Moduli spaces of bundles: commutative and noncommutative

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The usual construction of moduli spaces of vector bundles parametrises certain vector bundles by the points of a suitable scheme.

Regarding the points as the simple sheaves on this scheme, we are at least locally classifying vector bundles by the 1-dimensional modules for a commutative ring. This is usually fine with regard to the Hom-spaces but bad with regard to the Ext-spaces since all Ext-groups vanish between simple modules over a commutative ring.

One possible cure for this would be to parametrise vector bundles (locally) by the finite-dimensional modules for a noncommutative ring. I show how this works completely for moduli of vector bundles over a smooth projective curve leading to a construction of (noncommutative) fine moduli spaces together with a Fourier-Mukai theorem. This gives a simple construction for the standard moduli spaces.

I end by discussing my earlier results on rationality of moduli spaces as theorems on noncommutative rationality of these noncommutative spaces.