One-dimensional Gorenstein local rings with decreasing Hilbert function

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Let $(R, \mathfrak{m})$ be a one-dimensional Cohen-Macaulay local ring. By definition, the Hilbert function H_R of R is that of its associated graded ring $\bigoplus_{i>0} \mathfrak{m}^i/\mathfrak{m}^{i+1}$. The study of H_R is a classical topic in local algebra, in particular it is interesting to understand how H_R and the properties of R are related. For instance, when R is Gorenstein, M.E. Rossi asks if it has always non-decreasing Hilbert function and, in fact, in the last ten years several authors found families of one-dimensional Gorenstein local rings that has non-decreasing Hilbert function, especially in the case of numerical semigroup rings. In this talk we show that in general the answer to the above question is negative; more precisely, given $m, h \geq 1$ with $h \notin \{14 + 22k,\, 35 + 46k \,|\, k \in \mathbb{N}\}$, we explain how to construct one-dimensional Gorenstein local rings such that $H_R(h) - H_R(h+1) > m$. A key tool is numerical semigroup theory.

This is a joint work with Anna Oneto and Grazia Tamone.

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