Fitting ideals and multiple-points of surface parameterizations

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Parameterized algebraic surfaces are ubiquitous in geometric modeling and the determination of their singular loci is an important problem. Given a parameterization ϕ from $\mathbb{P}^2$ to $\mathbb{P}^3$ of a rational algebraic surface $\mathcal{S}$, we will see in this talk that the sets of points on $\mathcal{S}$ whose preimage consists in k or more points, counting multiplicity, can be described in terms of Fitting ideals of some graded parts of the symmetric algebra associated to the parameterization ϕ.

More precisely, we will show that the drop of rank of a certain elimination matrix $M(\phi)$ at a given point $P \in \mathbb{P}^3$ is in relation with the fiber of the graph of ϕ over P. Thus, the Fitting ideals attached to $M(\phi)$ provide a filtration of the surface which is in correspondence with the degree and the dimension of the fibers of the graph of the parameterization ϕ. We will also comment on the link with the double-point formulas that have been extensively studied in the field of intersection theory for finite maps.

This is joint work with Nicolas Botbol (University of Buenos Aires) and Marc Chardin (University of Paris VI).