

Barcelona Course, May 2010: Topics on Projective Varieties

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Prerequisites

The course will require some knowledge of the following subjects, for which I recommend one of the following books [10, 11, 12, 21].

Affine and projective varieties. Morphisms and rational maps. Smooth and singular points of a variety. Zariski tangent space and tangent cones. Linear systems and maps to projective spaces. Blow-ups. Basics on intersection theory: in particular, intersection of divisors on a smooth variety.

Multilinear algebra and related projective varieties: Veronese varieties, Segre varieties, Grassmannians.

Basics on cohomology. Flatness and semicontinuity.

Basics on curves and surfaces. Rational surfaces: minimal rational surfaces. Complete intersections and scrolls. The Riemann–Roch theorem.

Basics on vector bundles.

Basics on deformation theory.

Programme

In the course I will treat some topics about the geometry of projective varieties. Next I give some details about the programme, with some references which I may follow during the course. It is however not necessary any specific knowledge about them.

Projective varieties

Joins of varieties and secant varieties. Defective varieties. Terracini's lemma [18, 19, 22]. Interpolation and defectivity [6]. Degenerations of varieties and of secant varieties [7].

Weakly defective varieties. Defectivity and tangential projections [2]. Second fundamental form. Dual varieties and contact loci. Osculating spaces [18, 19].

Classification of defective (and weakly defective) surfaces [2]. Classification of defective threefolds [3, 4]. QEL varieties [13, 14, 20].

Defectivity and non-defectivity of classical varieties (Veronese, Segre, Grassmannians) [1]. The Waring problem for polynomials and tensors, rank and border rank for tensors [6, 17, 16].

The technique of degeneration of projections. OADP varieties [8]. Tangent cones to secant varieties. Varieties with minimal secant degree [9].

Theorem of Barth–Larsen. Hartshorne’s conjectures. Fulton–Hansen connectedness theorem. Zak’s theorems on tangencies and on linear normality. Severi varieties and their classification. Scorza varieties [18, 19, 22]. Extension of Zak’s theorems to higher secant varieties [5].

Some results about Hartshorne’s conjecture.

References

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