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The inverse problem for a SODE on a Lie algebroid

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The inverse problem of the calculus of variations addresses the question of whether or not a system of second order ordinary differential equations (SODE for short) is equivalent to a regular Lagrangian system. Some necessary and sufficient conditions are the well-known Helmholtz conditions given in [1]. The classical version of the problem is concerned with second order systems defined on a tangent bundle, but many mechanical systems are not defined on a tangent bundle but for instance on a quotient by a symmetry Lie group and then the equations of motion are not the standard Euler-Lagrange equations but the Lagrange-Poincaré equations. These are recovered as Euler-Lagrange equations if we work in the more general setting of Lie algebroids [2].
We will address the inverse problem in this context, adapting a new characterization of the classical inverse problem given in terms of Lagrangian submanifolds in [3]. In the Lie algebroid setting the tangent bundles will be replaced by appropriate prolongations of the Lie algebroid. The lack of the Poincaré lemma in this context implies that the Helmholtz conditions, derived using the differential of the prolongation of the Lie algebroid, are not enough to guarantee the existence of a Lagrangian [4].

References

[1] Jesse Douglas: Solution of the inverse problem of the calculus of variations. *Trans. Amer. Math. Soc.* 50 (1941), 71–128.

[2] Alan Weinstein: Lagrangian mechanics and groupoids. *Mechanics day*, Fields Institute Proc. v7 AMS (1995), 207–231.

[3] María Barbero-Liñán, Marta Farré Puiggalí, David Martín de Diego: Isotropic submanifolds and the inverse problem for mechanical constrained systems. *J. Phys. A: Math. Theor.* 48 (2015) 045203 (35pp).

[4] María Barbero-Liñán, Marta Farré Puiggalí, David Martín de Diego: Inverse problem for Lagrangian systems on Lie algebroids and applications to reduction by symmetries. *Monatsh. Math.* 180 (2016), no. 4, 665–691.