Knowledge licensing in a model of R&D-driven endogenous growth

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Abstract

In this paper I present an endogenous growth model where the engine of growth is in-house R&D performed by high-tech firms. I model knowledge (patent) licensing between high-tech firms. I show that if there is knowledge licensing, high-tech firms innovate more and economic growth is higher than in cases when there are knowledge spillovers or there is no exchange of knowledge between high-tech firms. However, in case when there is knowledge licensing the number of high-tech firms is lower than in cases when there are knowledge spillovers or there is no exchange of knowledge.

Keywords: Knowledge licensing; Intra-firm R&D; Competitive pressure; Endogenous growth

JEL classification: O30; O41; L16

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1 Introduction

A number of growth models treat private firms’ intentional investments in R&D as the driver of long-run growth and welfare (e.g., Romer, 1990; Aghion and Howitt, 1992; Smulders and van de Klundert, 1995). In these models R&D generates knowledge that can be used for subsequent innovations. Knowledge is non-rival and is partly non-excludable. It is excludable to the extent that firms can use it in order to secure (at least a temporary) monopoly position in the product market. It is not excludable, however, to the extent that there are knowledge spillovers and R&D builds on a pool of knowledge. In this sense these growth models abstract from the role of knowledge (patent) licensing and from the details about the exchange of knowledge in the economy. Nevertheless, licensing and establishing consortia for exchanging patents is common in high-tech industries (e.g., Hagedoorn, 1993, 2002).

Moreover, it has been argued extensively that exchanging patents plays a significant role for innovation in these industries (e.g., Grindley and Teece, 1997; Shapiro, 2001; Clark et al., 2000). For instance, yet at the beginning of the previous century some of the major players in the Radio, Television and Communication Equipment industry in the United States used their patents to block each other’s advance until the establishment of RCA Corporation, a patent consortium. Meanwhile, high-tech industries are the top private R&D performers and there is a large body of anecdotal and rigorous empirical evidence that they make a significant contribution to economic growth (e.g., Helpman, 1998; Jorgenson et al., 2005).

In this paper I present an endogenous growth model where high-tech firms engage in intra-firm (or in-house) R&D and that drives long-run growth. High-tech firms have exclusive rights to the type of their product. In a high-tech firm the innovation enhances firm/product-specific knowledge which reduces the firm’s marginal costs or increases the quality of its product. High-tech firms finance their R&D expenditures from operating profits. They set prices and compete strategically in their output market. My point of departure is that I model knowledge (patent) licensing between high-tech firms. The knowledge generated in a high-tech firm cannot be used for free, but it can be licensed. Given that each high-tech firm produces a distinct type of good, for a high-tech firm the knowledge of other high-tech firms is complementary.

1 In terms of 2-digit ISIC (Rev. 3), according to OECD STAN data high-tech industries as measured by R&D intensity are, for example, 24, 29, 30, 31, 32, 33, 34, 35, and 72.

2 Currently, there are virtually no comprehensive data for measuring the size of the market for patents and other types of intellectual property. According to some estimates (Robbins, 2009) in the US in 2002 corporate domestic income from licensing patents and trade secrets was $50 billion. It has grown at an average rate of 10 percent per year in 1994–2004 period. In comparison, total private R&D expenditure was approximately $200 billion according to OECD data.
to its own. If a high-tech firm licenses the knowledge of another it can combine that knowledge with its own and improve its in-house R&D process since the latter builds on the knowledge that the firm possesses.

In such a setup I show how market concentration, intensity of competition as measured by the elasticity of substitution between high-tech goods, and type of competition (Cournot or Bertrand) can matter for innovation in the high-tech industry and aggregate performance. I contrast the inference from this setup to the inference from setups where there is no exchange of knowledge between high-tech firms and/or there are knowledge spillovers (i.e., firms obtain others’ knowledge for free). Further, it is often conjectured that the use of high-tech goods such as phones and PCs entails positive externalities, which lower the transaction costs and increase the efficiency of users (e.g., [Leif 1984]). I assess how innovation in the high-tech industry and aggregate performance depend on the magnitude of such externalities.

I show that when there is an exchange of knowledge between high-tech firms in the form of licensing or spillovers, innovation in the high-tech industry and economic growth increase with the number of high-tech firms. The driver behind this result is the relative price distortion which is due to price setting by high-tech firms. This distortion adversely affects the demand for high-tech goods. A higher number of firms implies lower mark-ups and lower distortion. This increases the demand for high-tech goods and implies higher output and investments in R&D in the high-tech industry. However, if there is no exchange of knowledge between high-tech firms, then increasing the number of firms has two effects on innovation in the high-tech industry. One is the lower distortion, which is positive. Another is negative and is due to lower amount of R&D inputs available per high-tech firm. When the number of high-tech firms is relatively low the positive effect dominates, whereas for a relatively high number of firms the negative effect dominates. This negative effect when there is knowledge exchange between high-tech firms is offset by more complementary knowledge available by high-tech firms.

I further show that in all the setups that I consider, innovation in the high-tech industry and economic growth increase with the intensity of competition. Tougher competition, which is defined as the type of competition with lower mark-ups (Bertrand vs. Cournot; [Sutton 1991]), also implies more innovation in the high-tech industry and higher growth. These results are in line with the results of [Smulders and van de Klundert 1995] and [van de Klundert and Smulders 1997], and hold because both more intensive and tougher competition reduce mark-ups.

[O’Donoghue and Zweimüller 2004] have a similar result in a Schumpeterian growth model. [Vives 2008] shows that such a result can also hold in partial equilibrium for various types of demand functions.
and the relative price distortion.

The availability of complementary knowledge also motivates innovation in the high-tech industry. High-tech firms innovate more and the economy grows at a higher rate if there is an exchange of knowledge between high-tech firms than if there is no exchange. This is because R&D builds on a bigger pool of knowledge if there is an exchange of knowledge. Moreover, if there is no knowledge exchange high-tech firms might not innovate at all if there are many of them in the market. The driver behind this result is the scarcity of R&D inputs available per high-tech firm if there are many such firms. High-tech firms also innovate more and the economy grows at a higher rate when there is knowledge licensing compared to the case when there are knowledge spillovers between high-tech firms.

The higher magnitude of positive externalities from the use of high-tech goods implies lower innovation in the high-tech industry. Nevertheless, economic growth increases with the magnitude of these externalities. Innovation declines because the higher magnitude of positive externalities brings no additional internalized benefit to high-tech firms and in equilibrium it implies a higher rate of interest. In turn, economic growth increases since the higher magnitude of externalities implies a higher contribution of innovation in the high-tech industry to growth.

Finally, I endogenize the number of high-tech firms assuming cost-free entry. Innovation in the high-tech industry and economic growth are the lowest in case when there is no exchange of knowledge between these firms. In turn, innovation in the high-tech industry and economic growth are the highest in case when there is knowledge licensing between these firms. This happens, however, in expense of the number of high-tech firms (or the variety of high-tech goods.) In other words, the number of high-tech firms is the lowest in case when there is knowledge licensing and the highest in case there is no exchange of knowledge.

Increasing the intensity and/or toughness of competition reduces the number of firms. When there is an exchange of knowledge between high-tech firms this has no effect, however, on allocations, innovation in the high-tech industry, and economic growth. Meanwhile, allocations change and innovation and economic growth tend to increase with the intensity and toughness of competition if there is no exchange of knowledge between high-tech firms.

This paper is related to the endogenous growth literature (e.g., Romer 1990; Aghion and Howitt 1992; Smulders and van de Klundert 1995) where the positive growth of the economy on a balanced growth path is a result of technological and

\footnote{The positive relation between innovation and different types of competitive pressure is consistent with the empirical findings of, for example, Blundell et al. (1999) and Nickell (1996).}
preference factors. In particular, it is related to studies which in an endogenous growth framework suggest how the aggregate performance can be affected by imperfect competition in an industry where the firms engage in in-house R&D (e.g., van de Klundert and Smulders, 1997). It contributes to these streams of studies while showing how knowledge licensing in such an industry can affect innovation in that industry and the aggregate performance. It also contributes while showing how the positive externalities from the use of the goods of such an industry can affect the decentralized equilibrium outcomes.

Methodologically, this paper is also related to the multi-sector growth literature (e.g., Ngai and Pissarides, 2007; Acemoglu and Guerrieri, 2008), which analyzes the sources and the implications of sectoral growth differences. Primarily it is related to Vourvachaki (2009), which analyzes the impact of information and communication technologies on aggregate performance while focusing on inter-sector interactions. In contrast, this paper focuses on the inter-firm interactions and in-house R&D process in the high-tech industry. The contributions to this literature are the same as those to the endogenous growth theory.

Further, there is a number of papers that model knowledge (patent) and technology licensing in standard Schumpeterian growth framework and show how patent policy and international technology licensing can affect innovation and growth (e.g., O’Donoghue and Zweimüller, 2004; Yang and Maskus, 2001; Tanaka et al., 2007). In these papers licensing happens between incumbents and entrants given that in standard Schumpeterian growth framework incumbents have no incentives to innovate. Licensing does not explicitly aid R&D process and licenses are essentially permits for production. In such a framework in order to maintain incentives for licensing, these papers assume that either firms collude in the product market or licensees can access larger market (e.g., one of the countries bans FDI). The share in collective profits and licensing fees compensate incumbents’ loss of the product market (and costs of technology transfer) and are either exogenous or exogenously determined by patent policy. In contrast, this paper has a non-tournament framework where incumbents innovate because that allows stealing market share and licensing happens between incumbents. Firms have incentive to license knowledge from other firms because that aids their R&D process. Further, license fees are determined by the structure of the market for knowledge, which can depend on patent policy, and supply and demand conditions. To that end, the framework and analysis of this paper can be thought to be complementary.

There is also a large body of firm- and industry-level studies that analyze the implications of patent licensing, patent consortia or pools, and knowledge exchange
between firms on innovation and market conduct (e.g., Gallini 1984; Gallini and Winter 1985; Shapiro 1985; Katz and Shapiro 1985; Bessen and Maskin 2009).

This paper analyzes such issues at the aggregate level in a dynamic general equilibrium framework which assumes an undistorted market for knowledge/patents. This assumption allows to have tractable inference and can be justified to the extent that this paper aims to address long-run issues, for example. In turn, the dynamic general equilibrium framework allows to endogenize the growth rate of the economy and the effect of knowledge licensing on, for example, interest rate which affects the incentives to perform R&D. Licensing in this paper *ceteris paribus* motivates R&D. This, in turn, implies higher growth rate and higher rate of interest which reduces the incentives to perform R&D.

The next section offers the model. Section 3 analyzes the features of dynamic equilibrium. Section 4 concludes.

# 2 The model

## Households

The economy is populated by a continuum of identical and infinitely lived households of mass one. The representative household is endowed with a fixed amount of labor $(L)$. It inelastically supplies its labor to firms which produce final goods and to high-tech firms. The household has a standard CIES utility function with an intertemporal substitution parameter $\frac{1}{\theta}$ and discounts the future streams of utility with rate $\rho$ ($\theta, \rho > 0$). The utility gains are from the consumption of amount $C$ of final goods. The lifetime utility of the household is

$$U = \int_0^{+\infty} \frac{C_i^{1-\theta} - 1}{1 - \theta} \exp(-\rho t) \, dt. \quad (1)$$

The household maximizes its lifetime utility subject to a budget constraint,

$$\dot{A} = rA + wL - C, \quad (2)$$

where $A$ are the household’s asset holdings [$A(0) > 0$], $r$ and $w$ are the market returns on its asset holdings and labor supply.

The optimal rule that follows from the household’s optimal problem is the standard Euler equation,

$$\frac{\dot{C}}{C} = \frac{1}{\theta} (r - \rho). \quad (3)$$
This, together with budget constraint (2), describes the paths of the household’s consumption and assets/savings.

**Final goods**

Final goods are homogeneous, $Y$. The household’s demand for final goods is served by a representative producer. The production of final goods requires labor and $X$, which is a Dixit-Stiglitz composite of high-tech goods $\{x_i\}_{i=1}^N$, with an elasticity of substitution $\varepsilon$.

*Ceteris paribus* the increasing demand of $X$ creates externalities in final goods production, which are measured by $\tilde{X}$. These externalities increase the productivity of the final goods producers. For example, these externalities stand for network effects that stem from using high-tech goods such as PCs and phones.

The production of the final goods has a Cobb-Douglas technology and is given by

$$
Y = \tilde{X}X^\sigma L_Y^{1-\sigma}, \quad (4)
$$

$$
X = \left( \sum_{i=1}^N x_i \right)^{\frac{\varepsilon}{\varepsilon+1}}, \quad (5)
$$

where $L_Y$ is the share of the labor force employed in final goods production.

For ease of exposition, the problem of the representative final goods producer is divided into two steps. In the first step the representative producer decides on the optimal combination of $L_Y$ and $X$ in $Y$ and in the second step it decides on the optimal amounts of high-tech goods $x$ in $X$.

Therefore, in the first step the representative producer solves the following problem.

$$
\max_{L_Y,X} Y - wL_Y - P_X X
$$

s.t.

(4),

where $P_X$ is the price (marginal value) of $X$ and $Y$ is the numeraire. The optimal rules that follow from this problem describe the final goods producer’s demand for
labor and the optimal amount of $X$ for the production of $Y$,

$$[L_Y] : wL_Y = (1 - \sigma) Y, \quad (6)$$

$$[X] : P_X X = \sigma Y. \quad (7)$$

In the second step the producer solves

$$\max_{\{x_i\}_{i=1}^N} P_X X - \sum_{i=1}^N p_{x_i} x_i,$$

s.t.

$$\quad (5),$$

where $p_{x_i}$ is the price of $x$. This implies that the demand for a high-tech good is

$$[x_j] : x_j = X \left( \frac{P_X}{p_{x_j}} \right) \varepsilon. \quad (8)$$

From this expression follow two equilibrium conditions,

$$P_X X = \sum_{i=1}^N p_{x_i} x_i, \quad (9)$$

$$P_X = \left( \sum_{i=1}^N p_{x_i}^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}, \quad (10)$$

where the first means that there is no waste and the second implies that $P_X$ is an index of $p_{x_i}$.

Further, I assume that the measure of externalities $\tilde{X}$ is given by

$$\tilde{X} = X^\mu,$$

where the parameter $\mu$ measures the strength of these externalities and

$$1 - \sigma > \mu \geq 0. \quad (\text{9})$$

**High-tech goods**

At any time $t$ there are $N(t)$ producers in the high-tech industry.$^6$

$^5$ It is necessary to have $1 - \sigma > \mu$ in order for the production function of final goods $\Phi$ to be concave in $X$ in the Social Planner’s problem.

$^6$ In order to avoid complications arising from integer constraints I allow $N$ to be real number.
Production

Each high-tech firm owns a design (blueprint) of distinct high-tech good \( x \), which it produces. The production of a high-tech good requires labor input \( L_x \). The production function of a high-tech good \( x \) is

\[
x = \lambda L_x,
\]

where \( \lambda \) measures the producer’s knowledge of production process or quality of the high-tech good. This knowledge is firm/product-specific since each high-tech firm produces a distinct good.

High-tech firms are price setters in their output market and discount their future profit streams \( \pi \) with the market interest rate \( r \). I assume that high-tech firms cannot collude in the output market.

Knowledge accumulation

High-tech firms can engage in R&D for accumulating knowledge and increasing \( \lambda \). This can be interpreted as a process innovation that increases productivity (the firms are able to produce more of \( x \)), or as a quality upgrade (the firms are able to produce the same amount of higher quality \( x \)). Knowledge is non-rival so that potentially it can be used at the same time in multiple places/firms.

In this section I offer three different settings of knowledge accumulation/R&D process. The differences stem from whether and how knowledge is exchanged between high-tech firms.\(^7\)

Hereafter, when appropriate for ease of exposition I describe the properties of the high-tech industry while taking as an example high-tech firm \( j, j \in (1, N] \). In order to improve its knowledge \( \lambda_j \) the firm needs to hire researchers/labor \( L_{rj} \). Researchers use the current knowledge of the firm in order to create better knowledge.

Knowledge licensing: This is the benchmark setup (S.1). Knowledge in this setup can be licensed. In the market for knowledge the licensors (or the suppliers of knowledge) have the bargaining power in the sense that they can make a ‘take it or leave it’ offer.

\(^7\)The functional forms of the knowledge accumulation processes are selected so that they ensure a balanced growth path.

\(^8\)In these setups each high-tech firm engages in in-house R&D and there is no R&D cooperation. Appendix E.1 of the online version of this paper analyzes the case when firms cooperate in R&D and compete in the product market.
The benefit from licensing knowledge is that it can be used in the in-house R&D process. If high-tech firm $j$ decides to license knowledge from other high-tech firms, its researchers combine that knowledge with the knowledge available in the firm in order to produce new knowledge. The knowledge available in the firm is an essential input in the knowledge accumulation process of the firm. Moreover, it is the only essential input. This implies that the high-tech firm does not need to acquire knowledge from other firms in order to advance its own. However, it needs to have its knowledge for building on it. This is in line with that high-tech firms produce distinct goods\(^9\).\(^{10}\)

The knowledge accumulation/R&D process is given by

$$\dot{\lambda}_j = \xi \left[ \sum_{i=1}^{N} (u_{i,j} \lambda_i)^\alpha \right] \lambda_j^{1-\alpha} L_{r_j},$$  \hspace{1cm} (12)

where $\xi$ is an exogenous efficiency level, $u_{i,j}$ is the share of knowledge of firm $i$ ($\lambda_i$) that firm $j$ licenses, and $u_{j,j} \equiv 1$\(^{11}\).

It can be shown that in (12), the elasticity of substitution between the different types of knowledge that the high-tech firm can license is equal to $\frac{1}{1-\alpha}$. It can also be shown that the elasticity of substitution between the high-tech firm’s knowledge and any of the different types of knowledge that it can license is lower than $\frac{1}{1-\alpha}$. This restates the importance of the firm’s knowledge for its knowledge accumulation process.

In this knowledge accumulation process because of summation the productivity of researchers increases linearly with knowledge licensed from an additional high-tech firm. Such a formulation can be justified if there are significant complementarities between the knowledge of high-tech firms.\(^{12}\) Further, it might seem brave to assume that knowledge accumulation in a single firm can have non-decreasing returns.\(^{13}\) This assumption allows to focus on the effect of market structure of the high-tech industry on innovation in that industry through competitive pressure. It

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9 One way to think about this setup is that high-tech firms can use their knowledge (patents) and build around others’ knowledge.

10 I assume that license contracts do not allow licensees to re-license the acquired knowledge.

11 In the online version of this paper, Appendix E.2 incorporates knowledge spillovers and depreciation in this R&D process.

12 Rivera-Batiz and Romer (1991) and Grossman and Helpman (1995) have a similar additive structure for knowledge in the R&D process in the context of knowledge spillovers between countries. Peretto (1998a) and Peretto (1998b) have a similar structure in the context of knowledge spillovers between firms in an industry.

13 In this respect, a high-tech firm can be a firm that started with tabulating machines and reached the point of producing supercomputers and artificial intelligence systems (e.g., IBM).
can be relaxed setting \( u_{j,j} \equiv 0 \) in square brackets in \((12)\). In such a case in this model knowledge licensing (or exchange of knowledge) is a necessary condition to ensure non-decreasing returns to knowledge accumulation and positive growth in the long-run.\(^{14}\) The knowledge accumulation process \((12)\) can also be viewed as a simplification leading to tractable results. It ensures that there exists a balanced growth path, for example.

One way to think about this setup is that each high-tech firm can license the patented knowledge of other firms in order to generate its patented knowledge that helps to improve its production or output. The firm does not use the knowledge that it licensed directly in the production of its high-tech good because that knowledge needs to be combined with its own knowledge, and that requires investments in terms of hiring researchers (and time). The latter seems plausible for technologically sophisticated (e.g., high-tech) goods.

**Knowledge spillovers:** In this case \((S.2)\) there are knowledge spillovers between high-tech firms. In high-tech firm \(j\) the researchers combine the knowledge that spills over from other high-tech firms with the knowledge available in the firm while generating new knowledge. In order to maintain symmetry I assume also that the researchers do not fully internalize the use of the current knowledge available in the firm and have external benefits from it. Similar to the previous setup, this assumption allows to focus on the effect of market structure of the high-tech industry on innovation through competitive pressure.\(^{15}\)

The knowledge accumulation process is

\[
\dot{\lambda}_j = \xi \tilde{\Lambda} \lambda_j^{1-\alpha} L_{r_j}, \tag{13}
\]

where I assume that in equilibrium \(\tilde{\Lambda}\) is given by

\[
\tilde{\Lambda} \equiv \sum_{i=1}^{N} \lambda_i^a \tag{14}
\]

An interpretation of this case is that there are knowledge externalities/spillovers within high-tech firms and there is a market for knowledge where the potential licensees have a right to make a ‘take it or leave it’ offer. The licensees under this assumption receive the knowledge at no cost (i.e., there are knowledge spillovers) if

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\(^{14}\)Appendix E.3 of the online version of this paper offers the main properties of the model if \( u_{j,j} \equiv 0 \).

\(^{15}\)Appendix E.3 of the online version of this paper relaxes this assumption and offers the main properties of the model.

\(^{16}\)Peretto (1998a) and Peretto (1998b) model the knowledge accumulation process similar to \((13)\) and \((14)\), though these papers assume that \(\alpha = 1\).
the supply of knowledge is not elastic. The supply is necessarily inelastic if licensors
do not have trade-offs and/or costs associated with licensing knowledge. It seems
natural to assume that once knowledge is created its supply entails virtually no
costs. Meanwhile, there would be no trade-offs if licensors do not take into account
that the knowledge they license is used for business stealing: the licensees use it in
order to reduce their prices and steal market share. I assume that licensors do not
take into account this effect.

Such an assumption is not new to this line of literature. Many papers (e.g.,
van de Klundert and Smulders [1997]) assume that the originators of knowledge
spillovers (here, high-tech firms) do not internalize the effect of spillovers (here,
licensed knowledge) on other’s knowledge accumulation and production processes.
This assumption helps to avoid complications in differential games arising from
the dependence of the current choice on the entire future (or history) of states. 17
Further, in the frames of this model this assumption is necessary in order to give
such a market-based interpretation to knowledge spillovers, which links this setup
(S.2) with the previous one (S.1).

In this model, similar to λ, the design of a high-tech good can be interpreted
as knowledge/patent. In order to guarantee that high-tech firms have incentives to
innovate it needs to be assumed that (at least for sometime) the knowledge on the
design of high-tech goods does not spill over or cannot be used by other firms without
appropriate compensation. Any high-tech firm, nevertheless, could sell the design of
its high-tech good at market value: the discounted sum of the profit streams earned
selling the high-tech good. 18 Therefore, the market structure of knowledge on the
production process or the quality of high-tech goods λ where the licensors have a
right to make a ‘take it or leave it’ offer seems to be more appropriate in such a
setup.

In this model λ can also be viewed as a patent on the production process or the
quality of the product. Such market-based interpretations are then appropriate if,
for example, there is strong enforcement of intellectual property rights and patent
infringements are detectable. Given the recent history of the high number of patent
infringement lawsuits in high-tech industries, both assumptions seem to be plausi-
ble. Meanwhile, the existence of lawsuits can indicate that the market structure of
knowledge/patents where buyers have a right to make a ‘take it or leave it’ offer

17 In this model, under this assumption high-tech firms do not realize that the knowledge which
they accumulate enters the knowledge accumulation process of other high-tech firms and from the
next instance augments their rivals’ productivity. If they realized that, then by integrating over
the (future) changes of knowledge of their rivals they could track how their current investment in
knowledge affects the productivity and market share of their rivals in the future.

18 This simply implies that the name of the high-tech firm does not matter.
may not be realistic.

**No exchange of knowledge:** In this case (S.3) there is no exchange of knowledge between high-tech firms. Moreover, in the process of generation of new knowledge the researchers do not fully internalize the use of previous knowledge available in the firm and have external benefits from it.

The knowledge accumulation is given by

\[ \dot{\lambda}_j = \xi \tilde{\lambda} \lambda_j^{1-\alpha} L_{r_j}, \]  

(15)

where \( \tilde{\lambda} \) stands for the external benefits and I assume that in equilibrium

\[ \tilde{\lambda} \equiv \lambda_j^{19} \]  

(16)

Clearly, in a symmetric equilibrium this case can also be interpreted as if there is an exchange of knowledge between high-tech firms in terms of spillovers and these spillovers are from average knowledge, i.e.,

\[ \tilde{\lambda} \equiv \left( \frac{1}{N} \sum_{i=1}^{N} \lambda_i \right) ^{\alpha}. \]

The spillovers from average knowledge, however, mask the existence of complementarity between the knowledge available in different firms. This is because the spillovers from an additional firm bring no additional benefit. Therefore, I prefer avoiding this interpretation. In this respect, the knowledge accumulation process (15) and (16) can be interpreted as if the exchange of knowledge between high-tech firms is banned (or there is no appropriate institutional framework for it). It is clear that (12) and (13) reduce to (15) if there is no knowledge exchange between high-tech firms [i.e., (12) and (15) are equivalent if \( u_{i,j} = 0 \) for any \( i \neq j \) and limiting case \( \alpha = 0 \); (13) and (15) are equivalent if \( \lambda_i = 0 \) for any \( i \neq j \)].

**Optimal problem**

The revenues of high-tech firm \( j \) are gathered from the supply of its high-tech good and in the case when there is knowledge licensing (S.1) from the supply of its knowledge \( (u_{j,i}\lambda_j; \forall i \neq j) \). The costs are the labor compensations and license fees in case when there is knowledge licensing. The high-tech firm maximizes the present discounted value \( V \) of its profit streams subject to (8), (11), and either (12), or (13),

\[^{19}\text{van de Klundert and Smulders (1997) have a similar formulation for the knowledge accumulation process.}\]
or (15). Under Cournot competition, the high-tech firm chooses the supply of its product (i.e., $L_{x_j}$) given the (inverse) demand for it. In contrast, under Bertrand competition the firm chooses the price of its product (i.e., $p_{x_j}$) given the demand for it.

Formally, the problem of the high-tech firm is

Formally, the problem of the high-tech firm is

$$
\max_{\text{Cournot}}: \quad L_{x_j} \left( x_{j} \right) \quad \text{subject to} \quad \left( x_{j} \right) \quad \text{and either} \quad \left( \begin{array}{c} \text{(8)} \end{array} \right) \quad \text{or} \quad \left( \begin{array}{c} \text{(12)} \end{array} \right), \quad \text{or} \quad \left( \begin{array}{c} \text{(13)} \end{array} \right), \quad \text{or} \quad \left( \begin{array}{c} \text{(15)} \end{array} \right).
$$

where $t$ is the entry date and

$$
\pi_j = p_{x_j} x_j - w \left( L_{x_j} + L_{r_j} \right) + \left[ \sum_{i=1}^{N} p_{u_{j,i} \lambda_j} \left( u_{j,i} \lambda_j \right) - \sum_{i=1}^{N} p_{u_{i,j} \lambda_i} \left( u_{i,j} \lambda_i \right) \right].
$$

In profit function $\pi_j$ the term in square brackets stands for knowledge licensing, and $p_{u_{j,i} \lambda_j}$ and $p_{u_{i,j} \lambda_i}$ are the prices of $u_{j,i} \lambda_j$ and $u_{i,j} \lambda_i$.

The solution of the optimal problem implies that the supply of high-tech good $x_j$ and the demand for labor for knowledge accumulation are

$$
\left[ L_{x_j} \right]: \quad w = \lambda_j p_{x_j} \left( 1 - \frac{1}{e_j} \right),
$$

$$
\left[ L_{r_j} \right]: \quad w = q_{\lambda_j} \frac{\dot{\lambda_j}}{L_{r_j}},
$$

where $e_j$ is the elasticity of substitution between high-tech goods perceived by the high-tech firm and $q_{\lambda_j}$ is the shadow value of knowledge accumulation.

The perceived elasticity of substitution ($e_j$) varies with competition type. It can be shown that under Bertrand competition

$$
\varepsilon_j^B \equiv e_j = \varepsilon - \left[ \frac{\left( \varepsilon - 1 \right) p_{x_j}^{1-\varepsilon}}{\sum_{j=1}^{N} \left( \left( \varepsilon - 1 \right) p_{x_j}^{1-\varepsilon} \right)} \right],
$$

13
and under Cournot competition

\[ e_j^C \equiv e_j = \varepsilon \left\{ 1 + \left[ (\varepsilon - 1) \frac{\sum_{i=1}^{N} x_i^{\varepsilon-1}}{x_j^{\varepsilon}} \right] \right\}^{-1}. \quad (22) \]

The terms in square brackets in (21) and (22) measure the impact of other high-tech firms on the demand of high-tech firm \( j \). In other words, they measure the extent of strategic interactions between high-tech firms. Moreover, these terms indicate the difference between the perceived elasticity of substitution (\( e \)) and the actual elasticity of substitution (\( e_j \)). Therefore, they indicate some of the distortions in the economy which stem from imperfect competition with a finite number of high-tech firms. In a symmetric equilibrium, when the number of firms increases, these distortions tend to zero since the terms in square brackets tend to zero.

If there is knowledge licensing (S.1) the returns on knowledge accumulation are

\[
\left[ \lambda_j \right] : \frac{\dot{q}_{\lambda_j}}{q_{\lambda_j}} = r - \left( \frac{e_j^k - 1}{e_j^k} p_{x_j} L_{x_j} + \frac{\partial \lambda_j}{\lambda_j} \frac{\partial \lambda_j}{\partial \lambda_j} + \sum_{i=1, i \neq j}^{N} \frac{p_{u_{i,j}} \lambda_j u_{j,i}}{q_{\lambda_j}} \right), \quad k = C, B, \quad (23)
\]

where the first term in brackets illustrates the benefit from accumulating knowledge in terms of increased output. The second term illustrates the benefit in terms of higher amount of knowledge available for subsequent knowledge accumulation,

\[
\frac{\partial \lambda_j}{\partial \lambda_j} = \xi \left[ 1 + (1 - \alpha) \sum_{i=1, i \neq j}^{N} \left( \frac{u_{i,j} \lambda_j}{\lambda_j} \right)^\alpha \right] L_{r_j} \quad (24)
\]

In turn, the third term in brackets illustrates the benefit in terms of increased amount of knowledge that can be licensed.

The demand for and the supply of knowledge in this case are

\[
\left[ u_{i,j} \right] : p_{u_{i,j}} = q_{\lambda_j} \xi \alpha \left( \frac{\lambda_j}{u_{i,j} \lambda_j} \right)^{1-\alpha} L_{r_j}, \quad \forall i \neq j \quad (25)
\]

\[
\left[ u_{j,i} \right] : u_{j,i} = 1, \quad \forall i \neq j \quad (26)
\]

which means that the firm has an elastic and downward sloping demand for knowledge and licenses/supplies all its knowledge.

If there are knowledge spillovers between high-tech firms (S.2) the returns on
knowledge accumulation are given by (23) but

\[ p_{u,i,j} = 0, \forall i, \]  \hspace{1cm} (27)

and

\[ \frac{\partial \lambda_j}{\partial \lambda_j} = \xi (1 - \alpha) \left[ \sum_{i=1}^{N} \left( \frac{\lambda_i}{\lambda_j} \right)^\alpha \right] L_{r_j}. \]  \hspace{1cm} (28)

The first expression means that the licensees receive knowledge (patents) for free. In turn, there is a difference between (24) and (28) because in S.1 case there are no knowledge externalities within high-tech firms.

In turn, if there is no exchange of knowledge between high-tech firms (S.3) the returns on knowledge accumulation are given by (23) where the third term is absent and

\[ \frac{\partial \lambda_j}{\partial \lambda_j} = \xi (1 - \alpha) L_{r_j}. \]  \hspace{1cm} (29)

The expression for the price of knowledge (25) indicates that the licensees pay a fixed fee for it. The fee is equal to their marginal valuation of the knowledge that they acquire. This valuation includes all future benefits from using that knowledge for augmenting their current knowledge. Therefore, the licensors appropriate all the benefit from licensing knowledge (i.e., they make the ‘take it or leave it’ offer). With a continuous accumulation of knowledge, as given by (12), at each and every instant the licensees acquire new knowledge at a fixed fee.

It is clear from (23) that I have assumed that the firm does not take into account the effect of accumulating knowledge on the price of knowledge \( p_{u,i,j} \). From (25) it follows that \( p_{u,i,j} \) declines with \( \lambda_j \). In this sense, I focus on a perfect market for knowledge (where the price of knowledge is equal to its marginal product and the licensors appropriate all benefit). An alternative assumption would be that the firm internalizes this effect. In such a circumstance there is an additional term in (23): the derivative of \( p_{u,i,j} \) with respect \( \lambda_j \).

Even though taking into account this effect changes the incentives of accumulating knowledge, it does not affect the supply of knowledge (26). This is because supply entails no costs and/or trade-offs 20.

In these frames the assumption that the licensors of knowledge do not take into account that their knowledge is used for business stealing amounts to assuming that firm \( j \) takes \( q_{j,\lambda} \) for any \( i \) different than \( j \) as exogenous. This is in line with assuming

\[ ^{20}\text{In the online version of this paper, Appendix E.4 derives the model under this alternative assumption. It shows that high-tech firms innovate less if they take into account the effect of knowledge accumulation on the price of knowledge. This is because innovating decreases their returns on knowledge licensing.} \]
that it takes $p_{a_j, i_j}$ as exogenous.

Finally, in equilibrium there is no difference if high-tech firms license their knowledge in return to wealth transfer or knowledge of other firms (plus-minus a fee). Therefore, knowledge licensing between high-tech firms can also be thought to resemble patent consortia or pools.

**Firm entry**

I consider two regimes of "entry" into the high-tech industry. In the first regime there are exogenous barriers to entry (i.e., there is no entry) and all firms in the market are assumed to have entered at time $t_0 (t = 0)$. In the second regime there are no barriers to entry into the high-tech industry. Moreover, the entry entails no costs.$^{21}$

In order to support symmetric equilibrium, I assume that the entrants into the high-tech industry have the highest productivity available at that date. Further, I assume that high-tech firms do not coordinate on their entry and exit strategies.

### 3 Features of the dynamic equilibrium

I restrict the attention to a symmetric equilibrium in the high-tech industry.

For a variable $Z$ denote its growth rate by $g_Z$. Further, for subsequent analysis it is useful to define functions $I_{S,1-2}^N$ and $I_{S,2-3}^1$ as

$$I_{S,1-2}^N = \begin{cases} 1 & \text{for } S.3, \\ N & \text{otherwise} \end{cases}$$

and

$$I_{S,2-3}^1 = \begin{cases} 0 & \text{for } S.1, \\ 1 & \text{otherwise}. \end{cases}$$

The growth rate of knowledge/productivity in cases when there is an exchange of knowledge between high-tech firms (S.1-2) is given by (12), (13), and (14). In case when there is no exchange of knowledge (S.3) it is given by (15). Using $I_{S,1-2}^N$ it can be written as

$$g_{\lambda} = \xi I_{S,1-2}^N L_r, \quad (30)$$

for all S.1-3 cases.

$^{21}$This assumption is required for tractability of results. Appendix E.5 of the online version of this paper offers a model where the entry entails endogenous costs for the cases there is inter-firm exchange of knowledge (S.1-2) and high-tech firms cooperate in R&D.
The (internal) rate of return on knowledge accumulation can be derived from the optimal rules of the high-tech firm (19), (20), and (23)-(29). It is given by

\[ g_{q_\lambda} = r - g_\lambda \left( \frac{L_x}{L_r} + 1 - \alpha I_{S,2-3}^1 \right). \] (31)

This expression determines the allocation of labor to R&D in a high-tech firm relative to the allocation of labor to production. Here, this ratio does not (explicitly) depend on competitive pressure in the high-tech industry. This is because high-tech firms decide on the division of labor between production and R&D internally.

From the high-tech firm’s demand for labor for production (19), the representative final goods producer’s optimal rules (6)-(7), and the relation between \( P_X X \) and \( p_x x \) (9) follows a relationship between \( NL_x \) and \( L_Y \),

\[ NL_x = \frac{\sigma}{1 - \sigma} b^k L_Y. \] (32)

where

\[ b^k = \frac{e^k - 1}{e^k}. \] (33)

This relationship shows the effect of price setting by high-tech firms. In symmetric equilibrium the perceived elasticities of substitution are

\[ e^B = \varepsilon - \frac{\varepsilon - 1}{N}, \] (34)

\[ e^C = \frac{\varepsilon}{1 + \frac{\varepsilon - 1}{N}}. \] (35)

Therefore, competition is tougher and mark-ups are lower if high-tech firms compete in prices, \( e^B > e^C \). Moreover, mark-ups decline with the number of firms \( N \) and \( \varepsilon \). This implies that the ratio \( \frac{L_Y}{NL_x} \) declines with \( N \) and \( \varepsilon \) and toughness of competition. This is because as the competitive pressure in the high-tech industry increases the relative price of \( x \) declines and final goods producers substitute \( X \) for \( L_Y \).

From (32) it is clear also that \( \frac{L_Y}{NL_x} \) declines with \( \sigma \) and does not depend on \( \mu \). The first result follows from that higher \( \sigma \) implies higher marginal product of \( X \) and lower marginal product of \( L_Y \). The second result stems from the assumption that efficiency gains due to external effects are Hicks-neutral.

The relationship between \( NL_x \) and \( L_Y \) (32) together with labor market clearing condition,

\[ L = L_Y + N (L_x + L_r), \] (36)
implies a relationship between $NL_x$ and $NL_r$,

$$NL_x = D^k (L - NL_r), \quad (37)$$

where

$$D^k = \frac{\sigma (e^k - 1)}{e^k - \sigma}. \quad (38)$$

Meanwhile, in the final goods market since either there is no entry or entry entails no costs and the assets in this economy are the high-tech firms it has to be the case that

$$Y = C, \quad (39)$$

which means that all final output is consumed.

**Entry regime 1: Exogenous barriers to entry**

I take $N > 1$ and allow profits $\pi$ in (18) to be negative. This is needed in order to characterize the behavior of labor force allocations and growth rate of knowledge for any $N > 1$, $\varepsilon$, and type of competition, and can be supported by subsidies, for example.

**Decentralized equilibrium**

Since there are exogenous barriers to entry the number of firms is fixed,

$$\dot{N} = g_N = g_{eN - 1} = 0.$$

In turn, from (39) it follows that consumption and final output grow at the same rate,

$$g_C = g_Y. \quad (40)$$

Let the consumers be sufficiently patient so that $\theta \geq 1$, which is a standard stability condition in multi-sector endogenous growth models and seems to be the empirically relevant case.

**Proposition 1** Let the following parameter restriction hold for any sufficiently small $N$:

$$\xi D^k I^{N}_{N1-2} L > \rho. \quad (41)$$

$D^k$ measures the effect of competitive pressure in the high-tech industry on allocations of labor force. Appendix E.6 of the online version of this paper shows that in the limiting case when $\sigma = 1$ competitive pressure in the high-tech industry does not matter for these allocations. This is because in such a case there are no relative price distortions.
In such a case, in decentralized equilibrium in all S.1-3 cases the economy makes a discrete "jump" to balanced growth path where labor force allocations and growth rates of knowledge/productivity and final output are given by

\[
NL_r^{NE} = \frac{N}{\xi I_{S,1-2}^N} \frac{\xi D^k I_{S,1-2}^N L - \rho}{(\theta - 1) (\sigma + \mu) + \alpha I_{S,2-3}^1 + D^k},
\]

\[
NL_x^{NE} = D^k \left[ \frac{(\theta - 1) (\sigma + \mu) + \alpha I_{S,2-3}^1}{(\theta - 1) (\sigma + \mu) + \alpha I_{S,2-3}^1 + D^k} \right] L + \frac{N}{\xi I_{S,1-2}^N} \rho,
\]

\[
L_Y^{NE} = \frac{1 - \sigma}{\sigma b^k} D^k \left[ \frac{(\theta - 1) (\sigma + \mu) + \alpha I_{S,2-3}^1}{(\theta - 1) (\sigma + \mu) + \alpha I_{S,2-3}^1 + D^k} \right] L + \frac{N}{\xi I_{S,1-2}^N} \rho,
\]

and

\[
g_y^{NE} = (\sigma + \mu) g_\lambda^{NE},
\]

\[
g_\lambda^{NE} = \frac{\xi D^k I_{S,1-2}^N L - \rho}{(\theta - 1) (\sigma + \mu) + \alpha I_{S,2-3}^1 + D^k}.
\]

**Proof.** See Appendix P.1 of the online version of this paper. ■

I use \(NE\) superscript for equilibrium labor force allocations and growth rates to denote the case when there is no entry. Parameter restriction (41) ensures that the inter-temporal benefit from allocating labor force to R&D outweighs its cost.

**Proposition 2** If parameter restriction (41) does not hold high-tech firms do not innovate. Therefore, the economy is static \((g_\lambda = g_Y = 0)\) and the labor force allocations in all S.1-3 cases are given by

\[
NL_r^{NE} = 0,
\]

\[
NL_x^{NE} = D^k L,
\]

\[
L_Y^{NE} = \frac{1 - \sigma}{\sigma b^k} NL_x^{NE}.
\]

Parameter restriction (41) may not hold for large \(N\) if there is no exchange of knowledge between high-tech firms (S.3) since when \(I_{S,1-2}^N = 1\) the left-hand-side of the inequality tends to zero as \(N\) increases. In case when there is no exchange of knowledge, therefore, if \(N\) is sufficiently large then the economy is on balanced growth path where \(g_\lambda = g_Y = 0\) and labor force allocations are given by (47)-(49). In this respect, if parameter restriction (41) holds for any sufficiently small \(N > 1\) then it always holds in cases when there is exchange of knowledge between high-tech
firms (S.1-2). This is because when $I_{S,1-2}^N = N$ the left-hand-side of the inequality increases with $N$.

Without loss of generality, hereafter, I assume that (41) holds for any finite $N$ and does not hold in case when there is no knowledge exchange between high-tech firms (i.e., $I_{S,1-2}^N = 1$) if $N$ is arbitrarily large/infinite ($N = +\infty$).

**Proposition 3** Let parameter restriction (41) hold. If high-tech firms choose not to engage in R&D then labor force allocations are given by (47)-(49). Moreover, the value of high-tech firms is higher if none of the high-tech firms engages in R&D.

**Proof.** See Appendix P.1 of the online version of this paper. □

I further assume that high-tech firms cannot collude and not innovate (for example, because of antitrust regulation or non-sustainability of collusion). In this respect, the reason why in decentralized equilibrium each high-tech firm prefers to engage in R&D is that R&D reduces its marginal cost. Therefore, *ceteris paribus* it allows the firm to lower its price and capture more market.

**Social optimum**

The hypothetical Social Planner selects the paths of quantities so that to maximize the lifetime utility of the household (1). The Social Planner internalizes all externalities and solves the following problem.

$$\max_{L_x, L_r} U = \frac{\int_0^{+\infty} C_t^{1-\theta} - 1}{1 - \theta} \exp (-\rho t) \, dt$$

subject to

$$C = \left( N \frac{\lambda}{\sigma} \frac{\lambda L_x}{\lambda} \right)^{\sigma+\mu} \left[ L - N (L_x + L_r) \right]^{1-\sigma},$$

$$\dot{\lambda} = \xi I_{S,1-2}^N \lambda L_r,$$

$$\lambda (0) > 0.$$  

The Social Planner’s optimal choices for $L_x$ and $L_r$ are given by

$$[L_x] : \quad NL_x = D^{SP} (L - NL_r),$$

$$[L_r] : \quad q L_x I_{S,1-2}^N = \frac{(1 - \sigma) N}{L - L_x + L_r} C_t^{1-\theta},$$

where

$$D^{SP} = \frac{\sigma + \mu}{1 + \mu},$$
and I use $SP$ superscript in order to make a distinction between the outcomes of Social Planner’s choice and decentralized equilibrium outcomes. Meanwhile, its returns on knowledge accumulation are given by

$$[\lambda] : \dot{q}_\lambda = q_\lambda \rho - [q_\lambda \xi I^N_{S,1-2} L_r + (\sigma + \mu) \lambda^{-1} C^{1-\theta}] .$$  \hspace{1cm} (56)

The optimal choice of $L_x$ together with labor market clearing condition (36) implies that

$$NL_x = \frac{1 + \mu}{1 - \sigma} D^{SP} L_Y.$$  \hspace{1cm} (57)

This relation is the counterpart of (32) in decentralized equilibrium.

**Proposition 4** Let the following parameter restriction hold for any sufficiently small $N$:

$$\xi D^{SP} I^N_{S,1-2} L > \rho.$$  \hspace{1cm} (58)

In such a case, the Social Planner chooses labor force allocations such that the economy, where there is "no entry", makes a discrete jump to balanced growth path, where

$$NL^{NE,SP}_r = \frac{N}{\xi I^N_{S,1-2}} \left( \frac{\xi D^{SP} I^N_{S,1-2} L - \rho}{L} \right),$$  \hspace{1cm} (59)

$$NL^{NE,SP}_x = D^{SP} \frac{(\theta - 1)(\sigma + \mu)L + \frac{N}{\xi I^N_{S,1-2}} \rho}{(\theta - 1)(\sigma + \mu) + D^{SP}},$$  \hspace{1cm} (60)

$$L^{NE,SP}_Y = \frac{1 - \sigma}{\sigma + \mu} D^{SP} \frac{(\theta - 1)(\sigma + \mu)L + \frac{N}{\xi I^N_{S,1-2}} \rho}{(\theta - 1)(\sigma + \mu) + D^{SP}}.$$  \hspace{1cm} (61)

and

$$g^{NE,SP}_Y = (\sigma + \mu) g^{NE,SP}_\lambda,$$  \hspace{1cm} (62)

$$g^{NE,SP}_\lambda = \frac{\xi D^{SP} I^N_{S,1-2} L - \rho}{(\theta - 1)(\sigma + \mu) + D^{SP}}.$$  \hspace{1cm} (63)

**Proof.** See Appendix P.1 of the online version of this paper.  \hspace{1cm} $\blacksquare$

Parameter restriction (58) necessarily holds as long as (41) holds since $D^{SP} > D^k$. As in decentralized equilibrium, this inequality states that the benefit from R&D outweighs its cost.

Given that $C$ in (51) satisfies Inada conditions no corner solutions in terms of $NL_x$ or $L_Y$ satisfy the Social Planner’s optimal problem.
Proposition 5  Meanwhile, it is straightforward to show that if (58) holds no corner solutions in terms of $NL_r$ satisfy the Social Planner’s optimal problem. In case, however, parameter restriction (58) does not hold the Social Planner sets

$$NL_r = 0,$$  \hspace{2cm} (64)

and the remaining labor force allocations according to

$$NL^{NE,SP}_x = D^{SP}L, \hspace{2cm} (65)$$

$$L^{NE,SP}_Y = \frac{1 - \sigma}{\sigma + \mu}D^{SP}L. \hspace{2cm} (66)$$

Proof. See Appendix P.1 of the online version of this paper. 

This parameter restriction does not hold if $N$ is arbitrarily large/infinite and there is no knowledge exchange in the economy (S.3). It holds, however, for any $N$ in cases when there is an exchange of knowledge (S.1-2) since I have assumed that so does (41).

I further assume that the Social Planner can choose between S.1-2 and S.3 cases. In terms of policies implemented by a government in decentralized equilibrium this corresponds to motivating or banning knowledge exchange in the economy. \footnote{An example for such policy/action is the establishment of the Radio Corporation of America (RCA Corporation) that fostered cross-licensing in the telecommunications industry in the US.} Clearly, the Social Planner prefers S.1-2 over S.3 since it could set the same labor force allocations and have higher economic growth in S.1-2 cases. Therefore, in this sense it is socially desirable to have knowledge exchange in the economy.

Comparative statics and comparisons

Within the decentralized equilibrium outcomes, first, I discuss the case that the number of high-tech firms $N$ is finite ($N < +\infty$). Next, I discuss the limiting case when the number of high-tech firms is infinite ($N = +\infty$) and, therefore, (41) does not hold if there is no exchange of knowledge between high-tech firms (S.3). In the end of the section I compare the decentralized equilibrium allocations and growth rates with the choice of the Social Planner.

Proposition 6 In all S.1-3 cases the growth rate of knowledge/productivity ($g_\lambda$) and the growth rate of final output ($g_Y$) increase with the elasticity of substitution between high-tech goods ($\varepsilon$). Moreover, $g_\lambda$ and $g_Y$ are higher under Bertrand competition which is tougher than Cournot competition.

Proof. These results follow from (33)-(35), (38), (45), and (46). \hfill \blacksquare
The driver behind these results are the relative price distortions, which are due to price setting by high-tech firms. These distortions increase the demand for labor in final goods production. Increasing the elasticity of substitution or the toughness of competition reduces these distortions. The reduction of distortions motivates final goods producers to substitute (a basket of) high-tech goods for labor. Higher demand for high-tech goods and higher amount of available labor increase the incentives of high-tech firms to conduct R&D. This increases \( g_\lambda \) and \( g_\gamma \).

**Corollary 7** In this respect, in all S.1-3 cases \( NL_r \) and \( NL_x \) grow and \( L_\gamma \) declines with the elasticity of substitution \( \varepsilon \) and toughness of competition.

**Proof.** This result follows from \((42)-(44)\).

The comparative statics with respect to the number of high-tech firms in cases when there is an exchange of knowledge (S.1-2) are different from the case when there is no exchange of knowledge (S.3). The results are summarized in the following proposition.

**Proposition 8** In cases when there is an exchange of knowledge between high-tech firms (S.1-2), labor force allocations \( NL_r \) and \( NL_x \) and growth rates \( g_\lambda \) and \( g_\gamma \) increase with the number of firms \( N \), whereas \( L_\gamma \) declines with it. If there is no exchange of knowledge (S.3), however, this result does not hold if the number of firms is relatively high.

**Proof.** These results follow from \((42)-(46)\).

The driver behind the first result is the reduction in relative price distortions (or the intensification of competition) that the higher number of high-tech firms brings with it. Meanwhile, the second result holds because increasing the number of high-tech firms if there is no exchange of knowledge between high-tech firms (S.3) has two effects. It reduces the relative price distortions and the amount of labor force that can be devoted to R&D [see \( \frac{I N_1}{N^2} \) term in \((46)\)]. The first effect motivates higher demand for \( NL_r \) and increases \( g_\lambda \), whereas the second effect reduces \( NL_r \) and \( g_\lambda \). The second effect is absent in cases when there is an exchange of knowledge between high-tech firms (S.1-2) because increasing the number of high-tech firms also increases the amount of complementary knowledge made available by these firms. Clearly, the result that these effects exactly offset each other hinges on the functional form assumptions for knowledge accumulation processes \((12), (13)\) and \((14)\).  

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24 One way to relax this assumption is to multiply \((12)\) and \((14)\) by a function \( f(N) \), where \( f' \leq 0 \). As long as \( f(N) D^k \) increases with \( N \) the results remain intact. If, however, \( f(N) D^k \) decreases with \( N \) at least for sufficiently large \( N \) then the results would become similar to the case when there is no exchange of knowledge.
Proposition 9

- In cases when there is an exchange of knowledge between high-tech firms (S.1-2) $g_\lambda$ and $g_Y$ are concave functions of the number of firms $N$.

- In case when there is no exchange of knowledge (S.3) the derivative of $g_\lambda$, as well as $g_Y$, with respect to $N$ is positive if $N$ is close to 1 and it is negative for any $N$ greater than 2.

Proof. These results follow from (46) and that in cases when there is knowledge exchange between high-tech firms $I_{S1-2}^N = N$, whereas $I_{S1-2}^N = 1$ if there is no knowledge exchange. ■

The first part of the proposition holds because competition intensifies more from adding a firm if there are few high-tech firms. Meanwhile, the second part of the proposition holds because in case when there is no exchange of knowledge (S.3) at the higher levels of market concentration/lower levels of competition ($N \approx 1$) the positive effect of higher competition is dominant. Meanwhile, at the lower levels of market concentration/higher levels of competition ($N > 2$) the negative effect is dominant. The full characterization of the behavior of $g_\lambda$ and $g_Y$ for $N \in (1, 2)$ is not so straightforward, however. This is because of high non-linearity of $g_\lambda$ in that interval. In the neighborhood of $N = 1$ the growth rate of knowledge/productivity $g_\lambda$ is increasing and concave in $N$ and after a tipping point from $(1, 2)$ it becomes convex and decreasing. 25

Proposition 10

- In all S.1-3 cases labor force allocations $NL_r$ and $NL_x$ and growth rates $g_\lambda$ and $g_Y$ increase with $\sigma$, whereas $L_Y$ declines with it. In contrast, $g_\lambda$ and $N L_r$ decline with $\mu$ and $g_Y$, $NL_x$, and $L_Y$ increase with it.

- In cases when there are knowledge spillovers/externalities (S.2-3) $NL_r$, $g_\lambda$ and $g_Y$ decline with $\alpha$, whereas $NL_x$ and $L_Y$ increase with it.

Proof. These results follow from (42)-(46). ■

25This result implies that in case when there is no exchange of knowledge between high-tech firms (S.3) there is an inverted-U shape relationship between $g_\lambda$ and the number of firms $N$. This is in line with the results and evidence of Aghion et al. (2005) to the extent that in this model $N$ is one of the indicators of competitive pressure: $1/N$ measures market concentration. A similar result can be obtained also in cases when there is an exchange of knowledge between high-tech firms (S.1-2) assuming fixed management costs as in van de Klundert and Smulders (1997) or that (12) and (14) increase less than linearly with $N$. 24
The first result holds because higher $\sigma$ increases the marginal product of high-tech goods bundle $X$ and reduces the marginal product of labor force in final goods production $L_Y$. Therefore, the demand for $L_Y$ declines and labor force is reallocated to $NL_x$ and $NL_r$. According to (45) and (46) this implies that $g_\lambda$ and the growth rate of final output $g_Y$ increase with $\sigma$. In contrast, higher $\mu$ does not affect the balance between the demand for $X$ and $L_Y$ and in this sense does not alter the production and R&D incentives of high-tech firms. Meanwhile, ceteris paribus it increases the growth rate of final output $g_Y$ and equilibrium interest rate $r$ [see (3)], which discourages investments in R&D. Lower $NL_r$ implies lower growth rate of knowledge/productivity $g_\lambda$. Finally, the second part of the proposition holds because in case there are knowledge spillovers/externalities as $\alpha$ increases the internalized returns on R&D decline and firms invest less in R&D. Therefore, more labor force is allocated to production activities, and $g_\lambda$ and $g_Y$ decline.

In order to preserve space, hereafter, unless stated otherwise, I exclusively discuss the results for the growth rate of knowledge/productivity $g_\lambda$ while keeping in mind that the growth rate of final output $g_Y$ is proportional to it.

**Corollary 11** If the number of high-tech firms is arbitrarily large/infinite

- when there is an exchange of knowledge between high-tech firms (S.1-2) labor force allocations and growth rate $g_\lambda$ are given by (42)-(44), and (46) where
  \[ D^k = D = \frac{\sigma (\varepsilon - 1)}{\varepsilon - \sigma}; \]

- when there is no exchange of knowledge between high-tech firms (S.3) $g_\lambda = 0$ and labor force allocations are given by (47)-(49) where $D^k = D$.

The first part of this corollary implies that when there is an exchange of knowledge between high-tech firms and $N = +\infty$ neither labor force allocations nor the growth rate of knowledge depend on the type of competition and the number of high-tech firms. It implies also that the remaining comparative statics stay intact in these cases. The second part of the corollary holds because when there is no exchange of knowledge between high-tech firms and $N = +\infty$ parameter restriction (41) does not hold. It can be shown that in this case, $NL_x$ increases and $L_Y$ declines with $\sigma$ and $\varepsilon$ and both $NL_x$ and $L_Y$ do not depend on the type of competition and parameters $\alpha$ and $\mu$.

**Corollary 12** For both finite and infinite number of high-tech firms the comparison
between S.1-3 cases yields the following relationships.

\[ \begin{align*}
NL_{r,S}^{E,S_1} &> NL_{r,S}^{E,S_2} > NL_{r,S}^{E,S_3}, \\
NL_{x,S}^{E,S_3} &> NL_{x,S}^{E,S_2} > NL_{x,S}^{E,S_1}, \\
L_{Y}^{E,S_3} &> L_{Y}^{E,S_2} > L_{Y}^{E,S_1}, \\
g_{Y}^{E,S_1} &> g_{Y}^{E,S_2} > g_{Y}^{E,S_3}.
\end{align*} \] (67)

This means that in decentralized equilibrium with no entry high-tech firms innovate the most in case when there is knowledge licensing (S.1). High-tech firms innovate the least if there is no exchange of knowledge between these firms (S.3). Therefore, for a given \( N \) the growth rate of final output is the highest in case when there is knowledge licensing and the lowest in case when there is no exchange of knowledge between high-tech firms.

In order to further highlight the contrast between all knowledge accumulation/R&D setups (S.1-3) the following figure plots \( g_{Y} \) for parameter values \( \theta = 4, \rho = 0.01, \sigma = 0.3, \mu = 0.01, \varepsilon = 4, L = 1, \xi = 1, \) and \( \alpha = 0.1 \) and Cournot and Bertrand types of competition.

Comparisons between decentralized equilibrium and socially optimal results: Different types of competitive pressure matter for these decentralized equilibrium outcomes because of market interactions between high-tech firms. They do not matter, however, for the outcomes of the Social Planner’s problem (59)-(63).

\[ \text{The growth rate of productivity in S.1-3 cases} \]

\[ \text{Number of high-tech firms (N)} \]

The parameter values were selected so that the growth rate of final goods is neither too high nor too low.
Proposition 13  In contrast to the decentralized equilibrium results $NL_r^{SP}$, $NL_x^{SP}$, $g_\lambda$, and $g_Y$ increase with $\mu$ and $L_Y^{SP}$ declines with this parameter.

Proof. This result follows from (59)-(63).  

This result holds because the Social Planner internalizes $\mu$ and higher $\mu$ implies higher marginal product of $X$.

Corollary 14  For both finite and infinite $N$ the comparison between decentralized equilibrium growth rates and allocations and socially optimal growth rates and allocations yields the following relationships.

$$
NL_r^{NE,SP,S,1-2} > NL_r^{NE,S,1},
$$
$$
NL_x^{NE,SP,S,1-2} \geq NL_x^{NE,S,3},
$$
$$
NL_x^{NE,SP,S,1-2} \geq NL_x^{NE,S,2},
$$
$$
NL_x^{NE,SP,S,1-2} > NL_x^{NE,S,1},
$$
$$
L_Y^{NE,SP,S,1-2} < L_Y^{NE,S,1},
$$

and

$$
g_\lambda^{NE,SP,S,1-2} > g_\lambda^{NE,S,1},
$$

where $\leq$ indicates that the relation depends on model parameters.

This means that in decentralized equilibrium the economy innovates less than it is socially optimal and therefore grows at a lower rate. Moreover, in decentralized equilibrium it fails to have socially optimal labor force allocations. The driver behind these results are the relative price distortions and externalities. Due to these distortions final goods producers substitute labor for high-tech goods which lowers the output of high-tech firms and the number of researchers that high-tech firms hire. The externalities in R&D have an effect of similar direction. If such externalities are present then high-tech firms do not fully internalize the returns on R&D. This reduces their incentives to invest in R&D and they hire lower number of researchers. Meanwhile, externalities in final goods production increase interest rate $r$. Since high-tech firms do not take into account these externalities they invest less than it is socially optimal. Final goods producers also do not take into account these externalities. Therefore, they demand less than socially optimal amount of high-tech goods.

The differences between socially optimal and decentralized equilibrium growth rates and labor force allocations in terms of the relative price distortions and externalities in final goods production are summarized by $D^k$ and $D^{SP}$. It is easy to
notice that for sufficiently high \( N \)

\[
\lim_{\mu \to 0} D^{SP} = \lim_{\varepsilon \to +\infty} D^k.
\]

This equality holds because for sufficiently high \( N \) the limiting case \( \varepsilon = +\infty \) would imply perfect competition in the high-tech industry. In such a limiting case, however, in decentralized equilibrium high-tech firms make zero profits and have no market incentives to innovate.\footnote{In this respect, the positive relationships between innovation in the high-tech industry and different types of competitive pressures hold as long as high-tech firms have sufficient profits to cover costs of innovation or are compensated for that.}

**Entry regime 2: Cost-free entry**

In this section I endogenize the number of firms assuming that entry cost is zero.

**Decentralized equilibrium**

From (18), (19) and (31) it follows that the profits of a high-tech firm are

\[
\pi = wL_x \left[ \frac{1}{e^k - 1} - \frac{g_\lambda}{r - g_q - (1 - \alpha I_{S:2}^1) g_\lambda} \right].
\]

Given that entry cost is zero the condition that endogenizes the number of high-tech firms is \( \pi = 0 \).

Denote

\[
\bar{\pi} = \frac{1}{e^k - 1} - \frac{g_\lambda}{r - g_q - (1 - \alpha I_{S:2}^1) g_\lambda}.
\]

Therefore,

\[
\pi = 0 \iff \bar{\pi} = 0.
\]

**Proposition 15** At time 0 \((t = 0)\) \( N \) makes a discrete jump to the balanced growth path equilibrium level.

**Proof.** See Appendix P.1 of the online version of this paper. \( \blacksquare \)

This implies that in decentralized equilibrium with cost-free entry the economy is on a balanced growth path (for any \( t > 0 \)), where

\[
\dot{N} = g_N = g_{\frac{\lambda}{e^k - 1}} = 0.
\]

Therefore, labor force allocations and growth rate of knowledge/productivity are given by (42)-(44) and (46), where the number of high-tech firms \( N \) is endogenous.
In turn, \( N \) can derived from the zero profit condition (69) and \( g \) that solves the capital market equilibrium (46). The growth rate of productivity \( g \) that solves the zero profit condition (69) is

\[
g = e^{k - 1 - \alpha L_{k-3} - (\theta - 1)(\sigma + \mu)}, \tag{70}
\]

Let

\[
\varepsilon - 1 - \alpha - (\theta - 1)(\sigma + \mu) > 0,
\]

which implies that \( g \) can be positive for sufficiently large \( N \) or, equivalently, there can exist decentralized equilibrium where high-tech firms innovate.

Hereafter, I call \( g \) from (70) \( ZP \) – zero profit, and \( g \) from (46) \( CME \) – capital market equilibrium. If \( \alpha > 0 \) and/or \( \theta > 1 \) the number of high-tech firms \( N \) that satisfies

\[
e^{k - 1 - \alpha - (\theta - 1)(\sigma + \mu)} = 0
\]

is strictly greater than 1. Denote it by \( N^* \). For \( N \in (1, N^*) \) it can be shown that \( g \) in (70) or \( ZP \) is negative, decreasing, and convex function of \( N \) and

\[
\lim_{N \to N^*^-} g = -\infty.
\]

Meanwhile for \( N > N^* \) it can be shown that \( ZP \) is positive, decreasing, and convex function of \( N \) and

\[
\lim_{N \to N^*^+} g = +\infty.
\]

**Proposition 16** In decentralized equilibrium with endogenous entry it cannot happen so that \( N \in (1, N^*) \).

**Proof.** This is because for \( N \in (1, N^*) \) high-tech firms do not innovate, which implies that the profit of each firm is

\[
\pi = wL_x \frac{1}{e^{k - 1}} > 0.
\]

Therefore, there will be entry that will increase the number of high-tech firms above \( N^* \). ■

Both \( CME \) and \( ZP \) are continuous functions of \( N \) for \( N > N^* \), the values of \( CME \) are finite for any \( N > 1 \), and \( ZP \) is arbitrarily large around \( N^* \). Therefore, at least for \( N \) sufficiently close to \( N^* \) it has to be the case that \( ZP \) is higher \( CME \). This means that there exists decentralized equilibrium where high-tech firms innovate.

If \( ZP \) crosses \( CME \) from above then the decentralized equilibrium determined by the intersection is stable in the sense that the entry of firms reduces \( \bar{\pi} \) in (68) and
exit increases it. The number of firms and the growth rate of productivity can be solved from the intersection of CME and ZP in such a case. Moreover, if at time 0 \( t = 0 \) the number of high-tech firms is higher than (and in S.3 case sufficiently close to) the number determined by the intersection of ZP and CME then high-tech firms will exit the market till ZP and CME are equal. Considering such a setup, or exit of high-tech firms instead of entry, can support the zero entry costs assumption.

In order to have meaningful equilibrium in each of S.1-3 cases \([i.e., \quad (69) \text{ holds}]\) I further assume that the parameters are such that there exists \( N^{**} \) where ZP crosses CME under Cournot competition in case when there is no exchange of knowledge (S.3). Given that \((46)\) shifts up and \((70)\) shifts down with the elasticity of substitution \(\varepsilon\) this can be equivalent to assuming that the elasticity of substitution \(\varepsilon\) is sufficiently high. It implies that ZP crosses CME in all the remaining S.1-3 cases.\(^{28}\)

The previous section showed that if there is an exchange of knowledge (S.1-2) the growth rate of knowledge \(g_\lambda\) from \((46)\), or CME, is monotonically increasing function of \(N\).

**Proposition 17** In cases when there is an exchange of knowledge between high-tech firms (S.1-2) ZP crosses CME from above and the number of high-tech firms under Bertrand and Cournot types of competition can be found from

\[
e^k = \frac{\xi \sigma L \left[ 1 + \alpha I_{S-2} + (\theta - 1) (\sigma + \mu) \right]}{\xi \sigma L - \rho}, \tag{71}\]

where \(k = C, B\) and \(e^b\) and \(e^c\) are given by \((34)\) and \((35)\).

If there is no exchange of knowledge (S.3), however, CME is not a monotonic function for all \(N\). It is monotonically increasing function in the neighborhood of \(N = 1\) and monotonically decreasing after some \(N \in (1,2)\). Moreover, it is continuous and finite for any \(N\) and negative for \(N = 1\) and \(N = +\infty\). Therefore, given that ZP is a monotonically decreasing function and it is positive for any \(N\), ZP crosses CME at least twice.

**Proposition 18** If there is no exchange of knowledge between high-tech firms then the number of firms under Bertrand and Cournot types of competition can be found from

\[
e^k = \frac{\xi \sigma \frac{1}{N} L \left[ 1 + \alpha + (\theta - 1) (\sigma + \mu) \right]}{\xi \sigma \frac{1}{N} L - \rho}. \tag{72}\]

\(^{28}\)van de Klundert and Smulders (1997) offers a model which resembles the case when there is no exchange of knowledge between high-tech firms (S.3). The authors assume parameter values such that ZP crosses CME from above. Clearly, such a set of parameter values is restrictive for cases when there is an exchange of knowledge between high-tech firms (S.1-2).
It is straightforward to show that (72) is a quadratic equation in \( N \). This means in case there is no exchange of knowledge between high-tech firms (S.3) \( ZP \) crosses \( CME \) twice. It does so from above and from below. The smaller root of (72) corresponds to the stable equilibrium where \( ZP \) crosses \( CME \) from above. Meanwhile, the bigger root corresponds to the case when \( ZP \) crosses \( CME \) from below and the equilibrium is not stable. Denote it by \( N_{2}^{**} \). If the economy starts with a number of firms greater or equal to \( N_{2}^{**} \) then \( \pi \) does not decline to zero as \( N \) increases. In order to rule this out I further assume that the economy starts with a number of high-tech firms that is lower than \( N_{2}^{**} \). Therefore, depending on whether \( ZP \) is higher or lower than \( CME \), firms exit or enter till (the point where) \( ZP \) crosses \( CME \) from above.

**Social optimum**

In this case the hypothetical Social Planner solves the optimal problem (50) and chooses \( N \).

**Proposition 19** The Social Planner’s optimal choice for \( N \) in case when there is an exchange of knowledge (S.1-2) is given by

\[ [N]: \frac{\frac{\sigma + \mu}{\varepsilon - 1} C^{1-\theta}}{N} \geq 0 \]

or simply

\[ N = +\infty, \]

(73)

whereas if there is no exchange of knowledge (S.3) it is given by

\[ [N]: \frac{\frac{\sigma + \mu}{\varepsilon - 1} C^{1-\theta}}{N} \geq q_{s} \xi \lambda L_{r}, \]

(74)

The former result (73) holds because if there is an exchange of knowledge then \( I_{S.1-2}^{N} = N \) and the Social Planner has no trade-offs while increasing \( N \). In contrast, if there is no exchange of knowledge then \( I_{S.1-2}^{N} = 1 \) and it has a trade-off. Higher \( N \) implies lower growth rate.

---

29 The functional forms of knowledge accumulation process in cases when there is an exchange of knowledge between high-tech firms (S.1-2) help to avoid this assumption.

30 In cases when there is an exchange of knowledge (S.1-2) the Social Planner selects at time zero \( N = +\infty \) because of the assumption that firm entry or creating high-tech goods entails no costs. If there were costs associated with entry (or costs associated with maintaining the goods/firms as in van de Klundert and Smulders 1997) the Social Planner might not select at time zero (or at any time) \( N = +\infty \).
In order to be able to solve the optimal control problem in cases when there is an exchange of knowledge \((I_{S_{1-2}}^N = N)\) with first order conditions \(C\) needs to be rescaled by \(N\) so that at time zero \(C < +\infty\) (i.e., \(C\) needs to be divided to \(N^{-\rho}\)).

**Proposition 20** The Social Planner selects labor force allocations and \(N\) such that the economy makes a discrete jump to balanced growth path.

- If there is an exchange of knowledge, on this path labor force allocations and growth rate of knowledge \(g\) are given by (59)-(61) and (63) and \(N = +\infty\).
- If there no exchange of knowledge and (74) is binding then
  \[
  N = \frac{\xi (\sigma + \mu) \varepsilon - 1 - (\theta - 1) (\sigma + \mu)}{\varepsilon (1 + \mu) - (1 - \sigma)} L, \\
  g_{\lambda}^{CFE;SP;S:3} = \frac{\rho}{\varepsilon - 1 - (\theta - 1) (\sigma + \mu)},
  \]
  where \(CFE\) stands for cost-free entry.

**Proof.** See Appendix P.1 of the online version of this paper. ■

If there no exchange of knowledge and (74) is binding labor force allocations can be derived from (52), (53), (57), and (76), where the expression (76) is the counterpart of \(ZP\) (70) with \(N = +\infty\) and \(\alpha = 0\). Comparing the lifetime utility of the household it can be shown, however, that the Social Planner prefers to set \(N = +\infty\) also in case when there is no exchange of knowledge. Therefore, (74) does not bind. The following proposition summarizes this result.

**Proposition 21** The optimal condition (74) is not binding and the Social Planner sets

\[
N = +\infty, \\
g_{\lambda}^{CFE;SP;S:3} = NL_{r}^{CFE;SP;S:3} = 0, \\
NL_{x}^{CFE;SP;S:3} = DS^{P}L, \\
L_{Y}^{CFE;SP;S:3} = \frac{1 - \sigma}{\sigma + \mu} DS^{P}L,
\]

**Proof.** See Appendix P.1 of the online version of this paper. ■

As shown in the Social optimum section of Entry regime 1 this implies that the Social Planner prefers the case when there is an exchange of knowledge (S.1-2) over the case when there is no exchange of knowledge (S.3).
This result is not stemming from the cost-free entry assumption. Even if there were fixed costs associated with entry the Social Planner could set the number of firms in case when there is an exchange of knowledge (S.1-2) equal to the number of firms it finds optimal in case when there is no exchange of knowledge (S.3). In such a circumstance according to (63) it would have higher growth rate and, therefore, welfare in case when there is an exchange of knowledge (S.1-2).

Comparative statics and comparisons

The following proposition establishes the comparative statics results for the number of high-tech firms.

Proposition 22

- In all S.1-3 cases, there are fewer high-tech firms in equilibrium under Bertrand competition than under Cournot competition. Further, the number of firms declines with $\varepsilon$ and increases with $\mu$.

- In cases when there are knowledge spillovers/externalities (S.2-3) the number increases with $\alpha$. It does not depend on $\alpha$ in case when there is knowledge licensing (S.1).

Proof. See Appendix P.1 of the online version of this paper. ■

The number of firms declines with toughness of competition and $\varepsilon$ since tougher competition and higher $\varepsilon$ imply lower mark-ups, which reduces $\bar{\pi}$ for a given $N$. In turn, it increases with $\mu$ since higher $\mu$ implies lower R&D investments (fixed costs), which increases $\bar{\pi}$ for a given $N$. Higher $\alpha$ in cases when there are knowledge spillovers/externalities (S.2-3) also implies lower R&D investments. The comparative statics with respect $\sigma$ depend on model parameters.

Proposition 23 In cases when there is an exchange of knowledge between high-tech firms

- $g_{\lambda}$ and labor force allocations do not depend on the type of competition, $\varepsilon$, and $N$.

- $g_{\lambda}$ and $NL_{\varepsilon}$ decrease with $\alpha$ and $\mu$. $NL_{x}$ increases with these parameters, and $g_{Y}$ declines with $\alpha$ but increases with $\mu$. 

33
Proof. The first part of the proposition holds because in cases when there is an exchange of knowledge $e^k$ (71) does not depend on the type of competition, $\varepsilon$, and $N$. See Appendix P.1 of the online version of this paper for the second part of this proposition.

The analytical derivations for comparative statics with respect to $\sigma$ and for labor force allocation to final goods production $L_Y$ are not trivial. Numerical simulations where $L$ is normalized to 1 and the remaining parameters are from the following intervals

$$
\begin{align*}
\theta & \in [1,10], \rho \in [0.01, 0.1], \sigma \in [0.01, 0.99], \\
\mu & \in [0.01, 0.99], \xi \in [0.1, 10], \alpha \in [0.01, 0.99],
\end{align*}
$$

(80)

show that

Table 1: Numerical comparative statics for cases when there is an exchange of knowledge between high-tech firms (S.1-2)

<table>
<thead>
<tr>
<th>$g_\lambda$</th>
<th>$NL_r$</th>
<th>$NL_x$</th>
<th>$L_Y$</th>
<th>$g_Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>$\mu$</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>$\pm$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>$\pm$</td>
</tr>
</tbody>
</table>

Note: The values of parameters are from intervals (80) and satisfy parameter restrictions. Grids are equally spaced and each has 5 points.

where $+$ means positive relationship, $-$ negative, and $\pm$ that the relationship depends on model parameters.

If there is no exchange of knowledge between high-tech firms (S.3) it is not straightforward to derive the relationship between the toughness of competition and growth rate of productivity $g_\lambda$. This is because of high non-linearity of $CME$ in this case. Nevertheless, it is possible to show that if $ZP$ crosses $CME$ from above in the region of $N$ where $CME$ is monotonically increasing then the growth rate of productivity $g_\lambda$ is higher under Bertrand competition. The comparison of the labor allocations using analytical techniques also is not trivial. Numerical simulations show that under Cournot competition $g_\lambda$, $NL_r$ are lower and $NL_x$ is higher than under Bertrand competition. Meanwhile, depending on model parameters $L_Y$ can be both higher and lower.

The analytical derivation of comparative statics for labor force allocations and growth rates of final output and knowledge with respect to parameters $\varepsilon$, $\sigma$, $\mu$, and
\( \alpha \) also are not trivial in case when there is no knowledge exchange between high-tech firms. Numerical simulations show that

Table 2: Numerical comparative statics for cases when there is no exchange of knowledge between high-tech firms (S.3)

<table>
<thead>
<tr>
<th>( g_\lambda )</th>
<th>( N_L )</th>
<th>( N_L^x )</th>
<th>( L_Y )</th>
<th>( g_Y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon )</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>( \pm ) +</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>( \pm )</td>
<td>( \pm )</td>
<td>( \pm )</td>
<td>( \pm )</td>
</tr>
<tr>
<td>( \mu )</td>
<td>( \pm )</td>
<td>( \pm )</td>
<td>( + )</td>
<td>( \pm )</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>( \pm )</td>
</tr>
</tbody>
</table>

Note: The values of parameters are from intervals \([0, 1]\) and satisfy parameter restrictions. Grids are equally spaced and each has 5 points.

The following proposition summarizes the comparisons between different settings for R&D process

**Proposition 24** The growth rate of knowledge/productivity \( g_\lambda \) is higher in case when there is knowledge licensing (S.1) compared to the case when there are knowledge spillovers between high-tech firms (S.2). Moreover, it is higher in case when there are knowledge spillovers between high-tech firms (S.2) compared to the case when there is no exchange of knowledge (S.3), i.e.,

\[
g_{\lambda}^{CFE,S:1} > g_{\lambda}^{CFE,S:2} > g_{\lambda}^{CFE,S:3}.
\]

**Proof.** The first inequality follows from that \( g_\lambda \) declines with \( \alpha \). In turn, the second inequality follows from \((67)\) given that \( ZP \) is monotonically decreasing function of \( N \) and \( CME \) in cases there is an exchange of knowledge (S.1-2) is monotonically increasing function of \( N \). \( \blacksquare \)

Given that R&D investments are fixed costs, this implies that there are more high-tech firms in case when there are knowledge spillovers between these firms (S.2) than in case when there is knowledge licensing (S.1). Moreover, there are more high-tech firms in case when there is no exchange of knowledge between these firms (S.3) compared to the case when there are knowledge spillovers (S.2), i.e.,

\[
N_{CFE,S:3}^{CFE} > N_{CFE,S:2}^{CFE} > N_{CFE,S:1}^{CFE}.
\]

The differences between labor force allocations in cases there is an exchange of knowledge (S.1-2) and there is no exchange of knowledge (S.3) depend on model parameters.
These results indicate that high-tech firms innovate more in cases when there is an exchange of knowledge compared to the case when there is none. Moreover, these firms innovate more in case when there is knowledge licensing compared to the case there are knowledge spillovers/externalities. Meanwhile, using (42)-(44), (46), (59)-(61), and (63) it can be shown that in all S.1-3 cases in decentralized equilibrium with cost-free (endogenous) entry into the high-tech industry the economy invests in R&D less than it is socially optimal. Therefore, it grows at a lower than socially optimal rate. Further, it fails to have socially optimal number of high-tech firms.

Policies leading to the first best outcome in decentralized equilibrium

In this section I offer policies that if implemented in decentralized equilibrium lead to the first best outcome. I assume that there is knowledge licensing in decentralized equilibrium. This can amount to assuming that the government has motivated knowledge exchange between high-tech firms that happens in a market where the licensors have the right to make a ‘take it or leave it’ offer (i.e., they have the bargaining power). In this respect, such an action is one of the necessary policy instruments for increasing welfare in decentralized equilibrium. \(^{31}\)

I assume that the set of policy instruments includes marginal taxes on or subsidies to purchases of high-tech goods \((\tau_x)\) and high-tech firms’ expenditures on buying knowledge \((\tau_\lambda)\). It also includes lump-sum transfers to high-tech firms \((T_\pi)\) and households \((T)\). The latter balances government expenditures.

Under such a policy from the final goods producer’s problem it follows that (8) and (9) need to be rewritten as

\[
x_j = X \left( \frac{P_x}{(1 - \tau_x) p_{xj}} \right)^{\varepsilon},
\]

\[
P_x X = (1 - \tau_x) \sum_{i=1}^N p_{xi} x_i.
\]

In turn, the profit function of high-tech firm \(j\) is

\[
\pi_j = p_{xj} x_j - w (L_{xj} + L_{rj})
\]

\[
+ \left[ \sum_{i=1, i \neq j}^N p_{n_{ij} \lambda_j} (u_{ij} \lambda_j) - (1 - \tau_\lambda) \sum_{i=1, i \neq j}^N p_{n_{ij} \lambda_i} (u_{ij} \lambda_i) \right] + T_\pi.
\]

\(^{31}\)This is because in case there is no knowledge exchange between high-tech firms there is no set of (orthodox) policy instruments in terms of welfare transfers that in decentralized equilibrium equates labor force allocations and growth rate of knowledge to their socially optimal counterparts.
Therefore, the demand for knowledge of the high-tech firm (25) needs to be rewritten as

\[
[u_{i,j}] : (1 - \tau) p_{u_{i,j}, i} = q_i \xi \alpha \left( \frac{\lambda_j}{u_{i,j} \lambda_i} \right)^{1-\alpha} L_{r,j}, \forall i \neq j.
\]

Considering symmetric equilibrium and combining these optimal rules with (6), (7), (24) and labor market clearing condition (36) gives the counterparts of the relation between \( NL_x \) and \( L_Y \) (32), returns on knowledge accumulation (31), and the relation between \( NL_x \) and \( NL_r \) (37):

\[
NL_x = \frac{1}{1-\tau_x} \frac{\sigma}{1-\sigma} b^k L_Y, \tag{81}
\]

\[
g_{q\lambda} = r - g_\lambda \left( \frac{L_x}{L_r} + 1 + \alpha \frac{N-1}{N} \frac{\tau_\lambda}{1-\tau_\lambda} \right), \tag{82}
\]

\[
NL_x = D^{GO} \left( L - NL_r \right), \tag{83}
\]

where \( D^{GO} \) is the counterpart of \( D^k \),

\[
D^{GO} = \left[ (1 - \tau_x) \frac{1-\sigma}{\sigma} b^k + 1 \right]^{-1},
\]

and I use \( GO \) to denote decentralized equilibrium with government.

**Proposition 25** Let the marginal tax rates be constant. In such a case, labor force allocations and the growth rate of knowledge \( g_\lambda \) are

\[
NL_r = \frac{1}{\xi} \frac{\xi D^{GO} L - \rho}{(\theta - 1) (\sigma + \mu) + D^{GO} - \alpha \frac{N-1}{N} \frac{\tau_\lambda}{1-\tau_\lambda}},
\]

\[
NL_x = D^{GO} \left[ (\theta - 1) (\sigma + \mu) - \alpha \frac{N-1}{N} \frac{\tau_\lambda}{1-\tau_\lambda} \right] L + \frac{1}{\xi} \rho \left( \frac{\theta - 1}{\sigma + \mu} + D^{GO} - \alpha \frac{N-1}{N} \frac{\tau_\lambda}{1-\tau_\lambda} \right),
\]

\[
L_Y = (1 - \tau_x) \frac{1-\sigma}{\sigma b^k} NL_x,
\]

\[
g_\lambda = \xi NL_r.
\]

**Proof.** See Appendix P.1 of the online version of this paper. ■

Therefore, in order to have socially optimal growth rate and allocations it is sufficient to have

\[
NL_r = NL^{SP}_r, NL_x = NL^{SP}_x.
\]

**Proposition 26** In order to achieve such an outcome it is sufficient to subsidize
the purchases of high-tech goods,

\begin{align*}
\tau_{\lambda} &= 0, \\
\tau_x &= \frac{e^k \mu + \sigma}{e^k (\sigma + \mu)},
\end{align*}

(84)

This policy equates \( D^{GO} \) to \( D^{SP} \). It is enough to subsidize the demand for high-tech goods because the returns on knowledge accumulation are fully appropriated \(^{27}\).

Although under this policy labor force allocations and growth rate of knowledge in decentralized equilibrium are equal to their socially optimal counterparts, welfare is not. This is because in decentralized equilibrium there is lower number of firms/high-tech goods. The policy instrument that can correct for this is \( T_{\pi} \). It is straightforward to show that it is sufficient to set

\[ T_{\pi} = wL_x \tau_{\pi}, \]

(85)

where \( \tau_{\pi} \) is such that for any finite \( N \) the profits of high-tech firms are greater than zero, but for \( N = +\infty \) profits are zero.

**Proposition 27** The rate \( \tau_{\pi} \) can be derived from a zero profit condition and is given by

\[ \tau_{\pi} = \frac{\varepsilon - 1 + D^{SP}}{(\varepsilon - 1) \left[ (\theta - 1) (\sigma + \mu) \xi D^{SP} L + D^{SP} \right] \times \left[ \frac{\varepsilon - 1 - (\theta - 1) (\sigma + \mu)}{\varepsilon - 1 + D^{SP}} \xi D^{SP} L - \rho \right]. \]

(86)

**Proof.** See Appendix P.1 of the online version of this paper. \( \blacksquare \)

The expression in the second line of \( \tau_{\pi} \) needs to be positive in order to have \( N > 1 \) in (75). Therefore, \( \tau_{\pi} \) is greater than zero implying that entry into high-tech industry needs to be subsidized. Such subsidies are in the spirit of R&D subsidies in Romer’s (1990) model to the extent that entry can be thought to be a result of R&D that generates new types of high-tech goods.

The result that \( \tau_{\pi} \) is greater than zero is not stemming from the cost-free entry assumption. Even if entry into the high-tech industry entailed positive costs then still it could be that at least in very long-run the Social Planner sets \( N = +\infty \).
whereas in decentralized equilibrium the market is saturated for $N < +\infty$. This is because as $\lambda$ grows the marginal product of $N$ increases and the Social Planner would prefer increasing $N$ (see for further discussion Jerbashian, 2011).

4 Conclusions

The model presented in this paper incorporates knowledge (patent) licensing into a stylized endogenous growth framework, where the engine of growth is high-tech firms’ in-house R&D. The inference from this model suggests that if there is knowledge licensing high-tech firms innovate more and economic growth is higher than in cases when there are knowledge spillovers and/or there is no knowledge exchange between these firms. The results also suggest that innovation in the high-tech industry and economic growth increase with the intensity and toughness of competition in that industry. Such an inference holds also for the number of high-tech firms if there is an exchange of knowledge between these firms in the form of licensing or spillovers. Increasing the number of high-tech firms increases innovation in the high-tech industry and the growth rate of the economy. However, if there is no exchange of knowledge between high-tech firms, then increasing the number of firms can also discourage innovation in the high-tech industry and reduce economic growth.

Innovation in the high-tech industry declines with the magnitude of externalities which stem from the use of high-tech goods. However, the rate of economic growth increases with it. Further, the existence of such externalities creates a wedge between resource allocations in decentralized equilibrium and socially optimal allocations. In this model, this implies that the existence of externalities also creates a wedge between growth rates in decentralized equilibrium and the socially optimal growth rate.

If entry (or exit) is endogenous, innovation in the high-tech industry and economic growth are again higher in case when there is knowledge licensing. However, this happens in expense of lower number of high-tech firms. More intensive and/or tougher competition reduce the number of high-tech firms. If there is an exchange of knowledge between these firms the intensity and toughness of competition do not affect, however, allocations, innovation in the high-tech industry, and economic growth. In contrast, allocations change and innovation and economic growth tend to increase with the intensity and toughness of competition if there is no exchange of knowledge between high-tech firms.

Appendices E.4 and E.5 of the online version of this paper show how $\tau_\lambda$ can be used together with $\tau_\pi$ in cases when there is continuous entry into high-tech industry ($g_N > 0$) or high-tech firms do not take the price of knowledge as exogenous. If $\tau_\lambda \neq 0$ then subsidy rate $\tau_\pi$ is not given by (86).
In the model offered in this paper, a policy consisting of four instruments can be sufficient for achieving the first best outcome in decentralized equilibrium. The policy gives the bargaining power in the market for knowledge to the licensors so that they appropriate all the benefit. Further, it subsidizes the purchases of high-tech goods so that it offsets the negative effect of price setting by high-tech firms and takes into account the externalities from the use of high-tech goods. Finally, it subsidizes entry into the high-tech industry and uses lump-sum taxes to cover all these subsidies.
References


