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Correction on "The importance of being the upper bound in the bivariate family"

Carles M. Cuadras[1] and Ali Dolati[2]

[1] *University of Barcelona and* [2] *Shimz University*

1 Introduction

Let us perform two corrections on Cuadras (2006) and add a short note. First, the cumulative distribution function of the Pareto distribution should be $F(x) = 1 - (x/a)^{-c}$ if $x > a$, instead of $1 - (x/a)^{c}$. This misprint does not alter the formulas for the Lorenz curve, Gini coefficient and the role of the upper bound for evaluating the social inequality.

Second, a general expression for a measure of stochastic dependence between two random variables with cdf H was proposed in the same paper. For uniform marginals this measure reduces to

$$A(U, V) = c \int_{\mathbf{I}^2} (C_H(u, v) - uv)\, d\mu,$$

where c is a normalizing constant, μ is a suitable measure and C_H is the copula related to H. Then it is said that Kendall's τ and Spearman's ρ_s are coefficients of dependence computed from the copula C_H by using $d\mu = dC_H$ and $d\mu = dudv$, respectively. However, the first statement is not true. Thus, while the second equation in

$$\begin{aligned} \tau &= 4 \int_{\mathbf{I}^2} (C_H(u, v) - uv) dC_H(u, v) \\ &= 4 \int_{\mathbf{I}^2} C_H(u, v) dC_H(u, v) - 1, \end{aligned}$$

is correct, the first one is incorrect.

As it was proved by Behboodian *et al.* (2005), if we impose the condition $A(U,V) = 1$ for the upper bound $C^+ = \min\{u,v\}$, then $c = 6$ and $A(U,V)$ is maximum. Therefore from

$$\begin{aligned} \rho_s &= 12 \int_{\mathbf{I}^2} C_H(u,v)dudv - 3, \\ 3\tau &= 12 \int_{\mathbf{I}^2} C_H(u,v)dC_H(u,v) - 3, \end{aligned}$$

and

$$\int_{\mathbf{I}^2} C_H(u,v)dudv = \int_{\mathbf{I}^2} uvdC_H(u,v),$$

where C_H is any copula, this measure can be expressed as

$$\begin{aligned} A(U,V) &= 6 \int_{\mathbf{I}^2} (C_H(u,v) - uv)dC_H(u,v) \\ &= \frac{3\tau - \rho_s}{2}. \end{aligned}$$

Thus the above proposed measure can not give τ. Actually $A(U,V)$ combines both coefficients Kendall's τ and Spearman's ρ_s and is an example of average quadrant dependence measure.

Finally note that $A(U,V)$ is not a measure of concordance in the sense of Scarsini (1984). Specifically, it does not preserve the concordance ordering property, that is, if C_1, C_2 are two copulas such that $C_1 \leq C_2$ then not necessarily $A_1 \leq A_2$. See Section 5 and Example 5.1 in Behboodian *et al.* (2005) for further details. In fact, $A(U,V)$ is just a measure of association.

References

Behboodian, J., Dolati, A. and Úbeda-Flores, M. (2005). Measures of association based on average quadrant dependence. *Journal of Probability and Statistical Science*, 3, 161-173.

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