Polynomial functors, negative sets, and moduli of punctured Riemann spheres

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A polynomial functor (of one variable) is an endofunctor on the category of sets, built from disjoint unions, products, and exponentiation. They behave a lot like polynomials over the natural numbers: you can add and multiply them or substitute them into each other; you can also differentiate them, and there is a Leibniz rule and a chain rule, as I shall explain. Now, in order to model integer coefficients (and exponents) a notion of negative sets is required. I'll introduce Schanuel's category of polyhedral sets, for which a certain Euler measure provides a generalisation of cardinality that can take negative integer values. To finish with an interesting example, I'll describe the polynomial functors associated to the moduli spaces of Riemann spheres with $3+n$ punctures. The puncture operator corresponds precisely to differentiation, and in the setting of polyhedral sets, these functors are models for the familiar Laurent polynomials $-n!(-X)^{-n-1}$.