The Frobenius Structure of Local Cohomology

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For a local ring (R, m) of prime characteristic $p > 0$, the local cohomology modules with support in the maximal ideal $H^i_m(R)$ inherit a natural Frobenius action F. The talk with present results on the lattice of submodules N of the local cohomology that are invariant under the Frobenius action, that is $F(N) \subseteq N$. This lattice of submodules contains remarkable information about the structure of the ring R. A result of Karen E. Smith, from a decade or so ago, states that, under mild conditions, a Cohen-Macaulay ring R is F-rational (an analogue of rational singularities in positive characteristic) if and only if 0 is the only proper F-invariant submodule of the highest local cohomology module. We will explore the properties of the lattice of F-invariant submodules in detail. Part of the work is joint with Mel Hochster.