

Seminari de Geometria Algebraica 2016-2017

Divendres 25 de novembre a les 15:00, aula T2 FMI-UB

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## ON BLOWING UP THE WEIGHTED PROJECTIVE PLANE

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Let  $\mathbb{P}(a, b, c)$  be the weighted projective plane with pairwise coprime weights  $a, b, c$ , defined over an algebraically closed field  $\mathbb{K}$  of characteristic 0. This talk will focus on the Cox ring of the blowing-up  $X = X(a, b, c)$  of  $\mathbb{P}(a, b, c)$  at a general point  $x_0$ . The Cox ring of  $X$  is the following  $\mathbb{Z}^2$ -graded algebra [1]:

$$\mathcal{R}(X) = \bigoplus_{(d, -m) \in \mathbb{Z}^2} \left\{ f \in \mathbb{K}[x_1, x_2, x_3]_d : \begin{array}{l} V(f) \subseteq \mathbb{P}(a, b, c) \text{ has} \\ \text{multiplicity} \geq m \text{ at } x_0 \end{array} \right\},$$

where  $\mathbb{K}[x_1, x_2, x_3]_d$  denotes the vector space of degree  $d$  homogeneous polynomials with respect to the  $[a, b, c]$ -grading. The aim of this talk is to present a criterion for testing if  $\mathcal{R}(X)$  is finitely generated and to show for which values of  $a, b, c$  the Cox ring is generated by homogeneous elements of multiplicity at most 2. The proof of the criterion makes use of a theorem of Hu and Keel [3, Proposition 2.9], while for providing explicit generators of the Cox ring we apply [2, Algorithm 5.4].

This is joint work with J. Hausen and S. Keicher, <https://arxiv.org/abs/1608.04542>

- [1] Ivan Arzhantsev, Ulrich Derenthal, Jürgen Hausen, and Antonio Laface, *Cox rings*, Cambridge Studies in Advanced Mathematics, vol. 144, Cambridge University Press, Cambridge, 2015.
- [2] Jürgen Hausen, Simon Keicher, and Antonio Laface, *Computing Cox rings*, Math. Comp. **85** (2016), no. 297, 467–502.
- [3] Yi Hu and Sean Keel, *Mori dream spaces and GIT*, Michigan Math. J. **48** (2000), 331–348.