Hermitian vector bundles and extension groups on arithmetic varieties

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This is joint work with J. B. Bost. We define and investigate arithmetic extensions and their extension groups in the context of Arakelov geometry. We define the size of an arithmetic extension of hermitian vector bundles over an arithmetic curve and give an upper bound for this size in terms of the minimal slope of an associated hermitian vector bundle. We discuss three examples of arithmetic extensions: The arithmetic Atiyah extension, the arithmetic Hodge extension, and the arithmetic Schwarz extension. We give an upper bound for the Faltings height of a semistable elliptic curve over a number field in terms of the Artin conductor of the curve, the size of its associated arithmetic Hodge extension, and a constant which depends only on the degree and the discriminant of the number field. We discuss the non-triviality of the Hodge extension in the function field case and for elliptic curves over the algebraic closure of Q. In the function field case, we prove non-triviality of the Hodge extension by relating it to the Kodaira-Spencer morphism. Finally we investigate projective structures on curves and the associated arithmetic Schwarz extension.