Some results on the topology of complex algebraic varieties

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These lectures will be concerned about some relations among the geometry of the linear series and the topology of a complex projective variety X. On one hand we have some classical results and applications on the connectivity and the fundamental group of X and on the other hand topological methods apply to the theory of linear series.

Most part of the material can be found in the Lazarsfeld books:

R. Lazarsfeld. Positivity in Algebraic Geometry I and II, Ergebnisse der Mathematik 48 & 49, Springer-Verlag, Berlin.

Total time: 8 hours.

Schedule: 5, 12, 26 of April and 3 of May, from 15 to 17h.

Place: room B1, Faculty of Mathematics, University of Barcelona.

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Program

Lecture 1: **Fulton–Hansen connectedness theorem.**

After recalling some classical results, like Bertini theorem, we present a proof of the Fulton–Hansen theorem and we will give some applications.

References:
1) W. Fulton– J.Hansen. A connectedness theorem for projective varieties with applications to intersections and singularities of mappings. Ann. of Math. 2 11(1979) n1, 159-166
2) W. Fulton–R. Lazarsfeld. Connectivity and its applications in Algebraic Geometry. Algebraic Geometry (Chicago 1980), Lect. Notes in Math. Vol. 862, Springer Verlag Berlin, 26-92.

Lecture 2: **The fundamental group of the complement of a nodal curve.**

We will discuss the Fulton-Deligne solution of Zariski problem. It gives that the fundamental group of the complement of a nodal curve is abelian. In particular we present the Fulton proof on the algebraic fundamental group as an application of the Fulton–Hansen connectedness theorem. We will also discuss, if time will permit, the approach of Nori.

References:

3) W. Fulton. On the fundamental group of the complement of a node curve. Ann. of Math. 2 111(1980) n2, 407-409
4) M. Nori. Zariski's conjectures and related problems. Ann. Sci. Ecole Norm. Sup. (4) 16 1983 305-344.

Lecture 3: **Connectedness of degeneracy loci.**

We introduce the ample vector bundles and discuss the approach of Fulton–Lazarsfeld to the existence and connectedness of variety of special divisor of a curve.

Reference:

5) W. Fulton–R. Lazarsfeld. On the connectedness of degeneracy loci and special divisors. Acta Math. 1981, 271-283.

Lecture 4: **Vanishing theorems.**

We shortly recall some classical vanishing results. Then we consider the generic vanishing theorem of Green-Lazarsfeld and we speculate on its application to the Brill-Noether theory on higher dimensional varieties.

References:

6) M.Green–R. Lazarsfeld. Deformation theory, generic vanishing theorems and some conjectures of Enriques, Catanese, and Beauville, Invent. Math. 90 (1987) 1. 73-90
7) M.Green–R. Lazarsfeld   Higher Obstructions to deforming cohomology groups of line bundles, J. Amer. Soc. 4 (1991), 1, 87-103.
8) H. Clemens-C.Hacon. Deformation of the trivial line bundles and vanishing theorems. Amer. J.Math. 124 (2002) 4, 769-815
9) M. Mendes Lopes, R. Pardini and G.P. Pirola. Brill-Noether loci for divisors on irregular varieties arXiv:1112.6357   to appear on JEMS.