

# The fundamental group of surfaces with small $K^2$

Rita Pardini

Let  $S$  be a smooth minimal complex surface of general type. The numerical invariants of  $S$  satisfy the inequalities:

$$2\chi - 6 \leq K^2 \leq 9\chi.$$

It has now been clear for a long time that the smaller the ratio  $K^2/\chi$  is, the simpler the fundamental group of  $S$  is. The Severi inequality, which states that the Albanese image of a surface with  $K^2 < 4\chi$  has dimension at most 1, can be seen as an instance of this general principle. I will give a proof of the Severi inequality using the slope inequality for fibered surfaces. Then I will show how the Severi inequality and the slope inequality can be used to reprove quickly earlier results, due to several authors, on the fundamental group of surfaces with  $K^2 < 3\chi$ . Finally I will describe some recent joint work with Margarida Mendes Lopes, which extends these results.