# Geometry and Topology of Manifolds

## 2013-2014

### Objectives

- To define orientability of topological manifolds and of smooth manifolds.
- To determine the multiplicative structure of singular cohomology.
- To calculate with differential forms on manifolds.
- To prove Stokes' theorem on manifolds and related results.
- To prove that de Rham cohomology coincides with real singular cohomology.
- To learn Poincaré duality and some of its consequences.
- To discuss the existence of nonzero sections of vector bundles on manifolds.

### **Evaluation of learning outcomes**

Continuous assessment:

- Problem solving or short assignments: 50%.
- Partial and final written exams: 50%.

*Examination-based assessment* is optional and will consist of a single written exam. All students are admitted to re-evaluation, which will also consist of a written exam. The final grade will be the highest score between evaluation and re-evaluation.

#### **Teaching blocks**

- 1. <u>Singular cohomology</u>. Homotopy invariance. Mayer-Vietoris exact sequence. Cellular complexes. Universal coefficients. Products.
- 2. <u>Manifolds</u>. Manifolds with boundary and without boundary. Smooth structures. Tangent bundle and cotangent bundle.
- 3. <u>Cohomology of differential forms</u>. Differential forms on manifolds. De Rham cohomology. Integration of forms with compact support. Stokes' Theorem. De Rham's Theorem.
- 4. <u>Poincaré duality</u>. Orientation classes. Compact support cohomology. Duality. Consequences and applications.

### Reading and study resources

R. Bott, L. W. Tu, *Differential Forms in Algebraic Topology*, Graduate Texts in Mathematics, vol. 82, Springer, New York, 1986 (1st ed. 1982).

G. E. Bredon, *Topology and Geometry*, Graduate Texts in Mathematics, vol. 139, Springer, New York, 1993.

M. P. do Carmo, Differential Forms and Applications, Universitext, Springer, Berlin, 1994.

M. J. Greenberg, J. R. Harper, *Algebraic Topology: A First Course*, Mathematics Lecture Note Series 58, Addison-Wesley, Reading, 1981.

A. Hatcher, Algebraic Topology, Cambridge University Press, Cambridge, 2002.

K. Jänich, Vector Analysis, Undergraduate Texts in Mathematics, Springer, New York, 2001.

S. Morita, *Geometry of Differential Forms*, Translations of Mathematical Monographs, vol. 201, American Mathematical Society, Providence, 2001.

J. W. Vick, *Homology Theory: An Introduction to Algebraic Topology*, Graduate Texts in Mathematics, vol. 145, Springer, New York, 1994 (1st ed. Academic Press, 1973).