F-thresholds, a conjecture on multiplicity of ideals and core of ideals.

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This is a joint work with C. Humeke, M. Mustaţă and S. Takagi.

We define F-thresholds $c^J(\mathfrak{a})$ for pair of ideals $(J, \mathfrak{a})$ satisfying $\mathfrak{a} \subset \sqrt{J}$ in a Noetherian local or graded ring $(R, \mathfrak{m})$. This notion was introduced by Mustaţă to describe "jumping numbers" of multiplier ideals (in rings of characteristic 0) by characteristic p method for regular rings.

We have a conjecture comparing multiplicity of J and $\mathfrak{a}$), when $J, \mathfrak{a}$ are $\mathfrak{m}$ primary ideals and J is a parameter ideal.

Conjecture A. $e(\mathfrak{a}) \geq \left(\frac{d}{c^J(\mathfrak{a})}\right)^d e(J)$, where $d = \dim R$ and $e(\mathfrak{a}), e(J)$ denotes the multiplicity of $\mathfrak{a}, J$.

This conjecture is a generalization of a theorem of de Fernex, Ein and Mustaţă which gives a lower bound of $e(\mathfrak{a})$ by log canonical threshold of $\mathfrak{a}$.

Conjecture A was proved in the case of graded rings when $J, \mathfrak{a}$ are generated by homogeneous parameters.

In this talk, I talk on the relationship of Conjecture A with the following conjecture concerning core of ideals, where for an ideal $\mathfrak{a}$, core($\mathfrak{a}$) is defined as the intersection of all the reductions of $\mathfrak{a}$. I will explain that for certain rings, Conjecture A is equivalent to the following conjecture.

Conjecture B. Let $\mathfrak{a}, J$ be $\mathfrak{m}$ primary ideals and J be a parameter ideal. If $J \supset \text{core}(\mathfrak{a})$, then $e(J) \leq e(\mathfrak{a})$?