Families of Albanese Morphisms I

Francesco Zucconi

If $\pi : \mathcal{X} \to B$ is a family of irregular varieties, the number m of effective parameters of the birational equivalence classes of the fibers is called $\mathrm{var}(\pi)$. We estimate $\mathrm{var}(\pi)$ in terms of the dimension n, the geometrical genus p_g and the irregularity q of the fiber. Under the assuption that the Albanese morphism of the fiber has degree 1 onto its image, we prove:

$$\mathrm{var}(\pi) \leq p_g + q(n+1) - \frac{(n+2)(n+1)}{2}.$$

We use this result to solve the longstanding Castelnuovo conjecture about the moduli of an irregular surface:

Theorem (Castelnuovo Inequality) *Let X be a smooth surface of general type with $q \geq 4$. If $\mathrm{alb} : X \to \mathrm{Alb}(X)$ is birational onto the image then $m \leq p_g(X) + 2q - 3$.*