Seminari de Geometria Algebraica 2009/2010 (UB-UPC)
Divendres 30 d'octubre a les 15h. a l'aula T2
http://atlas.mat.ub.es/sga

Galois closures and Lagrangian varieties

Lidia Stoppino

U. dell'Insubria, Itàlia

Let X be a smooth complex algebraic variety of dimension n and consider the homomorphism

$$\psi_k : \wedge^k H^0\big(X, \Omega^1_X\big) \to H^0\big(X, \Omega^k_X\big),$$

given by the holomorphic part of the cup product homomorphism $\wedge^* H^1(X, \mathbb{C}) \to H^*(X, \mathbb{C})$. The kernel of ψ_k carries several informations about the topology of X. A classical result due to Castelnuovo-de Franchis states that ker ψ_2 contains decomposable elements if and only if X admits a fibration over a curve of genus ≥ 2. The most important case where non-decomposable elements appear in ker ψ_2 is when X is what we call Lagrangian, i.e. it admits a finite map to an abelian variety A of dimension $2n$, such that there exists a 2-form ω on A whose pullback vanishes on X.
In this talk I will describe a joint work with F. Bastianelli and P. Pirola, whose main result is the following. Consider a finite morphism $\gamma : Z \to Y$ whose monodromy group $M(\gamma)$ is the full symmetric group, and let X be its Galois closure. I will show that under the assumption that Z is irregular and $h^{k,0}(Y) = 0$, there is a natural way to construct non-trivial elements in ker ψ_k. In particular this method allows the construction of Lagrangian varieties. I will describe a new family of non-fibred Lagrangian surfaces of general type constructed as Galois closures of degree 3 morphisms from abelian surfaces to rational ones.
