Mumford's modular families:
between coarse moduli and stacks.

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A central problem in the classification of algebraic varieties, when there are continuous families of the objects under study, is to find some kind of universal family that ideally should contain each isomorphism type of the given varieties just once. Well-known examples are the Hilbert schemes that parametrize closed subschemes of projective spaces; the Picard variety that parametrizes line bundles, and the variety of moduli of curves of genus g. In this talk I will discuss the precise meaning of the moduli problem, including the notions of coarse moduli space, fine moduli space, formal moduli, and stacks. In particular I will discuss Mumford's notion of a modular family, which is more "stack-like" than a coarse moduli space, but much easier to handle technically than a stack. Reference: my recent book on Deformation Theory, Springer GTM 257.