*  if for every positive integer  $n$ , there exists a structure  $M_n \in \mathcal{C}$  and sequences of tuples  $\bar{x}_i$ ,  $i = 1, \dots, n$ , and  $\bar{y}_\sigma$ ,  $\sigma = \emptyset, \dots, \{1, \dots, n\}$ , in  $M_n$  such that

$$M_n \models \phi(\bar{x}_i, \bar{y}_\sigma) \quad \text{if and only if} \quad i \in \sigma.$$

We show that "almost every" formula of the form  $\ulcorner w(x, y) = 1 \urcorner$ , where  $w(x, y)$  is a group word in variables  $x$  and  $y$ , has the independence property relative to the class of torsion-free *C'(1/6)-groups* (which are known to be *hyperbolic*). In particular, the following one does:

$$\ulcorner xy^7x^2y^6x^3y^5x^4y^4x^5y^3x^6y^2x^7y = 1 \urcorner.$$

We deduce in the case  $f(2) = 0$  that every group  $G$  with the same universal theory as an existentially closed *CSA<sub>f</sub>-group* has the independence property, because

every torsion-free hyperbolic group is *CSA* and hence embeds into a model of the first-order theory of  $G$ .

**LUDOMIR NEWELSKI**  
**Bounded orbits**

**Abstract:** We consider the action of the group  $G$  on its space of types. We discuss some consequences of existence of bounded orbits, including existence of  $G^{00}$  and Keisler measure. Some of these consequences are absolute, both in the set- and model-theoretic sense, but the existence of a bounded orbit seems not. We discuss what “bounded” means here and consider some intermediate notions.

**ABDEREZAK OULD HOUCINE**  
**Superstable groups acting on trees**

**Abstract:** We initiate a study of superstable groups acting on trees (simplicial or real). We will show that if a free product with amalgamation  $G = G_1 *_A G_2$  is superstable with the index of  $A$  in  $G_1$  different from 2, then  $G$  interprets a simple superstable non- $\omega$ -stable group. We show also that an action of an  $\omega$ -stable group on a simplicial tree is trivial, and in particular a free product with amalgamation or an HNN-extension is not  $\omega$ -stable. We study “minimal” superstable groups acting on trees and we give some of their properties.

**JONATHAN PILA**  
**Rational points of definable sets and diophantine problems**

**Abstract:** I will discuss problems and results concerning the distribution of rational points on definable sets in  $\mathbb{R}^n$ . I will describe a result, joint with Wilkie, that such a set  $X$  can have only “few” rational points, in a density sense, that do not lie on some positive dimensional connected semi-algebraic subset of  $X$ . I will give some applications, including a new proof, joint with Zannier, of the Manin-Mumford conjecture and some further results of Andre-Oort-Manin-Mumford type.

**ANAND PILLAY**  
**Semiabelian varieties over function fields, logarithmic derivatives, and exponentiation.**  
(Joint work with D. Bertrand)

**Abstract:** We study generalizations of Ax’s work to possibly nonconstant semi-abelian varieties over function fields.

**MIKE PREST**  
**Abelian categories and imaginaries**

**Abstract:** In the additive context it is natural to use imaginaries which inherit and preserve the underlying additive structure. Every small abelian category can

be realised as such a category of imaginaries, indeed there is an equivalence between small abelian categories + exact functors and definable additive categories + functors which preserve direct product and direct limits (equivalently interpretation functors). I will explain this and some of the surrounding results and questions.

### JEAN-PHILIPPE ROLIN

#### Some aspects of quasianalyticity in o-minimal structures

**Abstract:** We recall, via several examples, how the notion of quasianalyticity can be used in establishing the o-minimality of expansions of the real field. We describe various techniques leading to this property, and a possible new one which may help in some open problems.

### ZLIL SELA

#### Envelopes and Imaginaries in free (and hyperbolic) groups

**Abstract:** We apply some of the techniques that were used in obtaining quantifier elimination and stability in free and hyperbolic groups, to study definable equivalence relations over these groups. First, we prove that conjugacy, cosets of cyclic groups, and double cosets of cyclic groups are imaginaries. Then we prove geometric elimination of imaginaries assuming new sorts for these basic imaginaries.

### TAMARA SERVI

#### Wilkie's Theorem of the Complement for definably complete and Baire structures

(Joint work with A. Fornasiero)

**Abstract:** We consider definably complete and Baire expansions of ordered fields: every definable subset of the domain of the structure has a supremum and the domain can not be written as the union of a definable increasing family of nowhere dense sets. Every expansion of the real field is definably complete and Baire. So is every o-minimal expansion of a field. The converse is clearly not true. However, unlike the o-minimal case, the structures considered form an elementary class. In this context we prove the following version of Wilkie's Theorem of the Complement: given a definably complete and Baire expansion  $K$  of an ordered field with a family of  $C^\infty$  functions, if there are uniform bounds on the number of connected components of quantifier free definable sets, then  $K$  is o-minimal. We derive the o-minimality of every definably complete and Baire expansion of an ordered field with any family of definable Pfaffian functions. We apply this result to obtain a simple candidate for a recursive axiomatization of the theory of the real exponential field, and some related structures.

**SERGEI STARCHENKO****Borel-Moore homology for o-minimal structures**

**Abstract:** In this talk we give a definition of Borel-Moore Homology for sets definable in o-minimal expansion of fields. Our goal is to demonstrate that Model Theoretic tools can be useful in establishing some properties of this homology theory.

**ALEX WILKIE****Valuation inequalities**

**Abstract:** I review the valuation inequality for polynomially bounded, o-minimal structures and discuss a version for the complex holomorphic case.

**BORIS ZILBER****Zariski geometries**

**Abstract:** To be announced.