The Russell-Priest schema and Tarski's paradox of Quotation.

1. Introduction.

Our notion of paradox is quite vague and imprecise. When we try to characterize it, we come up with descriptions like this:

"This is what I mean by a paradox: an apparently unacceptable conclusion derived by apparently acceptable reasoning from apparently acceptable premises. Appearances have to deceive, since the acceptable cannot lead by acceptable steps to the unacceptable."[1]

As it stands, this characterization leaves open many questions and says nothing that may help us identify concrete paradoxes. It merely states a problem we come across often when reasoning about lots of different subjects and issues but it does not help us understand what the problem or its causes are in each case. This description covers a wide range of phenomena that, at first sight, seem to have little in common except for the befuddlement we are left in when going through the reasoning that originates the paradox and our awareness of it. A natural question to ask under these circumstances is whether it is possible to find a unified account of paradoxes, an account that presents all paradoxes as particular instances of a unique phenomenon. On the face of the huge variety of paradoxes we can find in the literature, a positive answer to this question seems, however, highly unlikely. The number of paradoxes is indeed overwhelming: the sorites' paradoxes, the paradox of the liar, Russell's paradox, the paradox of the concept "heterological", the paradox of the barber, Zeno's paradoxes against movement, Burali-Forti's paradox, Berry's paradox, Curry's paradox, Yablo's paradox ... Given their abundance and the evidence that paradoxes usually involve different concepts and set up different scenarios, there is little hope that we can find a unified account for all of them. Some paradoxes seem to be too different from one another to attain that goal and sometimes we are just too generous in labelling things under the title of "paradox". But, in spite of these initial drawbacks, we might aim perhaps at a more modest goal. If we could at least find some similarities among certain paradoxes and sort them in groups providing a unified description of them (a description grounded on structural

[1] R. M. Sainsbury 1988.

similarities, for instance), we would be in a better position to understand the phenomenon they represent and to simplify this chaotic jungle of inconsistencies where paradoxes seem to grow everywhere. Many philosophers have tried to carry out different versions of this project in the past. The object of the present paper is to revise a recent attempt to push further this enterprise, an attempt due to Graham Priest[2].

2. "Reflexive" paradoxes.

For historical reasons, a privileged subset of the paradoxes just mentioned has received especial attention over the last century. I am talking about paradoxes like Russell's paradox, Burali-Forti's paradox, Cantor's paradox or the liar paradox (and related ones). These paradoxes seemed to threaten the consistency of some of the most basic notions and principles we can find in fields such as set theory or semantics and have motivated remarkable efforts in those fields to avoid their effects. Of course, this contingent fact does not provide any ground to isolate these paradoxes from the rest, however they all seem to share a feature that would apparently justify to include them (together with other paradoxes not listed here) in a single group. All these paradoxes contain, as it were, some sort of "reflexivity". They all involve relations, (concepts, definitions, descriptions, etc.) that can be "applied to themselves", or sets (collections, classes, etc.) that "belong to themselves", or sentences (propositions, utterances, etc.) that "refer to themselves" and so on. Unfortunately, this common trait is of little help in order to understand and explain the problem posed by paradoxes since this alleged "reflexivity" is also displayed by relations, collections, sentences, etc. in contexts that do not seem particularly problematic or paradoxical. We define many things, among others what we mean by a "definition", a collection of all collections with more than five elements surely contains itself, since it is bound to be a collection with more than five members and the sentence 'This sentence contains five words' refers to itself. There is no apparent reason for which we should find paradoxes in these examples (where we find things that bare some reflexive relation to themselves) and therefore there is no reason to blame "reflexivity" for the problems posed by paradoxes. But if "reflexivity" is not in itself the problem, then it seems that the fact that those paradoxes involve some sort of "reflexivity" might be as contingent and of little taxonomical significance as the fact that they received much attention during a certain period of

[2] G. Priest 1994.

time. We need more evidence to show that we are justified to include these paradoxes in the same group. If we eventually managed to describe some structural similarities these would rely not just on reflexivity but rather in the way in which reflexivity interacts with other factors also present in these paradoxes. Reflexivity should be just one of the ingredients of a complex structure underlying all these paradoxes. Is it possible to find any such structure?

3. Russell's schema:

Russell discovered that many of the paradoxes we could describe in set theory shared a common structure[3]. He offered, in particular, a schema that fitted three well known paradoxes in set theory: Russell's paradox, Burali-Forti's paradox on the class of all ordinals and Mirimanoff's paradox concerning the notion of well-founded hierarchy of sets. According to Russell, these paradoxes arise whenever we can build a **set???** Ω whose elements satisfy some property φ, and a function δ from the power set of Ω to Ω such that the image of any X (for $X \subseteq \Omega$) under δ, satisfies the following conditions: $\delta(X) \notin X$, and $\delta(X) \in \Omega$. The paradox obtains when we apply δ to one of the subsets of Ω, namely, Ω itself, then we get $\delta(\Omega) \notin \Omega$ and $\delta(\Omega) \in \Omega$. Russell considered his schema as a version of Cantor's description of the paradox stemming from the assumption that there is a bijection between any set and its power set[4].

Russell's schema:

1) **Set**: $\Omega = \{y : \varphi(y)\}$

2) Function δ: $\wp(\Omega) \to \Omega$, such that for every $X \subseteq \Omega$:

$\delta(X) \notin X$.

$\delta(X) \in \Omega$.

3) Contradiction: Given that $\Omega \subseteq \Omega$, we have $\delta(\Omega) \notin \Omega$ and $\delta(\Omega) \in \Omega$.

[3] Russell 1905, see G. Priest 1994 for details.

[4] This is also mentioned in Priest 1994.

Under this schema Russell's paradox, for example, can be cast in the following way: with φ equal to the property "is not a member of itself", and δ the identity function, we set: $\Omega = \{y : y \notin y\}$ and $\delta(x) = id(x) = x$.[5]

Suppose now (for $X \subseteq \Omega$) that $\delta(X) \in X$, this is equivalent to $X \in X$, therefore we infer that $X \in \Omega$ (since $X \subseteq \Omega$ and $X \in X$). However if $X \in \Omega$, then $X \notin X$ and we reach a contradiction, we conclude that $\delta(X) \notin X$ against what we had initially supposed. But if $\delta(X) \notin X$ ($X \notin X$), we have that $X \in \Omega$, this is, $\delta(X) \in \Omega$. The contradiction comes when we consider $\delta(\Omega)$, since then we have $\delta(\Omega) \notin \Omega$ and $\delta(\Omega) \in \Omega$.

Likewise, Burali-Forti's, Mirimanoff's and Cantor's paradox can be expressed by means of this schema[6]. This supports partly the claim that these paradoxes are actually different versions of a problem with the same structure.

4. Priest's modification of Russell's schema:

Russell was convinced that all reflexive paradoxes had a common origin, he tried to identify the source of the problem when he stated the vicious circle principle in 1908[7]. But his principle seemed unconclusive to many and anyway Russell was unable

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[5] Notice that, in spite of being the identity function, δ is still a one to one function from $\wp(\Omega)$ to Ω. This is possible because, with $\Omega = \{y : y \notin y\}$, it is the case that $\wp(\Omega) \subseteq \Omega$. Suppose that $x \in \wp(\Omega)$, then $x \subseteq \Omega$ and we have two possibilities: either $x \in x$ or $x \notin x$. If $x \in x$, then $x \in \Omega$ (given that $x \subseteq \Omega$), if $x \notin x$, then $x \in \Omega$, therefore $\wp(\Omega) \subseteq \Omega$.

Actually it is the case that $\Omega \subseteq \wp(\Omega)$, and thus we can prove that $\wp(\Omega) = \Omega$. Suppose that $x \in \Omega$, given that (as we have shown) $\wp(\Omega) \subseteq \Omega$, either $x \in \wp(\Omega)$ or $x \notin \wp(\Omega)$. If $x \notin \wp(\Omega)$, then $x \in \Omega - \wp(\Omega)$. ***WRONG TRACK? =>*** *However $\Omega - \wp(\Omega) \in \wp(\Omega)$ ($\Omega - \wp(\Omega)$ is a subset of Ω),* ***WRONG BIT =>*** *<u>therefore $\Omega - \wp(\Omega) = \varnothing$</u>. Therefore it must be the case that $x \in \wp(\Omega)$, but then $\Omega \subseteq \wp(\Omega)$.*

This highlights the parallelism pointed out by Russell (1905) between his own paradox and Cantor's. If the Russell set existed, then we would have a *one to one* function from $\wp(\Omega)$ to Ω, the identitiy function, and a function from $\wp(\Omega)$ *over* Ω, f: $\wp(\Omega) \to \Omega$ such that $f(\{x\}) = x$ (for every $\{x\} \in \wp(\Omega)$), otherwise $f(y) = \Omega$ (for $y \in \wp(\Omega)$)
(If the Russell set existed, then we would have a bijection between that set and its power set, namely, identity function).

[6] An explicit formulation of these paradoxes under the structure described here can be found in Priest 1994, except for Cantor's paradox. In this case, given a set A and a bijection h between A and $\wp(A)$, we take as Ω a subset of A: $\Omega = \{x \in A: x \notin h(x)\}$. Next, we define a one to one funtion δ: $\wp(\Omega) \to \Omega$ (notice that $\Omega \subseteq A$ and therefore $\wp(\Omega) \subseteq \wp(A)$) such that, restricting h to Ω ($h[\Omega]$), we have $\delta(x) = h^{-1}(x)$, where Dom(δ) = Range ($h[\Omega]$). δ assigns to each x in $\wp(\Omega)$ a set y such that $y \notin x$, since $x = h(y)$ and $y \in \Omega$, therefore we have $\delta(x) \notin x$ and $\delta(x) \in \Omega$. Once again the contradiction can be expressed as $\delta(\Omega) \notin \Omega$ and $\delta(\Omega) \in \Omega$. This highlights the parallelism pointed out by Russell (1905) between his schema and Cantor's paradox.

[7] "Thus all our contradictions have in common the assumption of a totality such that, if it were legitimate, it would at once be enlarged by new members defined in terms of itself" (Russell 1908, 63). "These

to extend his analysis of set theoretic paradoxes to paradoxes involving semantic concepts like “truth”, “definable” or “satisfyable”. Under these circumstances it was at best dubious that we could reach a unified account for all these paradoxes in order to seek a common solution that could be stated, as Russell intended, along the lines of principles similar to the vicious circle principle. This is partly the reason why Ramsey’s distinction between two different families of reflexive paradoxes: set theoretic paradoxes and semantic paradoxes seems to have survived nowadays. Ramsey took these paradoxes to be different due to the kind of concepts they involved and rejected the russellian approach to the problem.

Following Russell’s steps, Graham Priest has recently defended that Ramsey’s distinction is wrong and that we can actually apply a modified version of Russell’s schema to the liar paradox and paradoxes like König’s Richard’s, Berry’s or the paradox of “heterologicality”. All these paradoxes involve semantical notions (“true”, “definable”, “satisfyable”) and fit a schema similar to Russell’s which can also be viewed as subsuming Russell’s schema.

Priest’s modified schema:

1) **<u>Set</u>**: $\Omega = \{y : \varphi(y)\}$ and $\psi(\Omega)$

2) Function δ: $\wp(\Omega) \rightarrow \Omega$ such that for every $X \subseteq \Omega$, if $\psi(X)$, then:

$\delta(X) \notin X$ (Transcendence)

$\delta(X) \in \Omega$ (Inclosure)

3) Contradiction: $\delta(\Omega) \notin \Omega$ and $\delta(\Omega) \in \Omega$.

Consider König’s paradox: We know that not all ordinals are definable in English, for the number of definitions we can construct in English is denumerable whereas the set of all ordinals (On) is not. Granting that On is definable, we can then define a subset of On: the collection of all non definable ordinals. Furthermore we know that any subset of an ordinal (and On is itself an ordinal) has a least element and therefore it makes sense to speak about “the least non-definable ordinal”. But if we count this definite description as a definition of the ordinal in question (given that On and the subset just mentioned are definable) then we have a contradiction[8].

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fallacies, as we saw, are to be avoided by what may be called the ‘vicious-circle rinciple’; i.e., ‘no totality can contain members defined in terms of itself’” (Russell 1908 75).

[8] Perhaps it is not clear that this description provides a definition of the mentioned ordinal, for those who would straightforwardly reject the paradox under this form, it would be possible to replace all

According to Priest this paradox fits his schema perfectly taking φ to be "is a definable ordinal" and ψ to be "is definable":

1) $\Omega = \{y : y \text{ is a definable ordinal}\}$ and Ω is definable.

2) δ: $\wp(\Omega) \rightarrow \Omega$ such that if X is definable and $X \subseteq \Omega$:

$\delta(X)$ = the least ordinal not in X (lon(X) for short).

We can easily check that δ satisfies the conditions of transcendence and inclosure (in Priest's own terminology). Obviously $\delta(X) \notin X$ since $\delta(X)$ is the least ordinal not in X but, given that X is definable, then also lon(X) is definable, hence $\delta(X) \in \Omega$.

The definite description lon(X) provides a definition of an ordinal if and only if X itself is definable. This condition is not ensured by Russell's schema and that is the reason why we need to supply it as a further constraint. This condition is represented by ψ in the modified schema. Priest shows that all semantical paradoxes (according to Ramsey's terminology) fit this schema and also all set theoretic paradoxes by taking ψ to be the trivial property "is identical to itself"[9]. In König's paradox (as in the rest), the contradiction, of course, is $\delta(\Omega) \notin \Omega$ and $\delta(\Omega) \in \Omega$.

Priest's modification of Russell's schema seems to be a really powerful tool to model paradoxes under a limited and always recurrent number of elements and structural patterns. All these paradoxes fit the structure described by the scheme, the question is then, how many paradoxes can we reduce to this scheme? I will consider two (families of) paradoxes in the next section.

4. Tarski's paradox of quotation:

In his famous article "The Concept of Truth in Formalized Languages", Tarski favours an interpretation of quotation marks as "syntactically simple expressions", this is, as "single constituents" of the names where they appear. This interpretation of quotation marks precludes us from reading them as functions that assign to any expression of our language a name of that expression in our language. Among the reasons offered by Tarski to favour his choice we find a very powerful one: if we

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occurrences of 'definable' in the argument with occurrences of 'describable', in this way the paradox is less controversial and still holds.

[9] Priest offers descriptions of the paradoxes due to König, Berry and Richard (which he calls definability paradoxes) and the paradoxes of the liar, liar chain, knower and "heterological", see Priest 1994. He also

consider quotation marks as a function from expressions to names we can derive a semantical paradox[10]. Consider the following sentence: $\forall p\ (c = \text{‘}p\text{’} \rightarrow \neg\, p)$; and the following fact: $c :=_{\text{ref.}} \forall p\ (c = \text{‘}p\text{’} \rightarrow \neg\, p)$ or, in other words, c refers to the sentence ‘$\forall p\ (c = \text{‘}p\text{’} \rightarrow \neg\, p)$’. If we consider as valid the following principle (which seems uncontroversial if we accept that quotation marks are functions): *$\text{‘}P\text{’} = \text{‘}Q\text{’} \rightarrow (P \equiv Q)$*, we obtain a paradox. Here is the reasoning[11]:

1. $c = \text{‘}\forall p\ (c = \text{‘}p\text{’} \rightarrow \neg\, p)\text{’}$	(Premiss 1)
2. $\text{‘}P\text{’} = \text{‘}Q\text{’} \rightarrow (P \equiv Q)$	(Premiss 2)
3. $\forall p\ (c = \text{‘}p\text{’} \rightarrow \neg\, p)$	(Supposition)
4. $c = \text{‘}\forall p\ (c = \text{‘}p\text{’} \rightarrow \neg\, p)\text{’} \rightarrow \neg\, \forall p\ (c = \text{‘}p\text{’} \rightarrow \neg\, p)$	(instance of 3)
5. $\neg\, \forall p\ (c = \text{‘}p\text{’} \rightarrow \neg\, p)$	(Modus Ponens: 1, 4)
6. $\neg\, \forall p\ (c = \text{‘}p\text{’} \rightarrow \neg\, p)$	(absurdum: 3-5)
7. $\exists p\, \neg\, (c = \text{‘}p\text{’} \rightarrow \neg\, p)$	(Def. $\neg$, $\forall$, $\exists$: 6)
8. $\exists p\ (c = \text{‘}p\text{’}\ \&\ p)$	(Def.$\rightarrow$, $\wedge$, $\neg$: 7)
9. $c = \text{‘}q\text{’}\ \&\ q$	(Supposition)
10. $\text{‘}q\text{’} = c = \text{‘}\forall p\ (c = \text{‘}p\text{’} \rightarrow \neg\, p)\text{’}$	(Def. ‘=’: 1, 9)
11. $q \equiv \forall p\ (c = \text{‘}p\text{’} \rightarrow \neg\, p)$	(premiss 2)
12. $\forall p\ (c = \text{‘}p\text{’} \rightarrow \neg\, p)$	(from 9 and 11)
13. $\forall p\ (c = \text{‘}p\text{’} \rightarrow \neg\, p)$	(Def. $\exists$: 9-12)
14. Contradiction: 13, 6	

As many people have pointed out, whether or not we read quotation marks as Tarski suggested,[12] this paradox is still distressing for, if we are not able to solve it, it shows that there is no consistent way of conceiveing of a function sign that produces a name of an expression when appended to the expression in question[13]. I am not concerned here with looking for solutions to this paradox or with examining what its

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shows that under some interpretation of the function δ, it is possible to frame these paradoxes within Russell's schema, however the property ψ plays then a crucial role in defining δ.

[10] For the whole Tarski 1931, pp 159-162

[11] Suppositions and their clousure are represented by tabulations.

[12] Many people have backed up with further arguments Tarski's reading of quotation marks. See {Reference}.

[13] See for example S. Haack 1978, pp. , and S. Soames 1999, pp. 86-92. Anyone interested on the topic can find here some reflections about the problem and further bibliography and information.

consequences are. All what I would like to do here is to check whether Tarski's paradox fits the Russell-Priest scheme or not.

We can intuitively read c as a sentence saying of itself that "it is not true" or, to be more accurate, as a sentence saying that if something is a name of sentence c (where c happens to be the sentence under consideration), then it is false. Consider now the folllowing sentences and their (sometimes several) names:

a. $a = b \rightarrow \neg$ true(a). d. $d = d \rightarrow \neg$ true(d).

b. $a = b \rightarrow \neg$ true(a). e. $\forall x\ (e = x \rightarrow \neg$ true(x)).

Sentences a/b, d and e are paradoxical and bare intuitively some similarities to c (especially e). The most relevant difference is that, apart from the absence of quantifiers in most of them, they all contain occurrences of the truth predicate. If we consider as the simplest version of the liar paradox the sentence l, where $l :=_{ref.} \neg$ true(l), it is clear that all these sentences can be modelled under the modified schema advanced by Priest as versions of the liar paradox. In this case (for 'φ' = 'is true' and 'ψ' = 'is definable') we would always have:

1) $\Omega = T = \{y : \text{true}(y)\}$ and T is definable.

2) δ: $\wp(T) \rightarrow T$ such that if X is definable and $X \subseteq T$: $\delta(X) = l$.

Now, depending on the liar sentence we want to build, we let: $l :=_{ref.} l \notin X$; or $l :=_{ref.} \forall y\ (l = y \rightarrow y \notin X)$; etcetera. Here we read $y \in X$ and $y \notin X$ as X(y) (this is, y is X) and $\neg$ X(y) (y is not X) respectively. 'l' is therefore a name of a sentence that says of itself that it is not X (or that, if something is called 'l', then it is not X; etcetera). Obviously, when we replace X with T (Ω), we obtain a liar sentence, a sentence of the form $l :=_{ref} l \notin T$ (or $l :=_{ref.} \forall y\ (l = y \rightarrow y \notin T)$, etcetera). It is easy to check that δ satisfies the conditions of transcendence and inclosure[14]:

1. Suppose for $X \subseteq T$ that, $\delta(X) \in X$,

2. $l \in X$ (def. δ(X): 1)

3. $l \in T$ ($X \subseteq T$: 2)

4. $\forall x\ (l = x \rightarrow x \notin X)$ ***(T-Scheme: 3)***

[14] Here I include a proof for both, liar sentences of the simplest form and liar sentences like e. The lines (and explanations) **resalted** must be added to show transcendence and inclosure for liar sentences like e. In the same way as we needed an additional principle ('P' = 'Q' $\rightarrow$ P $\equiv$ Q) to show Tarski's paradox, we need now the principle: $a = b \rightarrow P(a) \equiv Q(b)$ to carry out a proof of the Russell-Priest schema for e. We need in both cases the Tarskian T-Schema: $p \equiv x \in T$ ($p \equiv x$ is true), where 'p' is a variable ranging over sentences and 'x' a variable we must replace with a name of sentence p. Proofs for a/b and d are easy to extract from this reasoning.

5. l = l → l ∉ X ***(Instance: 4)***

6. 1 = 1 ***(Theorem)***

7. l ∉ X ***(Modus ponens: 5, 6)*** (T-Scheme:3)

8. We reach a contradiction (2, 7) and show transcendence: δ(X) ∉ X.

Next, we show inclosure:

1. l ∉ X (δ(X) ∉ X, def. δ)

2. l = t ***(supposition)***

3. t ∉ X ***("a = b → P(a) ≡ P(b)": 1, 2)***

4. l = t → t ∉ X ***(Introduction →: 1-3)***

5. ∀x (l = x → x ∉ X) ***(Introduction ∀: 4)***

6. l ∈ T ***(T-Scheme: 5)*** (T-Scheme: 1)

7. δ(X) ∈ T (def. δ: 6)

In the case of liar sentences like a/b, d or e the reasoning is, as we can see, a bit more complicated because they involve logical connectives and we need to draw some inferences before reaching the relevant contradiction, but all these sentences can be understood as examples of sentences that claim of themselves that they are not true. An objection to the possibility of reducing sentence c (Tarski's quotation marks paradox) to e[15] (its closest counterpart among the liar sentences) is that Tarski built this paradox to show that if we use quotation marks as function, we can derive a paradox even dispensing with the notion of truth. However, we do not need to appeal to the notion of truth in order to describe this paradox under the parameters fixed by Priest.

However, we do not need to appeal to the notion of truth in order to describe this paradox under the parameters fixed by Priest. All what we need is to set 'φ' = 'refers to 'p', for some sentence 'p'', 'ψ' = 'is definable'. Assuming that a formula with just one free variable can be interpreted as a predicate (the predicate whose extension is constituted by all those things that turn the formula into a true sentence when we substitute them for the free variable) and assuming, towards a contradiction, that quotation marks can be used in order to form names of expressions, we can express φ in the following way: $\exists_S$p (x = 'p' & 'p' ∈ S). Here we take S to be the set of sentences of our language. Under this construal of φ, Ω is the set of all names of sentences of the relevant language.

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[15] Or even to e', where e'$:_{\text{ref.}}$= ∀p (e' = 'p' → ¬ true ('p')).

1. $\Omega = \{x: \exists_S p\ (x = \text{‘p’}\ \&\ \text{‘p’} \in S)\}$ and Ω is definable.

2) δ: $\wp(\Omega) \to \Omega$ such that if X is definable and $X \subseteq \Omega$: $\delta(X) = c$.

Where, as before, c is the name of a sentence saying that, if c is the name of a sentence, then that sentence can be denied: $c :=_{ref.} \forall_S p\ ((c = \text{‘p’}\ \&\ \text{‘p’} \in X) \to \neg p)$[16]. Once again, δ satisfies the constraints of transcendence and inclosure.

1. Suppose, for $X \subseteq \Omega$, that $\delta(X) \in X$
2. $c \in X$ (def. $\delta(X)$: 1)
3. $c = \text{‘}\forall_S p\ ((c = \text{‘p’}\ \&\ \text{‘p’} \in X) \to \neg p)\text{’}$ (def. c)
4. $\text{‘}\forall_S p\ ((c = \text{‘p’}\ \&\ \text{‘p’} \in X) \to \neg p)\text{’} \in X$ (“$a = b \to P(a) \equiv P(b)$”: 2, 3)
5. $\forall_S p\ ((c = \text{‘p’}\ \&\ \text{‘p’} \in X) \to \neg p)$ (supposition, 5 for short)
6. $(c = \text{‘5’}\ \&\ \text{‘5’} \in X) \to \neg 5$ (Instance: 5)
7. $c = \text{‘5’}\ \&\ \text{‘5’} \in X$ (3, 4)
8. $\neg 5$ (Modus Ponens: 6, 7)
9. $\neg 5$ (Absurdum: 5-8)
10. $\exists p\ (c = \text{‘p’}\ \&\ p)$ (Theorem: 9)
11. $c = \text{‘q’}\ \&\ q$ (Supposition)
12. $\text{‘q’} = c = \text{‘5’}$ (11, 3)
13. $q \equiv 5$ ($\text{‘P’} = \text{‘Q’} \to (P \equiv Q)$)
14. 5 (from 11, 13)
15. 5 (Def. $\exists$: 11-14)
16. Contradiction 9, 15
17. We cancel here our supposition that $\delta(X) \in X$ and infer transcendence: $\delta(X) \notin X$. WRONG

EL PROBLEMA ÉS QUE HE CONFÒS X i Ω AMB SUBCONJUNTS DE S QUAN S ÉS EL CONJUNT DE TOTES LES ORACIONS DE CERT LLENGUATGE I x i Ω SÓN CONJUNTS DE NOMS D'ORACIONS DEL LLENGUATGE EN QÜESTIÓ. Sobra el supòsit que ‘$\forall p\ ((c = \text{‘p’}$ **& ‘p’ $\in$ S**$) \to \neg p)$

[16] I have slightly modified the earlier formulation in order to ensure that c is indeed the name of a sentence and to build the paradox hinging on the set S of sentences rather that T of “truths”. Due to this maneouvre I have also had to include the principle

Our proof of inclosure is much simpler, it just relies on the fact that $\delta(X)$ (this is, c) is indeed a name of the sentence '$\forall p$ ((c = 'p' & 'p' $\in$ X) $\rightarrow \neg$ p)' and that this sentence, namely, c, is a well formed sentence (according to some definition of sentence) and therefore belongs to S. We conclude then that $\delta(X) \in \Omega$ ($c \in \Omega$).

5. A further example: Paradoxes of denotation:

Barwise and Moss' paradox of denotation presents a term interpreted in the following way:

t_1 refers to 1 (if t_1 refers to 0)
t_1 refers to 0 (if t_1 refers to 1)
t_1 refers to 0 (if t_1 does not uniquely refer to anything)

Paradoxes of this kind can be represented in the following way: For a set Ω consisting of all referential expressions of a language L that actually denote something in a domain D by means of an interpretation I of the elements of L in D, suppose we have a linguistic term τ whose interpretation according to I is determined by the following rule:

$I(\tau) = 1$, but if $\tau \in X \subseteq \Omega$, then $I(\tau) \neq I(\tau)$

This is τ is interpreted as $1 \in D$ under I, however if the denoting term τ belongs to some set of terms X included in Ω (which, as we shall see, is the set of al expressions with a denotation in the language L), then the interpretation of τ is different from itself, this is, a contradiction follows.

Here we have:

$\Omega = \{\alpha \in L: \exists x\ (x \in D \ \&\ I(\alpha) = x)\}$ where Ω is definable.
For $X \subseteq \Omega$, $\delta(X) = \tau$, where X is definable.

Clearly, $\delta(X) \notin X$, for suppose $\delta(X) \in X$, then we would have $\delta(X) = \tau$, $I(\tau) = 1$and thus $I(\tau) \neq I(\tau)$ (def. of τ), this is $1 \neq 1$, which is obviously false. Therefore τ satisfies transcendence

However, $\delta(X) \in \Omega$, since $\delta(X) = \tau$, $I(\tau) = 1$and thus $\tau \in \Omega$, satisfying inclosure.
The paradox arises when we consider $\delta(\Omega)$ and $\tau =_{def.} I(\tau) = 1$, but if $\tau \in \Omega$, then $I(\tau) \neq I(\tau)$. Then we have $\delta(\Omega) \notin \Omega$ and $\delta(\Omega) \in \Omega$.

6. Curry paradoxes.

I want to hold here that this kind of paradoxes is closely related to Tarski's paradox of quotation.

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