p-adic unipotent differential systems

Abstract:

Beside the usual "archimedean" one, there are many non-trivial absolute values on the field of rational numbers. Up to equivalence, they are in bijection with prime numbers p and called p-adic. Given such a prime p, there exists a smaller algebraically closed field C_p which is complete with respect to the the p-adic absolute value.

Many arithmetic problems lead to (linear) differential systems over C_p. Those systems have two fundamental properties. First, they are (over-) convergent, which means that they have a full set of solutions with big radius of convergence (and "over" means that the coefficients too have a big radius of convergence) . Also, they have a Frobenius structure which is analogous to complex conjugation in the classical case.

Not all differential system have an arithmetic origin. It is a natural question to ask if a given system is (over-) convergent and/or if it has a Frobenius. Actually, the second property implies the first (at least locally).

So let us consider the simplest ton trivial case : unipotent differential equations (that have a nilpotent connexion matrix). Chiarellotto and myself showed that they are overconvergent (condition on coeficients imply condition on solutions) but do not have a Frobenius in general. In an independant way, Chiarellotto, Crew and Deligne showed that such a Frobenius structure exists after adding some more equations to the system. This was a conjecture of Dwork. I will explain how one can prove these theorems.