One-dimensional Gorenstein local rings with decreasing Hilbert function

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Let \((R, \mathfrak{m})\) be a one-dimensional Cohen-Macaulay local ring. By definition, the Hilbert function \(H_R\) of \(R\) is that of its associated graded ring \(\bigoplus_{i>0} \mathfrak{m}^i/\mathfrak{m}^{i+1}\). The study of \(H_R\) is a classical topic in local algebra, in particular it is interesting to understand how \(H_R\) and the properties of \(R\) are related. For instance, when \(R\) is Gorenstein, M.E. Rossi asks if it has always non-decreasing Hilbert function and, in fact, in the last ten years several authors found families of one-dimensional Gorenstein local rings that has non-decreasing Hilbert function, especially in the case of numerical semigroup rings. In this talk we show that in general the answer to the above question is negative; more precisely, given \(m, h \geq 1\) with \(h \notin \{14+22k, 35+46k \mid k \in \mathbb{N}\}\), we explain how to construct one-dimensional Gorenstein local rings such that \(H_R(h) - H_R(h+1) > m\). A key tool is numerical semigroup theory.

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