Analytic order of singular and critical points

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We deal with the following closely related problems: (i) For a germ of a reduced plane analytic curve, what is the minimal degree of an algebraic curve with a singular point analytically equivalent (isomorphic) to the given one? (ii) For a germ of a holomorphic function in two variables with an isolated critical point, what is the minimal degree of a polynomial, equivalent to the given function up to a local holomorphic coordinate change? Classically known estimates for such a degree d in these questions are $\sqrt{\mu}+1 \leq d \leq \mu+1$, where μ is the Milnor number. Our result in both the problems is $d \leq a\sqrt{\mu}$ with an absolute constant a.