# Open Problems in Algebraic Geometry, Line Arrangements and Symbolic Powers

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The folkloric and still open Bounded Negativity Conjecture proposes that for each smooth complex projective surface $X$ there is an $n_X$ such that $C^2 \geq n_X$ for every reduced curve $C$ on $X$. Three questions related to the Bounded Negativity Conjecture lead to other open problems: How singular can a plane curve be? What arrangements of lines in the plane have no simple crossings? For which sets of points $p_1, \ldots, p_r$ in the projective plane is there a homogeneous polynomial vanishing to order at least three at each point but which is not a sum of pairwise products of homogeneous polynomials vanishing at all of the points? (I.e, for which point sets is $I^{(3)} \not\subseteq I^2$, where $I$ is the ideal of the points?) I will motivate these questions and give examples and some recent results.