

Graded rings associated with contracted ideals

M.E. Rossi

The study of the ideals in a regular local ring (R, m) of dimension 2 has a long and important tradition dating back to the fundamental work of Zariski . More recent contributions are due to several authors including Cutkosky, Huneke, Lipman, Sally and Tessier among others. One of the main result in this setting is the unique factorization theorem for complete ideals proved originally by Zariski. Another important property of a complete ideal I is that its reduction number is 1 which in turns implies that the associated graded ring, the Rees Algebra and the Fiber Cone are Cohen-Macaulay, as well its Hilbert series is well understood. The class of contracted ideals plays an important role in the original work of Zariski as well as in the work of Huneke. Any complete ideal is contracted but not the other way round. The associated graded ring of a contracted ideal I need not be Cohen-Macaulay and its Hilbert series can be very complicated. Our goal is to study depth, Hilbert function and defining equations of the various graded rings (Rees algebra, associated graded ring and fiber cone) of homogeneous contracted ideals in the polynomial ring $R = k[x, y]$ over an algebraically closed field k of characteristic 0. We also present several equivalent characterizations of contracted ideals in the graded and local case.