The firm size distribution across countries and skill-biased change in entrepreneurial technology*

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Abstract

How and why does the firm size distribution differ across countries? This paper documents that features of the firm size distribution are strongly associated with income per capita. Richer countries have fewer entrepreneurs and fewer small firms. The average, dispersion and skewness of firm size are all larger in richer countries. A simple general equilibrium model of occupational choice with skill-biased change in entrepreneurial technology calibrated to U.S. data can account very well for these patterns. The crucial assumption is that some entrepreneurs benefit more from technological progress than others. Marginal entrepreneurs then switch to becoming employees as technology advances.

JEL codes: E24, J24, L11, L26, O30

Keywords: occupational choice, entrepreneurship, firm size, skill-biased technical change

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1 Introduction

How and why does the firm size distribution differ across countries and within country over time? Within the growing body of recent work on the firm size distribution (see e.g. Samaniego 2006, Gollin 2007, Restuccia and Rogerson 2008, Hsieh and Klenow 2009), most papers study the distribution in one or few countries at a single point in time. As a consequence, little is known about broad cross-country patterns. This paper documents a particular set of first order features of the cross-country data: changes in the firm size distribution with development. It documents how the firm size distribution differs across countries with different income levels and in U.S. history and then proposes a simple theory that is consistent with these patterns.

Governments in many countries spend resources to encourage the formation and to subsidize the operation of small businesses (for example the Small Business Administration’s Advantage Loan Initiatives in the U.S. and the “Ich-AG” (“Me, Inc.”) scheme in Germany), while at the same time, often implicitly, promoting large companies (“national champions”). In advocating such policies, often comparisons are drawn to firm size distributions in other places (e.g. “dynamic” emerging markets with many small firms), without much knowledge of the patterns to expect when comparing firm size distributions across countries. Information on patterns in firm size distributions across countries and on their determinants therefore provides a valuable baseline for putting such statements in perspective.

Comparing firm size distributions across countries is a challenge because of a lack of harmonized data. To overcome this problem, this paper uses data from the Global Entrepreneurship Monitor (GEM), a survey conducted in around 50 countries that focuses on obtaining internationally comparable information on entrepreneurs, in addition to historical U.S. data. To the best of my knowledge, this is the first paper using information from the GEM for macroeconomic analysis. Section 2 uses this data to review and extend to a broader cross-section two known facts about the firm size distributions, and to establish two new facts. The findings are: First, the entrepreneurship rate falls with per capita income across countries. Second, average firm size increases with per capita income. The first fact fits with the finding of Gollin (2007) that the self-employment rate falls with per capita income in ILO cross-country data. The second one extends Lucas’s (1978) results to more recent U.S. data and into the cross-country dimension. The next two facts are new: Third, the standard deviation of firm size increases with per capita income across countries. Fourth, the skewness of the firm size distribution also increases with per capita income across countries.

The data thus show a relationship between the level of development and features of the
firm size distribution. Lucas (1978) and Gollin (2007) provide explanations for the first two facts, but their models do not fit the other two. This paper shows that all four facts can be explained in an otherwise standard occupational choice model à la Lucas (1978) with two additional features: technological change not benefitting all potential entrepreneurs equally, and a positive relationship between an individual’s potential payoffs in working and in entrepreneurship.

I call the first feature skill-biased change in entrepreneurial technology. It is remarkable that in contrast to the very large literature on skill-biased technical change among workers, there is hardly any work on the importance of skills for the entrepreneurs who employ those workers, and in particular on their evolution over time. Technological change is taken for granted as the main historical driver of growth in developed economies. In the recent literature, several types of technological change apart from the neutral variety have received a lot of attention (see e.g. Greenwood, Hercowitz and Krusell (1997) on investment-specific technological change, Krusell, Ohanian, Rios-Rull and Violante (2000) on capital-skill complementarity, Katz and Murphy (1992) on skill-biased technical change and the demand for workers or Hornstein, Krusell and Violante (2005) on links among the three). However, there has been barely any work on how technological change affects entrepreneurs. Yet, entrepreneurs need to implement the technologies that they and their employees then operate, so the effect of technical change on entrepreneurs is of crucial importance for how technology subsequently affects labor demand, wages and employment.

In the simple theory of skill-biased change in entrepreneurial technology proposed here, aggregate technology affects occupational choice of entrepreneurs and thus the firm size distribution. When calibrated to U.S. data, the model is consistent with the cross-country evidence on the firm size distribution and development.

Anyone who has programmed a VCR or tried to set up a home computing network will appreciate that while technological progress brings productivity advances, it often goes along with increased complexity of technology. This is even more so for firms, and not just for large or “high-tech” ones. Consider the corner shop owner contemplating the installation of bar code scanners. This allows automating inventory control, but requires managing the related computing infrastructure. Or consider the owner of a car repair shop who needs to master the increasing amount of computing power of customers’ cars. This allows for faster diagnostic checks, but also requires mastering technology that is quite distinct from the core technologies used in that business.

As the menu of available technologies expands, raising aggregate productivity (assuming love of variety, as in Romer 1987), individual firms have to cope with increasing complexity of technology. To reflect this, the key assumption in the model, which otherwise is a standard
occupational choice model à la Lucas (1978), is that, while advances in the technological frontier give all firms access to a more productive technology, they do not affect all firms equally. Some firms can implement new technologies at lower cost, and therefore take more advantage of them. As a result, some firms remain close to the frontier and use a production process involving many, highly specialized inputs, while others fall behind the frontier, use a simpler production process, and fall behind in terms of relative productivity\(^1\).

The second crucial assumption is that agents differ in their labor market opportunities and that more productive workers can also manage more complex technologies if they become entrepreneurs. Occupational choice between employment and entrepreneurship closes the model. Because advances in the technological frontier do not benefit every potential entrepreneur equally, the position of the frontier then governs occupational choice. The more advanced the frontier, the greater the benefit from being able to stay close to it, as other firms fall behind. Because in equilibrium, advances in the frontier also raise wages, entrepreneurs’ outside option improves, and marginal entrepreneurs exit. The result is a “history”, explored in Section 4 in which high-productivity entrepreneurs are gradually drawn into the market and slowly expand their operations as their productivity improves more than others\(^1\). Their entry and growth raise labor demand and the wage, implying that low-productivity entrepreneurs eventually find employment more attractive and exit\(^2\).

The need for skills to deal with a broad array of technologies at the same time is in line with Lazear’s (2004, 2005) finding that entrepreneurs tend to have more general skills than employees. It also fits with evidence from the burgeoning recent literature on CEOs and CEO pay, which shows that the importance of general skills has risen of late (see e.g. Murphy and Zabojnik 2004, Rajan and Wulf 2006, Frydman 2007)\(^3\) These skills are usually measured as the variety of someone’s experience of different industries, companies, functions within companies (e.g. production, marketing, finance), and thus technologies. The main

\(^1\) Jovanovic and Rousseau (2008) document that from 1971 to 2006, the average yearly growth rates of the stocks of patents and trademarks in the U.S. were 1.9% and 3.9%, respectively, implying a substantial increase in variety. Similarly, every new classification of occupations in the U.S. from 1970 to 2010 lists more occupations than the preceding one (Scopp 2003). At the same time, Cummins and Violante (2002) find that the gap between the frontier and average technology in use has been increasing in the U.S. over the entire span of their data (1947-2000), implying that firms have not all benefitted equally from technology improvements.

\(^2\) As there is an across-the-board productivity increase in the model as the frontier advances, it also allows for certain tasks that used to be at the technological frontier to be achieved by entrepreneurs behind the frontier as technological advances. Think e.g. about multimedia; a professional can now do on a single computer what in earlier times would have required much more resources. Yet, the frontier moves on – the professional benefits, but entrepreneurs closer to the frontier now can use even more advanced technology.

\(^3\) Of course, CEOs and entrepreneurs do not fulfill exactly the same functions. Still, their job content is rather similar, with the main difference being the importance of the willingness to take risk. As this will not play a prominent role in this paper, CEOs are an informative group of comparison.
reasons for this phenomenon suggested by that literature are a growing need to master more technologies at the same time and broader responsibilities that come from flatter hierarchies made possible by advancing information technology. If entrepreneurs want to benefit from the new possibilities put on the menu by technological advances, they need to keep up with technological developments. The degree to which they can do so determines how much benefit they reap from technological progress.

While the effects of this development on organizational hierarchies and CEO pay have received a lot of attention recently, the general equilibrium implications have not been studied. Yet, they are substantial, as incentives for entrepreneurship determine not just individual occupational choice and entrepreneurs’ incomes, but also aggregate labor demand, the level of aggregate technology that is actually in place, output and, of course, the firm size distribution. Analyzing this and fitting observed cross-country patterns is the main contribution of this paper. While the assumptions made here on the use of technology are admittedly much simpler than those in the micro literature, they make it possible to transparently obtain a full set of general equilibrium results and compare these to the evidence.

The calibration exercise in Section 5 shows that the model fits the U.S. experience, including the history of average firm size, well. Although not targeted in the calibration, it also generates a trend in income concentration at the top very similar to that documented by Piketty and Saez (2006). More strikingly, using parameter values from the calibration to the U.S., the model matches not only the qualitative relationship between per capita income and the entrepreneurship rate, the fraction of small firms, average firm size, and firm size dispersion and skewness across countries, but actually delivers a good quantitative fit for most of these dimensions. In particular, the predicted changes in the entrepreneurship rate, in the fraction of small firms and in average firm size with per capita income are very close to those in the data. Because of its stylized nature, the model overpredicts the sensitivity of firm size dispersion and skewness to per capita income in part of the income distribution.

The simple model with skill-biased change in entrepreneurial technology proposed here hence fits cross-country patterns in the firm size distribution very well. It can thus provide

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4 This is qualitatively different from the need for employees to keep up with technology: employees need to apply a given technology, while entrepreneurs need to choose and coordinate the technologies used in a firm’s production process. So even if technological progress had de-skilling elements in the 19th century, as argued by James and Skinner (1985) and by Cain and Paterson (1986), replacing skilled workers with machinery still made increasing demands on entrepreneurs to understand and coordinate the new technologies that now were available in addition to the old ones. The setting here thus does not depend on complementarity between capital and workers’ skills; all that is needed is that keeping up with advancing technology is costly for entrepreneurs.

a baseline when thinking about differences in the firm size distribution across countries. In addition, it provides a convenient way of taking some results from the micro literature on entrepreneurs and skills to macroeconomics. (Section 5.4 discusses some alternative potential explanations; they cannot fit all the patterns in the data documented in Section 2.) Besides the references above, this paper is related to two further strands of literature. First, several papers have analyzed entrepreneurial choice and its macroeconomic implications; see Quadrini (2009) for an excellent review. An important recent paper here is Cagetti and De Nardi (2006). These authors fit a model of entrepreneurial choice to U.S. data with the aim of assessing the contribution of entrepreneurship and credit constraints to wealth inequality. Like other contributions to this literature, their model does not involve changes in entrepreneurial choice with development. Entrepreneurial choice and development has been analyzed by Banerjee and Newman (1993) and Lloyd-Ellis and Bernhardt (2000). However, these papers focus on the role of the wealth distribution when there are credit constraints, but do not feature an evolving role for skills as the present paper does.

Secondly, some papers have taken a similar view of skills, complexity or the role of the entrepreneur as this paper. Teulings (1995) relates skills to the ability to deal with complexity, but does not consider entrepreneurship. Lloyd-Ellis (1999) assumes that skill is required for implementing a technology, but focuses on the tradeoff between using skills for R&D or for implementation. Jovanovic and Rousseau (2008) also model a manager’s task as finding the right combination of heterogeneous inputs but focus on the quality of the match between a firm’s products and its workers’ skills, not on the evolution of entrepreneurial choice and the firm size distribution with development.

The paper is organized as follows. Section 2 describes the GEM dataset and documents relevant facts about entrepreneurship and the firm size distribution. Section 3 presents the model, and Section 4 shows how entrepreneurship and characteristics of the firm size distribution change with development. Finally, Section 5 presents a generalization of the model and quantitative results, and Section 6 concludes.

2 Entrepreneurship, the firm size distribution and development

Obtaining data on the firm size distribution across countries is notoriously hard because measurement is not harmonized across countries. The relatively new Global Entrepreneur-
ship Monitor (GEM) dataset is an exception.\(^6\) To the best of my knowledge, this is the first paper using GEM data across countries for macroeconomic analysis. As this is a new dataset and probably is not well known to macroeconomists, I briefly present it in the next subsection.

The remainder of the section then shows four facts on occupational choice and the firm size distribution across countries obtained using the GEM data: entrepreneurship and the self-employment rate fall with per capita income, and the mean, standard deviation and skewness of firm size all increase with per capita income. The first two facts are known yet worth revisiting briefly, while the last two are new.

### 2.1 The Global Entrepreneurship Monitor (GEM) survey

The GEM is an individual-level survey run by London Business School and Babson College now conducted in more than 50 countries. Country coverage has been expanding since its inception in 1999, with data for several years available for most countries. The micro data is in the public domain, downloadable at [http://www.gemconsortium.org/](http://www.gemconsortium.org/). Most developed economies are represented, plus a substantial number of transition and developing economies, ensuring that the data covers a wide variety of income levels.\(^7\)

The survey focuses on entrepreneurship. That is, while the survey overall is conducted by local research organizations or market research firms to be representative of a country’s population, it contains only limited demographic information (e.g. education) on non-entrepreneurs. It contains much richer information on entrepreneurs, including their firm's employment.

Importantly, the survey is designed to obtain harmonized data across countries. It is thus built to allow cross-country comparisons, the purpose for which it is used here. In addition, because it is an individual-level survey, it captures all types of firms and not just firms in the formal sector or above some size threshold. For studying occupational choice, this is evidently important. This feature makes the GEM data more adequate for the purposes of the analysis in this paper than firm- or establishment-level surveys such as the World Bank Group Entrepreneurship Survey, which covers only registered corporations, or Dun & Bradstreet data, which is reasonably representative of U.S. firms but does not cover many

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\(^6\)Another exception are some OECD publications such as Bartelsman, Haltiwanger and Scarpetta (2004) that provide information on some OECD countries and a limited number of other countries. Their numbers arise from an effort to harmonize national official data, while the GEM approach already involves harmonized data collection (though inevitably at a smaller scale).

\(^7\)Inclusion in the survey depends on an organization within a country expressing interesting and financing data collection.
small firms in other countries, especially in poorer ones.

To obtain data on entrepreneurship rates, I use country averages of the country-level data covering the years 2002-2008 available on the GEM website for 66 countries. Micro data is available for 1999 to 2005 and covers fewer countries. I use it to obtain statistics on the firm size distribution, for which no country-level numbers are reported. As the initial years of the survey may be less reliable, I use the micro data for the period 2001-2005. For this period, data is available for 47 countries, though not for all years for all countries. Pooling the available years for each country, the number of observations per country is between 2,000 in some developing economies and almost 80,000 in the UK, with a cross-country average of 11,700. This is sufficient for computing the summary statistics of the firm size distribution that I use in the following. Unfortunately, in many countries, there are not enough observations for obtaining reliable estimates for detailed size classes, so I rely on summary statistics for the entire distribution. I consider someone an entrepreneur if they declare running a firm that they own and they have already paid wages (possibly to themselves, for the self-employed). I then obtain firm size data for these firms, truncating the distribution at 1000 employees to reduce measurement error.

The GEM dataset is very useful because of the harmonized data collection. Moreover, it allows establishing all facts of interest using one single dataset. However, it is still important to know that results hold more generally, and are not due to specificities of the survey. Therefore, I compare the facts presented here to some results from other sources. In addition, Reynolds et al. (2005), Acs, Desai and Klapper (2008) and Ardagna and Lusardi (2008) show that observations from GEM data tend to align well with those based on other sources.

2.2 The facts

Figure plots statistics on entrepreneurship and the firm size distribution against 2005 real GDP per capita at purchasing power parity from the Penn World Tables (Summers and Heston 1991). Each subfigure illustrates one of the following four facts.

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8Because the GEM is a household survey, publicly listed firms with dispersed ownership are not included. Since these are particularly important in high-income countries and tend to be large, their inclusion would only strengthen the results reported below.

9By its sampling procedure, the survey captures few agricultural businesses (only 4% on average). As self-employment is typically higher and income per capita typically lower in agriculture (see e.g. Caselli 2005, Restuccia, Yang and Zhu 2008), the facts presented in the following would be even more pronounced if they could be produced using a reliable up-to-date measure of non-agricultural GDP per capita at PPP.

10All regression lines plotted in the figure are significant at least at the 5% level. Measures of fit are reported in the figure notes and are relatively high for a bivariate relationship in cross-sectional data.
Fact 1  The entrepreneurship rate falls with income per capita (see Figure 1(a)).

This fits with the finding of Gollin (2007) that the self-employment rate falls with income per capita in ILO data. Although the negative relationship between the entrepreneurship rate and per capita income is very robust, it does not seem to be well known. The reason for that is that the population of entrepreneurs under consideration matters. The fact holds for broad measures of entrepreneurship that include small firms. Because the share of small firms is larger in poorer countries (see Figure 3(a)), the relationship is reversed when considering only large firms or using sample selection criteria that exclude most small firms, e.g. incorporation. This is the case for instance in data from the World Bank Group Entrepreneurship Survey, which covers only registered corporations. This positive relationship is often attributed to differences in regulation; see e.g. Klapper, Laeven and Rajan (2006) and Barseghyan (2008). For studying occupational choice, focussing on registered firms is not sufficient and it is necessary to take into account all firms, as in the GEM or ILO data.

Fact 2  Average firm employment increases with income per capita (see Figure 1(b)).

This fact is of course closely related to fact 1, as high entrepreneurship rates must necessarily imply smaller average employment. Previously, this relationship has only been documented across a limited number of countries (Tybout 2000). In addition, Lucas (1978) reported that average firm size increased with per capita income over U.S. history (1900-70). Figure 2.2 shows that this time-series relationship persists. It reports measures of average firm size close to those used by Lucas (the two series labelled “BEA Survey of Current Business” and “Dun & Bradstreet”, both from Carter, Gartner, Haines, Olmstead, Sutch and Wright 2006) and more recent data. The most recent available series is from U.S. Census Business Dynamics Statistics (BDS). This is aggregated annual data based on the Longitudinal Business Database (LBD) maintained by the Census Bureau’s Center for Economic Studies which draws on, among other sources, the Business Register, Economic Censuses and IRS payroll tax records. As a result, the BDS covers employer firms accounting for 98% of U.S. private employment. In order to obtain average firm size for a broader measure of firms I also report average firm size when taking into account non-employer firms, or self-employed without employees. This measure is obtained by combining BDS data with data

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11 In U.S. history, the self-employment rate fell continuously until the mid-1970s, when it temporarily rebounded for a few years, mainly due to changes in tax rates (Blau 1987; see also Hipple 2004).
12 Interestingly, the share of small firms also falls over the whole sample period 1977-2009 in the U.S. Census BDS data presented below.
13 Strangely enough, this simple relationship often seems to escape policy discussions on promoting entrepreneurship.
Figure 1: The firm size distribution and per capita income – alternative measures.

Notes: Real GPD per capita for 2005 at purchasing power parity from the Penn World Tables (Summers and Heston 1991, Heston et al. 2009); entrepreneurship rate, average employment, standard deviation and skewness of employment from GEM data, http://www.gemconsortium.org. The measure of skewness is the fraction of firms with less than average size. Entrepreneurs are defined as survey respondents who declare running a firm that they own and who have already paid wages, possibly to themselves. Average firm size for Latvia is 60% above the next-highest value. This may indicate data problems; the observation is therefore excluded. All regression lines plotted in the figure are significant at least at the 5% level. The regression $R^2$ is 15.9% in panel (a), 24.0% in panel (b), 15.3% in panel (c) and 14.8% in panel (d).

While the five series shown in the figure cover slightly different populations of firms, they all show an increasing trend, except for the period 1900-1930. This upward trend of course occurs simultaneously with increasing per capita income. Firm size thus increases with per capita income both in

14Unfortunately, this series is rather short. This is because information on employment by the unincorporated self-employed is only available starting in 1995. Many thanks to Steven Hipple for providing some additional information.
U.S. history and across countries\cite{footnote15}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Average firm size (employment) over U.S. history, 1890-2006}
\end{figure}

Sources: Census Bureau Business Dynamics Statistics (BDS): data available at [http://www.ces.census.gov/index.php/bds](http://www.ces.census.gov/index.php/bds), when including non-employers, combined with Current Population Survey (CPS) data reported in Hipple (2010); Census Enterprise Statistics series: from various Census reports; BEA Survey of Current Business series: from Carter et al. (2006, Series Ch265); Dun & Bradstreet series: from Carter et al. (2006, Series Ch408). The first three sources also report total employment. For the last two series, employment is from Carter et al. (2006, Series Ba471-473 and Ba477). The Dun & Bradstreet firm counts exclude finance, railroads and amusements. Adjusting employment for this using Series Ba662, Dh31, Dh35, Dh53 and Df 1002 shortens the series without affecting the trend. Starting 1984, Dun & Bradstreet gradually cover additional sectors, at the cost of comparability over time, so I only plot data up to 1983. Series Ch1 in Carter et al. (2006), which draws on Internal Revenue Service data, also contains historical firm counts but is less useful because of frequent changes of definition, in particular for proprietorships.

\textbf{Fact 3} The dispersion of firm size in terms of employment increases with income per capita (see Figure 1(c)).

This is the fact for which the GEM data contribute most, as it seems impossible to obtain from other sources in a consistent way for more than a small number of countries. The figure shows a clear positive relationship between the standard deviation of firm size and per capita income\cite{footnote16}. The only previous mention of such a relationship I could find is Bartelsman et al.\cite{footnote15}

\footnotetext[15]{Jovanovic and Rousseau (2008) show that another measure of size, patents or trademarks per firm, has also increased from 1971 to 2006 (see also footnote \[1\]). For other countries, it is not easy to come by histories of average firm size. However, data reported in a special issue of \textit{Small Business Economics} reveal that average firm size also increased with development in several East Asian economies. This is the case in Indonesia (Berry, Rodriguez and Sandee 2002), Japan (Urata and Kawai 2002), South Korea (Nugent and Yhee 2002) and Thailand (Wiboonchutikula 2002). Only in Taiwan, the smallest of these countries, did it fall (Aw 2002). Finally, Moscarini and Postel-Vinay (2010) also use BDS data to analyze the cyclical behavior of employment creation by firm size.}

\footnotetext[16]{It also holds when using the interquartile ratio of firm size, a measure more robust to outliers (see Figure 3(b)).}
(2004), who show that firm size dispersion is substantially higher in industrialized countries compared to emerging markets, using OECD and World Bank data for a much smaller set of countries. This finding is important, because it indicates that larger average size in richer countries is not simply due to a shift to the right of the firm size distribution.\footnote{Hsieh and Klenow (2009) compute TFP dispersion in China, India and the U.S.. Apart from the fact that their numbers are hard to compare to the ones obtained here because they are restricted to manufacturing and refer to establishments, not firms, they are also effectively forced to impose a size cutoff because some variables are missing for small establishments in their otherwise very rich data. This affects measured dispersion. Comparing their Table I to Census BDS data shows that in the case of the U.S. in 2001 for instance, they need to exclude almost half the manufacturing establishments. The size distribution plotted in their Figure IX shows that these are mostly small establishments belonging to firms with less than 10 employees. While these issues are less important for the purpose of their paper, it is preferable to have firm data without a size cutoff and without the limitation to a single sector for analyzing occupational choice between wage work and entrepreneurship.}

**Fact 4** The skewness of firm size in terms of employment increases with income per capita (see Figure 1(d)).

The share of firms with size below the mean, significantly above 50\% in all countries, strongly increases with per capita income, implying increasing importance of the right tail of large firms. This pattern also arises when using alternative measures of skewness; see Figures 3(c) and 3(d).

Overall, richer countries thus feature fewer, larger firms, with a firm size distribution that is more variable and more skewed. Facts 2 and 3 thus are not simply due to a shift or a mean-preserving spread in the size distribution, but rather to the size distribution stretching out further to the right.

The first two facts have received some previous attention. Lucas (1978) in his seminal occupational choice framework explains Fact 2 by allowing for complementarity in production between the capital and labor inputs. More productive economies accumulate more capital, which with the complementarity raises wages more than profits, reducing the share of entrepreneurs and thus raising the average size of firms. Gollin (2007) explicitly introduces self-employment as an option and then uses a similar framework to fit self-employment rates across countries (Fact 1).

In each of these cases, the agents who choose entrepreneurship are the fraction of the population that is best at it. Increases in productivity raise the threshold and reduce that fraction. While this implies that in richer countries, there are fewer and larger firms, this mechanism does not explain Facts 3 and 4. To the contrary, a more homogeneous population of entrepreneurs may well reduce the standard deviation and skewness of firm size. Other
Figure 3: Entrepreneurship, the firm size distribution and per capita income.

Notes: Sources as in Figure 1. All regression lines plotted in the figure are significant at least at the 5% level. Denoting the $x^{th}$ percentile of the firm size distribution by $p_x$, the interquartile ratio is $p_{75}/p_{25}$, the 90/10 percentile skewness measure used in panel (c) is $((p_{90} - p_{50}) - (p_{50} - p_{10}))/((p_{90} - p_{10}))$, and the skewness measure in panel (d) is the third central moment applied to the data excluding the 5% largest firms to reduce the influence of outliers. The regression $R^2$ is 31.4% in panel (a), 24.4% in panel (b), 7.7% in panel (c) and 13.9% in panel (d).

Potential explanations for differences in average firm size are briefly discussed in [Section 5.4](#). They also have problems matching the patterns in dispersion and skewness. Facts 3 and 4 are thus important for evaluating different models that can fit Facts 1 and 2. The model developed in the next section addresses these points and thus is able to explain all four facts.
3 A simple model

The economy consists of a unit continuum of agents and an endogenous measure of firms. Agents differ in their endowment of effective units of labor $a \in [0, \bar{a}]$ that they can rent to firms in a competitive labor market. Refer to this endowment as “ability”. Differences in ability can be thought of as skill differences. They are observable, and the distribution of ability in the population can be described by a pdf $\phi(a)$.

Agents value consumption $c$ of a homogeneous good, which is also used as the numéraire. They choose between work and entrepreneurship to maximize consumption. The outcome of this choice endogenously determines the measures of workers and of firms in the economy.

Labor supply and wage income. Consumption maximization implies that individuals who choose to be workers supply their entire labor endowment. Denoting the wage rate per effective unit of labor by $w$, a worker’s labor income then is $wa$.

Labor demand and firm profits. Firms use labor in differentiated activities to produce the homogeneous consumption good. They differ in their level of technology $M_i$, which indicates the number of differentiated activities in a firm. It thus corresponds to the complexity of a firm’s production process, or the extent of division of labor in the firm. A firm’s level of technology depends on the entrepreneur’s skill in a way detailed below.

A firm’s production technology is summarized by the production function

$$y_i = X_i^\gamma, \quad X_i = \left(\int_0^{M_i} n_{ij}^{\sigma-1} \, dj\right)^{\frac{1}{\sigma-1}}, \quad \gamma, \sigma \in (0, 1), \sigma > 1,$$

where $y_i$ is output of firm $i$, $X_i$ is an aggregate of the differentiated labor inputs $n_{ij}$ it uses, and $M_i$ indicates the degree of complexity of its technology. The production function exhibits decreasing returns to scale. This can be interpreted to reflect any entrepreneur’s limited span of control, as in Lucas (1978). It also ensures that firm size is determinate, implying a firm size distribution given any distribution of $M$ over firms. The elasticity of substitution among inputs is given by $\sigma$. Given that $M$ differs across firms and that thus not all firms use all types of differentiated inputs, it is natural to assume that different inputs are substitutes ($\sigma > 1$). Heterogeneity in $M$ plays a role as long as they are imperfect substitutes.

$^{18}$Concave utility would not affect qualitative results. While in general, risk aversion is an important factor affecting entrepreneurial entry (see e.g. Kihlstrom and Laffont 1979, Vereshchagina and Hopenhayn 2009), the mechanism at the heart of this paper does not interact with it. An extension in Section 5 can be interpreted in terms of heterogeneity in risk aversion.
substitutes, as shown below. Importantly, the production function exhibits love of variety, and firms with larger $M$ are more productive.

The firm’s profit maximization problem can be solved using a typical two-stage approach: choose inputs $n_{ij}$ to minimize the cost of attaining a given level of the input aggregate $X_i$, and then choose $X_i$ to maximize profit. The solution to the latter will depend on a firm’s productivity $M_i$.

Denoting desired output by $\bar{y}$ and defining $\bar{X} = \bar{y}^{1/\gamma}$, the solution to the cost minimization problem yields the firm’s labor demand function for each activity $j$ as

$$n_j(M) = \left(\frac{w}{\lambda(M)}\right)^{-\sigma} \bar{X} \quad \forall j,$$

(2)

where $\lambda$ is the marginal cost of another unit of $X$. With constant returns to scale for transforming the differentiated labor inputs into $X$, $\lambda$ is independent of $X$ and equals $M^{1-\sigma} w$, and the demand for each $n_j$ becomes

$$n_j(M) = M^{\frac{\sigma}{1-\sigma}} \bar{X} \quad \forall j.$$

(3)

Because of greater specialization in firms using more complex technologies, their marginal cost of $X$, $\lambda$, is lower. As a consequence, they require less of each input to produce $\bar{y}$. Because a larger $M$ allows a firm to produce more output from a given quantity of inputs, I will in the following refer to $M$ as the firm’s productivity. While $M$ does not equal TFP, it maps one-to-one with TFP.

Choice of $\bar{X}$ to maximize profits yields optimal output and profits as

$$y(M) = \left(\frac{w}{\gamma}\right)^{\frac{1}{1-\gamma}} M^{\frac{1}{\gamma-1}} \bar{X}, \quad \pi(M) = (1-\gamma)y(M).$$

(4)

Both output and profits increase in $M$. They are convex in $M$ if $\gamma > \frac{\sigma-1}{\sigma}$ \(^{20}\) As this inequality holds for reasonable sets of parameter values (e.g. $\gamma = 0.9$ and $\sigma < 10$), I will from now on assume that it is satisfied.

\(^{19}\)The formulation in equation (1) is isomorphic to one where final goods firms use (a heterogeneous number of) differentiated intermediate products, intermediates are produced using a production function that is linear in labor, and there is perfect competition in each intermediate goods sector. Monopolistic competition in intermediate goods can also be accommodated easily and would just require a remapping of parameters. In the quantitative exercise in Section 5, a more general specification is chosen in which intermediates are produced using capital and labor with constant returns to scale.

\(^{20}\)A low $\gamma$ implies more quickly decreasing returns to scale. As a result, optimal size responds less to productivity, and benefits from being more productive are not as large, implying less convex $\pi(M)$. High $\sigma$ implies that inputs are more substitutable, so the benefit of being able to use more of them declines.
Skills and technology. Entrepreneurs run firms and collect their firm’s profits. The crucial activity involved in running a firm is setting up and overseeing a technology involving $M_i$ differentiated activities. Agents differ in their skill in doing this.

To capture this, suppose that an entrepreneur’s time endowment is fixed at 1, and that overseeing an activity takes $c(a, \bar{M})$ units of time, where $\bar{M} \geq 1$ is a measure of aggregate technology. Since profits increase in $M$, each entrepreneur chooses to oversee as many activities as possible given limited time. This implies that $M(a, \bar{M}) = 1/c(a, \bar{M})$. Also suppose that $\partial c/\partial a < 0$ and $\partial c/\partial \bar{M} < 0$. The first assumption implies that more able individuals can manage more complex production processes and thus run more productive firms. The second assumption implies that, conditional on an entrepreneur’s skill, any firm is more productive when situated in a technologically more advanced economy. This links aggregate output to $\bar{M}$. Finally, assume that $\partial^2 c/(\partial \bar{M} \partial a) < 0$. This is “skill-biased change in entrepreneurial technology”. It captures that, while all entrepreneurs benefit from improvements in aggregate technology $\bar{M}$, more skilled entrepreneurs benefit more.

For concreteness, suppose that $c(a, \bar{M}) = \bar{M}^{-a}$. This functional form is similar to the one often chosen for the marginal cost of innovation in the literature on endogenous growth with R&D. The presence of $a$ in the exponent is akin to introducing heterogeneity in the parameter that controls how existing knowledge affects the productivity of R&D in e.g. Jones (1995). More skilled entrepreneurs are better in drawing on existing knowledge. They are better in exploiting similarities and synergies between different activities, therefore can oversee more of them, and are more productive. As technology advances, the potential for exploiting synergies grows, and more skilled entrepreneurs benefit more from the new technologies.

With this specification, the most able entrepreneurs ($a = \bar{a}$) operate at the technological frontier, the least able ones ($a = 0$) at the lowest level, and intermediate ones at some distance to the frontier. The position of a firm relative to the frontier, $m(a, \bar{M}) = M(a, \bar{M})/M(\bar{a}, \bar{M}) = \bar{M}^{a-\bar{a}}$, is bounded between 0 and 1. Crucially, for low levels of the frontier, all firms are close to it. The higher the frontier, the more dispersed the levels of technology of potential firms. The actual distribution of technology among active firms

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21Galí (1995) uses a similar setup with choice of $M$ for a representative firms. Yet, this is a natural place to introduce heterogeneity. The positive correlation of potential profits and earnings $wa$ is in line with empirical evidence showing that more educated individuals earn both higher wages and make more profits as entrepreneurs; see e.g. Evans and Leighton (1989) and Hamilton (2000). Rosen (1982) makes a similar assumption. With a different sign of the derivative, radically different occupational choice outcomes are possible, as shown by Jovanovic (1994) in a related setting. However, results are quite rich even with the natural assumption in the text.

22In that paper, the marginal cost of a unit of knowledge is proportional to $A^{-\phi}$, where $A$ is existing knowledge and $\phi$ governs the contribution of $A$ to new knowledge creation.
depends on occupational choice.

**Occupational choice.** Occupational choice endogenously determines the distributions of workers’ ability and of firms’ technologies. Since both the firm’s and the worker’s problem are static, individuals choose to become a worker if \( w(\bar{M})a > \pi(M(a, \bar{M})) \). Given the wage rate and the state of aggregate technology, the known value of an agent’s ability thus is sufficient for the choice.\(^{23}\) A population ability distribution then implies, via labor market clearing, an occupational choice for each \( a \) and corresponding distributions of workers’ ability and firms’ productivity.

Because profits are continuous, increasing and convex in \( a \), while wages are linear in \( a \), it is clear that there is a threshold \( a_H \) above which it is optimal to become an entrepreneur. If \( a_H < \bar{a} \) (the upper bound on \( a \)), high-productivity firms are active in the economy. At the same time, from (4), \( \pi(M(0)) > 0 = w \cdot 0 \), so that agents with ability between 0 and a threshold \( a_L \) become entrepreneurs. Individuals with \( a \in (a_L, a_H) \) choose to become workers.

In the model, the activity of low-profit entrepreneurs – so abundant in the data – is due to the specific way in which technology and its relationship with ability is modelled here and need not necessarily arise with other ways of modelling heterogeneity in productivity and its relation to ability. Yet, while the specification chosen here delivers their existence somewhat directly, their occupational choice arises naturally in more general settings with heterogeneity in productivity and pre-entry uncertainty about a project’s merits, as shown in Poschke (2010b). More precisely, even if expected profits of the lowest-ability potential entrepreneur are zero or negative, this is not what matters because of the ability to reject bad projects. Once only sufficiently good projects are accepted, low-ability agents will choose entrepreneurship if projects that are preferred to employment exist and they are sufficiently likely to find them. That paper also provides empirical evidence on the phenomenon of low-ability entrepreneurship and its relationship with potential wages that fits with the setting adopted here.

For \( a_H < \bar{a} \), the resulting occupational choice pattern then is as depicted in Figure 4, which plots the value of entrepreneurship (solid line) and of employment (line with crosses) against \( a \). Low- and high-\( a \) agents become entrepreneurs, with intermediate-\( a \) individuals choosing to become workers.\(^{24}\) This pattern persists when also considering additional heterogeneity that is orthogonal to that in \( a \), e.g. differences in taste for entrepreneurship or in attitudes towards risk. This two-sided occupational choice pattern fits with evidence on

\(^{23}\)We abstract from entry costs, sunk investment, search or other issues that would make the problem dynamic without necessarily substantially affecting results. Poschke (2010b) analyzes occupational choice with search for a good project.

\(^{24}\)The lower threshold \( a_L \) is always interior (\( \in (0, \bar{a}) \)), as otherwise the labor market does not clear.
the propensity to be an entrepreneur across the education and wage distribution reported in Poschke (2010b). It differs from the pattern usually obtained in this type of model, e.g. the individuals with the highest entrepreneurial ability (Lucas 1978) or the lowest risk aversion (Kihlstrom and Laffont 1979) choosing entrepreneurship. The self-employed in Gollin (2007) also have relatively high entrepreneurial ability and potential wages.

Figure 4: The values of employment ($W(a)$) and entrepreneurship ($V(a)$)

Equilibrium. An equilibrium of this economy consists in a wage rate $w$ and an allocation of agents to activities such that, taking $w$ as given, agents choose optimally between work and entrepreneurship, firms demand labor optimally, and the labor market clears.

Denoting the density of firms over $a$ by $\nu(a)$, their total measure by $B$ and total effective labor supply by $N \equiv \int_{a_L}^{a_H} a\phi(a)da$, the equilibrium wage rate then is obtained from labor market clearing as

$$w = \gamma \left[ \frac{B}{N} \int \nu(a) M(a) \frac{1}{\sigma + 1} \frac{1}{\gamma} da \right]^{1-\gamma}. \quad (5)$$

The model is easy to extend to capital as an input, to the production of intermediate goods outside the firm, with perfect or monopolistic competition, and to other dimensions of heterogeneity, e.g. in tastes or in risk aversion. The quantitative exercise in Section 5 will employ such a more general model.
4 Development and the firm size distribution

In this model, technological improvements affect occupational choice and, through this channel, the firm size distribution.

4.1 The technological frontier and occupational choice

Changes in the technological frontier affect incentives to become a worker or an entrepreneur both through their effect on potential profits and on wages. As technology advances, some firms stay close to the advancing frontier, while others fall behind. As a result, profits as a function of ability change, the populations of firms and workers change, and the equilibrium wage rate changes. Using $M(a, \bar{M}) = \bar{M}^a$, recall that profits and the wage are given by

$$\pi(a, \bar{M}) = (1 - \gamma) \left( \frac{w}{\gamma} \right)^{\frac{\gamma}{1 - \gamma}} \bar{M}^a$$

$$w(\bar{M}) = \gamma \left[ \frac{B}{N} \int \nu(a) \bar{M}^a da \right]^{1 - \gamma},$$

where $\eta \equiv \frac{1}{\sigma - 1} \frac{\gamma}{1 - \gamma} > 1$. An individual decides to become an entrepreneur if

$$\pi(a, \bar{M}) > w(\bar{M})a.$$  

Because $\pi(0) > 0 = w \cdot 0$, $\pi', \pi'' > 0$, there are two thresholds $a_L, a_H, a_L < a_H$ at which agents are indifferent between the two activities. Agents with $a < a_L$ and $a > a_H$ become entrepreneurs and agents with $a \in [a_L, a_H]$ become workers.\(^{25}\)

How does occupational choice evolve as $\bar{M}$ rises? An agent of ability $a_0$ who is indifferent between entrepreneurship and wage work strictly prefers entrepreneurship after an increase in $\bar{M}$ if $\partial \pi(a_0, \bar{M})/\partial \bar{M} > a_0 \partial w(\bar{M})/\partial \bar{M}$. At points of indifference, this is equivalent to

$$\varepsilon(\pi(\cdot), \bar{M}) > \varepsilon(w(\cdot)a, \bar{M}),$$

where $\varepsilon(x, y)$ is the elasticity of $x$ with respect to $y$. These are given by

$$\varepsilon(\pi(\cdot), \bar{M}) = \eta a - \frac{\gamma}{1 - \gamma} \varepsilon(w, \bar{M})$$

$$\varepsilon(w(\cdot)a, \bar{M}) = \frac{\gamma}{\sigma - 1} \int \nu(a) a \bar{M}^a da \left[ \int \nu(a) \bar{M}^a da \right]^{-1}.$$  

\(^{25}\)There could also be only one tangency or no intersections of the two schedules. In these cases, there would be no workers. Clearly, they therefore cannot be equilibrium outcomes as they are not consistent with labor market clearing.
where the elasticity of earnings with respect to $\bar{M}$ is taken keeping the distribution of active firms, $\nu(a)$, constant. This will be relaxed below. Indifference is broken in favor of entrepreneurship if

$$\eta a > \frac{1}{1-\gamma} \varepsilon(w(\cdot)a, M).$$

(11)

Defining $\tilde{a}$ as the level of $a$ at which (11) holds with equality yields

$$\tilde{a} = \frac{\sigma - 1}{\gamma} \varepsilon(w(\cdot)a, M) = \int \nu(a) a \bar{M}^\eta da \left[ \int \nu(a) \bar{M}^\eta da \right]^{-1}.$$

(12)

Then, indifference of agents with $a > \tilde{a}$ is broken in favor of entrepreneurship, while indifference of agents with $a < \tilde{a}$ is broken in favor of wage work. Note that $\tilde{a} > 0$, that $\tilde{a}$ is equal to the productivity of active firms if all active firms have the same level of $a$, and that $\tilde{a}$ is below $a$ of the most productive firm if firms of different levels of $a$ coexist.

How does $\tilde{a}$ evolve with $\bar{M}$? The sign of $\partial \tilde{a} / \partial \bar{M}$, again keeping $\nu(a)$ constant, equals the sign of

$$\int \nu(a) \bar{M}^\eta da \cdot \int \nu(a) a^2 \bar{M}^\eta da - \left( \int \nu(a) a \bar{M}^\eta da \right)^2.$$

(13)

This expression is weakly positive and strictly so if there is dispersion in the productivity of active firms. As a consequence, $\tilde{a}$ increases in $\bar{M}$. This implies that even firms for who at some point $a > \tilde{a}$, so that they benefit from technological improvements, find that at some later point $a < \tilde{a}$, and they suffer. (Unless $a = \bar{a}$.)

These derivations neglected changes in $\nu(a)$. Clearly though, as optimal occupational choice changes, $\nu(a)$ also changes. How does this affect this reasoning? Let an “improvement” in $\nu(a)$ be any change in $\nu(a)$ that raises average $a$ of active firms. Both integrands in (10) increase in $a$, but the one in the numerator more so. Therefore, improvements $\nu(a)$ raise $\varepsilon(w(\cdot)a, M)$ and $\tilde{a}$. So: suppose that $\bar{M}$ increases. Then firms may adjust their occupational choice, depending on whether $a_L, a_H \gtrless \tilde{a}$. Since entrants have $a > \tilde{a}$ and exiters $a < \tilde{a}$, changes in occupational choice in reaction to changes in $\bar{M}$ will always constitute improvements in $\nu(a)$. Thus, $\tilde{a}$ increases in $\bar{M}$ both because of changes in the integrand and because of improvements in $\nu(a)$.

“Histories” in this economy can feature four phases. The economy moves through them as $\bar{M}$ and, with it, $\tilde{a}$ change.

1. $\tilde{a} < a_L, \tilde{a} < a_H$: This occurs if $\bar{M} = 1$ or not much larger. With $\bar{M} = 1$, profits do
not depend on $a$. Therefore, high-$a$ agents work. Entrepreneurs with $a = a_L$ are the most productive ones, and therefore, $\tilde{a} < a_L$. As $\dot{M}$ grows, two things happen: a) $\tilde{a}$ increases. b) Because $a_L, a_H > \tilde{a}$, firms at these thresholds benefit from the increase in $\dot{M}$, and $a_L$ rises and $a_H$ falls. At some point, $a_H$ reaches $\tilde{a}$, and the economy enters the next phase.

2. $\tilde{a} < a_L < a_H < \bar{a}$: Highly productive entrepreneurs enter. It is clear from above that this quickly pushes up $\tilde{a}$. It increases until at some point it “catches” and overtakes $a_L$. (Is it guaranteed that this happens? Yes: here the upper bound on $\tilde{a}$ is essentially $\bar{a}$, so it can increase ever further. $a_L$ is bounded because it has to be strictly below $a_H$, which is strictly below $\bar{a}$.) (Does this case exist? For it not to exist, and $a_L$ to fall directly as firms start entering at the top, it would need to be the case that $\tilde{a} = a_L$ the moment $a_H = \bar{a}$. But unless there is a mass point at $\bar{a}$, this is not possible, because $\tilde{a} < a_L$ while firms with $a_L$ are the most productive ones.)

3. $a_L < \tilde{a} < a_H < \bar{a}$: In this phase, the top group of firms benefits from productivity improvements, whereas the bottom firms are harmed. Now, $a_L$ falls with $\dot{M}$. $a_H$ also falls. However, $\tilde{a}$ continues to increase. Even firms with $a_H$ at some point are “caught”.

4. $a_L < a_H < \tilde{a} < \bar{a}$: This is the final phase. $a_L$ falls and $a_H$ rises with $\dot{M}$: marginal firms on both margins lose from technological improvements. The top firms continue to gain, but that group becomes smaller and smaller.

How do the entrepreneurship rate and the firm size distribution evolve?

1. $a_L$ increases, $a_H$ is above $\bar{a}$: the entrepreneurship rate increases. Average productivity, dispersion and skewness grow. (Show that skewness grows even if $\nu(a)$ doesn’t change.)

2. The entrepreneurship rate increases. Average productivity, dispersion and skewness first jump, then grow further.

3. The entrepreneurship rate falls. This is because for the labor market to clear, fewer firms have to enter at the top than exit at the bottom. (This presupposes constancy of employment, which isn’t given – show that this is really true.)

4. The entrepreneurship rate falls. Average firm size, dispersion and skewness increase because they increase in $\dot{M}$ even if $\nu(a)$ is constant.

**check if the functional form of $M(a)$ is really needed above.**

21
To see the effect of advances in the technological frontier, consider their elasticities with respect to $\bar{M}$.

$$\varepsilon(\pi(\cdot), \bar{M}) = \eta a - \frac{\gamma}{1 - \gamma} \varepsilon(w, \bar{M}) \quad (14)$$

$$\varepsilon(w(\cdot), \bar{M}) = \frac{\gamma}{\sigma - 1} \int \nu(a) a \bar{M}^{\eta a} da \left[ \int \nu(a) \bar{M}^{\eta a} da \right]^{-1} \quad (15)$$

An advance in the frontier has two effects on profits: it improves every firm’s technology (the first term), but it also raises the wage rate (the second term), which is a drag on profits. As the effect of higher wages is independent of $a$, it is clear that only firms with high enough $a$ benefit from aggregate technology improvements. Low-$a$ firms lose more to the wage increase than they gain from the productivity improvement. Wages, in contrast, unambiguously increase with advances in the technological frontier. As a consequence, the composition of the firm size distribution changes as technology advances.

Note that if all agents had the same ability $a$, both $\varepsilon(w, \bar{M})$ and $\varepsilon(\pi(a), \bar{M})$ would reduce to $\frac{a\gamma}{\sigma - 1}$. As a consequence, wages and profits would increase in sync with technological advances, and occupational choice would remain unaffected, i.e. the thresholds $a_L$ and $a_H$ constant. Only with heterogeneity in $a$ do some agents benefit more than others from advances in the frontier, and occupational choices change.

For an individual with ability $a$, an improvement in the frontier makes becoming an entrepreneur relatively more attractive if

$$\Delta \varepsilon(a, \bar{M}) \equiv \varepsilon(\pi(\cdot), \bar{M}) - \varepsilon(w(\cdot), \bar{M}) = \eta a - \eta \int \nu(a) a \bar{M}^{\eta a} da \left[ \int \nu(a) \bar{M}^{\eta a} da \right]^{-1} > 0. \quad (16)$$

Advances in the frontier thus affect the occupational choices of agents of different ability differently. For the most productive entrepreneurs ($a = \bar{a}$), $\Delta \varepsilon(\cdot)$ will always be positive. This is because for $a \in (0, \bar{a}]$ and for any $\nu(a)$, both integrals in (16) are strictly positive. In addition, $a\bar{M}^{\eta a}/\bar{M}^{\eta a} < \bar{a}$ for $a \in (0, \bar{a})$, implying that the ratio of integrals is between 0 and $\bar{a}$. Similarly, $\Delta \varepsilon$ is strictly negative for the worst entrepreneurs. This implies that as the technological frontier advances, the best entrepreneurs gain, and the worst ones lose.

Intuitively, whether a firm gains or loses depends on its productivity relative to a complicated moment of the productivity distribution. This is because advances in the frontier increase labor demand and wages, and thereby all firms’ costs. They also improve firms’ productivity – but only firms that can make use of most of the advance in the frontier benefit sufficiently from this. Low-$a$ firms are squeezed: they are fully exposed to wage increases due to advances in the frontier, while being unable to translate these into large improvements in
their own productivity.

What is more, as the frontier continues to advance, the winners become more concentrated. This is because

\[
\frac{\partial \Delta \varepsilon}{\partial \bar{M}} = -\frac{\eta^2}{\bar{M}} \int \nu(a) \bar{M}^{\alpha} da \int \nu(a) a^2 \bar{M}^{\alpha} da - \left[ \int \nu(a) a \bar{M}^{\alpha} da \right]^2 < 0. \tag{17}
\]

This implies that even firms that at low levels of \( \bar{M} \) benefit from increases in the frontier see these benefits reduced and eventually turn negative as the frontier advances further. Only for firms with \( a = \bar{a} \) is it certain that \( \Delta \varepsilon \) cannot turn negative. For firms with \( a = 0 \), in contrast, it is always negative. For high enough \( a \), \( \Delta \varepsilon \) is positive for low \( \bar{M} \), eventually turns negative and ultimately pushes \( \pi(a, \bar{M}) \) below \( \omega a \). The next section explores the evolution of occupational choice as captured by \( a_L \) and \( a_H \) and its implications for the firm size distribution and entrepreneurship.

### 4.2 A “history” of entrepreneurship and the firm size distribution

Historically, every successful development experience has been characterized by improvements in total factor productivity. This section explores the predictions of the model for occupational choice and the firm size distribution along a “history” of an advancing technological frontier. As the model is static, every \( \bar{M} \) induces an equilibrium occupational choice, summarized by the thresholds \( a_L \) and \( a_H \), and a firm productivity distribution implied by these choices. Let \( \bar{M} = \{ \bar{M}_0, \bar{M}_1, \ldots, \bar{M}_T \} \), \( \bar{M}_0 = 1 \), be a strictly increasing sequence of real numbers and refer to it as the history of \( \bar{M} \). Analyzing the equilibrium of the model economy for each element of \( \bar{M} \) then yields a “history” of occupational choice and the firm size distribution.\footnote{An alternative is to consider a history where \( \bar{M}_t \) grows over time at an exogenous rate \( g \). This is particularly relevant in the context of the extension with capital used in the next section. While growth in \( \bar{M} \) leads to changes in occupational choice and in the share of entrepreneurs, the setting is consistent with balanced growth since increases in \( \bar{M} \) constitute labor-augmenting technical progress and the aggregate production function exhibits constant returns to scale (King, Plosser and Rebelo 1988). Results in this section can thus also be interpreted as developments along the balanced growth path of an economy.}

The sequence \( \bar{M} \) can also be interpreted as a list of different countries’ technological states at a point in time. It then induces a cross-section of occupational choices and firm size distributions. This interpretation is pursued in the next section. To evaluate the quantitative fit, the model is slightly extended and calibrated in that section. This is not necessary for the qualitative history explored in the present section.

Figure 5 shows the evolution of occupational choice as \( \bar{M} \) increases. The left panel shows profits and wages as functions of ability for two levels of \( \bar{M} \). As in Figure 4, the straight
lines correspond to wages and the curved ones to profits, and $a_{Li}$ and $a_{Hi}$ ($i = 1, 2$) indicate
the choice thresholds.

![Graph](image)

(a) Occupational choice for two values of $\bar{M}$

(b) The evolution of the thresholds $a_L$ and $a_H$ with increasing $\bar{M}$

Figure 5: Occupational choice as $\bar{M}$ increases

The left panel illustrates how occupational choice changes with $\bar{M}$. Higher $\bar{M}$ raises the
productivity of all firms and thereby leads to higher wages: the wage line pivots up from the
straight dash-dot line to the straight dotted line. Higher productivity raises profits (they change from the dashed to the solid line), except for some firms of low-$a$ entrepreneurs for
who the productivity increase is so small that it is outweighed by the increase in wages. This
unambiguously makes entrepreneurship more attractive for the highest-ability agents, and
less so for the ones with the lowest ability. In the situation drawn in the figure, entrepreneurs
with $a$ just below $a_{H1}$ still benefit and agents at or just below $a_{L1}$ lose from higher $\bar{M}$. As
a result, $a_H$ falls from $a_{H1}$ to $a_{H2}$, and $a_L$ falls from $a_{L1}$ to $a_{L2}$. It is mainly higher labor
demand from top firms and the entry of new relatively productive firms between $a_{H1}$ and
$a_{H2}$ that drives wages up.

The right panel shows the values taken by $a_L$ and $a_H$ for a “history” of increasing $\bar{M}$.
Starting from low $\bar{M}$, increases in $\bar{M}$ reduce both $a_H$ and $a_L$, as in the left panel of the
figure. Despite entry from the top, the entrepreneurship rate falls here. This is because
entrants with $a$ around $a_H$ are larger than the exiting firms with $a$ around $a_L$ they replace.
Labor market clearing then requires there to be fewer entrants than exiting firms, implying
falling entrepreneurship rate.

Once most low-productivity firms are gone, firms with $a = a_H$, while run by relatively
high-ability individuals, actually have low productivity compared to other firms in the economy. From this point on, further advances in $\bar{M}$ raise profits less than wages for firms with $a = a_H$, and the upper threshold $a_H$ shifts up again. (Formally, this is because $\Delta \varepsilon(a_H, \bar{M})$ as defined in equation (16) turns negative with increasing $\bar{M}$, as shown in equation (17).) As $\bar{M}$ increases further, $a_L$ falls further, but approaches zero only asymptotically. The upper threshold $a_H$ also continues to rise, albeit at a slow pace. ($\partial \Delta \varepsilon(a, \bar{M})/\partial \bar{M}$, while always negative for $a < \bar{a}$, falls in absolute value as $\bar{M}$ increases.) As a result, for very high levels of the frontier, almost all active firms have high productivity.

Advancing technology does not lift all boats here. By assumption, the most able agents benefit most from advances in the technological frontier, as they can deal more easily with the increased complexity and use a larger fraction of the new technologies. Low-ability entrepreneurs benefit less. In fact, increasing wages due to higher productivity at top firms (wage earners always gain from technological improvements) mean that the least productive firms’ profits fall as technology improves. As a consequence, marginal low-productivity entrepreneurs convert to become wage earners, and eventually also do better, though not necessarily immediately. The lowest-ability agents ($a = 0$) always lose. Technology improvements thus have a negative effect on low-productivity firms that operates through wage increases.

Figure 6(a) depicts the consequences of this development: the entrepreneurship rate (solid) falls as technology improves. While high-productivity firms replace the exiting low-productivity ones, they operate at a larger scale, so their number is smaller.
This development in the model parallels the evidence from Section 2, which reported entrepreneurship rates that fall in income across countries and in U.S. history. The model also replicates observed patterns in average firm size (solid line in Figure 6(b)). If some agents are better placed than others to benefit from technological advances, they drive others out of the market. As a consequence, marginal small firms exit, average firm size grows (solid line), and fewer, more productive firms remain.

At the same time, firm size dispersion increases. (The figure shows the standard deviation of employment, dashed line.) This has two sources. Firstly, for any fixed thresholds \(a_L\) and \(a_H\), increases in \(\bar{M}\) imply increasing dispersion in productivity and therefore in employment. On top of that, entry of very productive firms increases dispersion – as long there are small firms around. As their proportion falls with development, this driver weakens, explaining the concavity of the line in the figure. The same factors drive the evolution of skewness (dash-dot line). \[27\]

Summarizing the model “time series”, the model thus is consistent with the facts reported in Section 2 that the entrepreneurship rate falls with per capita income and that average firm size and dispersion and skewness of firm size increase with per capita income.

5 Quantitative exercise: occupational choice and entrepreneurship across countries

How well do the historical experience of one country and cross-country patterns accord? This gives an indication of how relevant the mechanisms in the model are relative to other factors affecting entrepreneurship and the firm size distribution.

To explore this, I calibrate the model to the U.S. experience and then evaluate how well it fits across a broad set of countries; in particular, how well it mimics the empirical relationships shown in Section 2.

\[27\] How does this fit with Hsieh and Klenow’s (2009) finding of larger TFP dispersion in China than in the U.S. (keeping in mind the measurement issues discussed in [17]). The long left tail of the Chinese productivity distribution visible in their Figure I suggests a large distortion of the entry and exit margin when seen through the lens of standard heterogeneous firm models (Hopenhayn (1992); see also Samaniego (2006), Barseghyan (2008), Poschke (2010a) and Moscoso Boedo and Mukoyama (2010)). Given that their high size cutoff probably excludes from their data almost all the rather small firms run by low-ability entrepreneurs, this corresponds to a downward distortion of \(a_H\) in the present context. If this distortion is large in a poor country, the model can generate higher productivity dispersion of firms with \(a > a_H\) in the poorer country. Dispersion computed using firms of all sizes will however still be larger in the richer country, as it is in the GEM data.
5.1 Generalized model

For the quantitative exercise, it is useful to generalize the very stylized model from Section 3 slightly. I introduce three modifications: production of intermediates with capital and labor, heterogeneity in taste for entrepreneurship, and a more general specification of $M(a)$.

Capital. In the simple model in Section 3, the differentiated activities used for producing final output use labor only. The aggregate input $X$ has constant returns to scale in all labor inputs. Replace this by

$$X = \left( \int_0^{M_i} \left( n_j^\gamma k_j^{1-\alpha} \right)^{\frac{\sigma-1}{\sigma}} d j \right)^{\frac{1}{\sigma-1}},$$

i.e., production of intermediates with capital and labor. This allows setting $\alpha$ and $\gamma$ to match income shares in the data. Firms’ optimization is as in Section 3, taking the wage rate $w$ and the rental rate of capital $r$ as given. Households, who own the capital stock and rent it to firms, now face a capital accumulation decision. Their Euler equation, evaluated at the steady state of the economy they live in (thus, given its $M$), prescribes equating the rental rate of capital net of depreciation to the rate of time preference. Assuming a common rate of time preference $\rho$ and a common depreciation rate $\delta$, this implies $r = \rho + \delta$. The firm’s optimality condition for capital then pins down the aggregate capital stock.

Taste heterogeneity. In the model of Section 3, only agents with $a < a_L$ or $a > a_H$ become entrepreneurs. Given the one-to-one mapping between $a$ and $M$, this implies a bimodal firm size distribution with only low- and high-productivity firms, but no firms with intermediate productivity. This is clearly counterfactual. Incorporating heterogeneity in tastes for entrepreneurship into the model allows to “fill in” the hole in the middle of the firm size distribution, while also adding realism. Indeed, most empirical studies of entrepreneurship point to some role for heterogeneity in tastes or risk aversion for entrepreneurship (see e.g. Hamilton 2000).

Thus, suppose that agents differ in their taste for entrepreneurship $\tau$. Define this such that individuals choose entrepreneurship if $\tau \pi(a) > w \cdot a$. $\tau > 1$ then implies “enjoyment” of entrepreneurship. If agents enjoy entrepreneurship, they will choose it even if $\pi(a) < w \cdot a$. Whether on average agents enjoy entrepreneurship is an empirical question; therefore the distribution of $\tau$ has to be calibrated, and the mean could be different from 1. A mean below 1 indicates that on average, individuals do not enjoy entrepreneurship.

With this additional dimension of heterogeneity, there are entrepreneurs of all levels of
ability, and the productivity distribution can be unimodal if the ability distribution is so. However, individuals of high or low ability are still more likely to become entrepreneurs. Changes in $\bar{M}$ shift the relationship of $\pi(a)$ and $wa$ and therefore the taste threshold for entering entrepreneurship, resulting in an evolution of the proportion of agents with a given $a$ who are entrepreneurs.

Heterogeneity in risk aversion combined with a simple extension of the model would yield similar results. Suppose that wage income is certain and equals $wa$ every period. Business income is a function of the entrepreneur’s ability and of an iid shock every period. (This reflects the higher variance of income from entrepreneurship; fluctuating wages could easily be accommodated, too.) Define the shock such that profits are given by $s_\pi \pi(a)$, where $s$ has mean 1 and variance $\sigma_\pi^2$. Let the period utility function be $u(c) = c^{1-\rho}/(1 - \rho)$, where the coefficient of relative risk aversion $\rho$ can vary across people. Then higher risk aversion $\rho$ or variance of profits $\sigma^2_\pi$ make entrepreneurship less attractive. The parametrization of heterogeneity in $\tau$ in the next section can thus alternatively be interpreted as describing variation in risk aversion. Because the setting with risk aversion contains more free parameters and also raises issues of the dynamic behavior of profits, I will pursue the taste interpretation in the remainder of the paper.

The technological frontier and complexity. How much additional complexity do advances in the technological frontier comport? The simple specification of $M(a)$ chosen in Section 3 restricted this relationship. But it is of course an empirical issue. Therefore, in this section, let the time cost of using an activity be

$$c(a, \bar{M}) = \bar{M}^{-\frac{\lambda}{\bar{M}} - 1},$$  \hspace{1cm} (19)$$

implying that a firm’s technology be given by

$$M(a, \bar{M}) = \bar{M}^{\frac{\lambda}{\bar{M}} + 1},$$  \hspace{1cm} (20)$$

and that its position relative to the frontier is

$$m(a, \bar{M}) = \frac{M(a, \bar{M})}{M(\bar{a}, \bar{M})} = \bar{M}^{\frac{a - \bar{a}}{\bar{M}}}. \hspace{1cm} (21)$$

The lower $\lambda$, the faster low-ability entrepreneurs fall behind the technological frontier as it advances. Note that this relationship contains two parameters: $\lambda$ and $\bar{M}$, which is an important parameter in its own right. They enter equation (20) sufficiently differently that
both can be calibrated, using information from the U.S. time series.

5.2 Calibration

The model is calibrated to U.S. data. Some parameters can be set using standard numbers from the literature, while the remaining ones are calibrated to match a set of moments describing the U.S. economy. Note in particular that $\bar{M}$ has important effects on endogenous variables and can therefore be calibrated using U.S. data.

The share parameters $\gamma$ and $\alpha$ are set to generate a profit share of income of 10% and a labor share of two thirds. This implies a $\gamma$ of 0.9 and an $\alpha$ of 0.74. The elasticity of substitution among intermediate inputs is set to 4, which is about the 75th percentile of the distribution of $\sigma$ across 4-digit industries estimated by Broda and Weinstein (2006).

Setting the rate of time preference to 4% and the depreciation rate to 10% per annum implies a rental rate of capital of 14%.

For the remaining parameters, first suppose that the ability and taste distributions are lognormal. A lognormal ability distribution implies that the wage distribution would be lognormal if everyone was an employee. With taste heterogeneity, entrepreneurs will come from across the ability distribution, and the wage distribution will be close to lognormal. For tastes, a lognormal distribution also seems natural, as they affect payoffs multiplicatively. Letting $\ln a \sim N(\mu_a, \sigma_a)$ and $\ln \tau \sim N(\mu_\tau, \sigma_\tau)$ and normalizing $\mu_a$ to be zero, the remaining moments to be calibrated are $\sigma_a, \mu_\tau, \sigma_\tau, \lambda$ and $\bar{M}$.

Data and model moments are shown in Table 1. U.S. data is for the year 2000, or close years where data for that year is not available. To pin down the parameters, information about the firm size distribution, about the distribution of wages and about the link between the two is needed. Targets are chosen accordingly:

- Average employment is informative about $\mu_\tau$, the mean taste for entrepreneurship. In an analysis of occupational choice, the broadest possible set of firms run as full-time concerns should be considered, so the target combines information from the Census Businesses Dynamics Statistics (BDS) on employer firms with CPS data on the self-employed reported in Hipple (2010) that is informative about full-time entrepreneurs without employees. Wage inequality, measured as the ratio between the 90th and the 10th percentile of the wage distribution, is taken from Autor, Katz and Kearney (2008, Figure 2.A) and helps to pin down $\sigma_a$. Changes in $\sigma_\tau$ affect occupational specialization

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28 Results are robust to setting $\sigma$ substantially higher, to 6. This is although the sensitivity of profits with respect to $\bar{M}$ declines with $\sigma$ (see e.g. equation (4)).
29 Setting $\mu_a = 0$ is a normalization because changes in $\mu_a$ can be undone by changing $\lambda$ appropriately.
30 In fact, the five parameters have to be calibrated jointly. While the following discussion stresses the main informational contribution of individual targets, parameters and target choices actually interact.
choice, in particular for small firms where $\pi(a)$ and $wa$ are not far apart. A statistic that is informative about it is the share of firms with less than average employment, computed from the GEM data for the US to take advantage of the more detailed size information. (Using the less detailed Census BDS and CPS data would yield similar results.) As seen in the previous section, the level of $\bar{M}$ also affects the dispersion of the firm size distribution. To capture this, I target the interquartile range standardized by mean firm size, combining size class data from the Census Statistics of U.S. Businesses (SUSB) and from the CPS (for non-employers). This measure of dispersion is robust to outliers, something especially important with a distribution that is as skewed as the firm size distribution.31

Finally, to separate $\lambda$ and $\bar{M}$, information on changes over time is needed. It would be most straightforward to use e.g. average firm size in 1900 in addition to average firm size in 2000, but there is no single series that encompasses both dates. An alternative is to use the elasticity of average firm size with respect to output per worker. This can be computed using any of the average firm size series plotted in Figure 2.2. They imply elasticities between 0.12 and 0.57. While the Dun & Bradstreet series is longest (1890-1983), the figure suggests that it may overstate the increase in average firm size in the post-war period. To be conservative, I therefore target an elasticity of 0.34, which is in the middle of the range in the data. Moreover, this value is close to the ones implied by the recent BDS series (1988-2006) and by the BEA Survey of Current Business series when omitting the Great Depression years.

Table 1: Calibration: Data and model moments

<table>
<thead>
<tr>
<th></th>
<th>model</th>
<th>data</th>
</tr>
</thead>
<tbody>
<tr>
<td>average employment $\bar{n}$</td>
<td>11.5</td>
<td>11.5</td>
</tr>
<tr>
<td>firm size $\text{iqr}/\bar{n}$</td>
<td>0.26</td>
<td>0.26</td>
</tr>
<tr>
<td>fraction firms with $n &lt; \bar{n}$</td>
<td>0.85</td>
<td>0.85</td>
</tr>
<tr>
<td>$\ln 90/10$ wage ratio</td>
<td>1.64</td>
<td>1.62</td>
</tr>
<tr>
<td>$\varepsilon(\bar{n}, Y)$</td>
<td>0.34</td>
<td>0.34</td>
</tr>
</tbody>
</table>

Sources for data moments: average firm size Census Business Dynamics Statistics (BDS) and CPS data as reported in Hipple (2010); interquartile range (iqr) from Census Statistics of U.S. Businesses (SUSB) tabulations; fraction firms with $n < \bar{n}$ from GEM, see Section 2.1 wage ratio from Autor et al. (2008, Figure 2A); elasticity of average employment with respect to output per worker uses average firm size data plotted in Figure 2.2 combined with data on non-farm employment from the BLS and from Weir (1992, Table D3), reprinted in Carter et al. (2006), and data on non-farm output from the BEA (http://www.bea.gov/bea) and from U.S. Department of Commerce (1975, Series F128).

31 Many thanks to Lori Bowan at the Census Bureau for providing a table with 1997 firm counts in detailed size categories. To be able to use this larger detail, I use these Census SUSB data instead of the close BDS data discussed above for computing this statistic. Given the predominance of small firms in all firm counts, I would expect the BDS to yield a similar number.
Values of the calibrated parameters are reported in Table 2. On average, individuals do not like entrepreneurship (the implied average $\tau$ in the population is clearly below 1), and thus require a premium before they take it up. There is substantial variation, however. Also note that the $\bar{M}$ resulting from the calibration describes the U.S. level of technology in 2000. To evaluate cross-country patterns, it will be necessary to set other countries’ $\bar{M}$ relative to the U.S. level such that the output ratios match the data. The model-generated “time series” of average employment in the U.S. is plotted against non-farm output per worker in Figure 7. As the calibration fits the observed elasticity of 0.34 well, the series of average employment also fits well.\(^{32}\)

### Table 2: Calibrated parameter values

<table>
<thead>
<tr>
<th>from external sources:</th>
<th>$\gamma$</th>
<th>$\alpha$</th>
<th>$\sigma$</th>
<th>$\rho$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.9</td>
<td>0.74</td>
<td>4</td>
<td>0.04</td>
<td>0.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>from fitting U.S. target moments:</th>
<th>$\sigma_a$</th>
<th>$\mu_\tau$</th>
<th>$\sigma_\tau$</th>
<th>$\lambda$</th>
<th>$\bar{M}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.658</td>
<td>-1.609</td>
<td>0.999</td>
<td>35.583</td>
<td>1811.7</td>
</tr>
</tbody>
</table>

An interesting dimension that has not been targeted in the calibration is the evolution of income inequality. Overall income inequality in the model increases more than wage inequality, as entrepreneurs’ incomes lie at the extremes of the income distribution. Figure 8 reports the income shares of the top 10% and 1% in the U.S. income distribution for the data (from Piketty and Saez 2006, for 1950-2002) and for the model, plotted against U.S. GDP per capita relative to its level in 2002. It is not surprising that inequality in the model does not reach its level in the data, as the model has no mechanism generating a fat right tail of the income distribution. What is remarkable, however, is that the trend in the model essentially replicates the trend in the data. For instance, from the mid-1960s to 2002, the income share of the top 1% increased by 6.6 percentage points. The model captures two thirds of this increase. It only misses the jump in U.S. income inequality that is known to have occurred in the 1980s (at about 75% of 2002 GDP per capita).

\(^{32}\)While the model fits the short series of recent data well, it is evident that it could still fit rather well if a different average size target from one of the other sources covering a narrower population of firms were used.
5.3 Cross-country results

The model fits the U.S. experience quite well. To evaluate the fit with other countries, each country is assigned the $\bar{M}$ that replicates the output per capita ratio to the U.S. observed in the data. This $\bar{M}$ is then taken to be the country’s effective state of technology.\(^{33}\) Figure 9 plots the entrepreneurship rate, average firm size, firm size dispersion and share of firms below average size generated by the model for these levels of $\bar{M}$ against the data. The straight line in each graph is the OLS fit discussed in Section 2. The slightly curved lines are the outcomes generated by the model.

Given that it was calibrated to the U.S., the model fits the cross-country experience rather well. Of course, as shown in Section 4, it predicts that the entrepreneurship rate falls with per capita income, while average firm size and the dispersion and skewness of firm size increase with it.

Strikingly for such a stylized model, however, the quantitative performance is quite good. In particular, the predicted change in the entrepreneurship rate, average firm size and the share of small firms with per capita income are very close to the relationship in the data.

\(^{33}\)Strictly speaking, $\bar{M}$ of course also captures non-technological sources of income differences like distortions, just as total factor productivity does. It appears reasonable that these also affect entrepreneurs’ technological opportunities. Also note that cross-country differences in $\bar{M}$ are taken to be exogenous here; explaining them is beyond the scope of this paper.
Figure 8: Income concentration at the top, U.S., 1950-2002, model and data


(Figures 9(a), 9(b) and 9(c)).

The model does slightly less well in terms of predicting firm size dispersion and skewness. It predicts a somewhat too stark relationship between dispersion and income. This is mainly because it predicts that in the poorest countries, $a_H$ is very large and almost all firms are small (see also Figure 9(c)). In addition, skill differences do not affect optimal firm size much for low levels of technology, implying low size dispersion even among these small firms. This actually fits with the conditions in many poor countries (see the cluster around Brazil in Figure 9(c)), but neglects other factors like government promotion of certain firms that allow for the existence of some large firms in countries like China, Venezuela or Russia. The model of course cannot pick this up. For the same reason, the model seriously understates skewness in poor countries. Nevertheless, the predicted skewness-income relationship is pretty close to the empirical one in developed economies.\(^{33}\)

\(^{33}\)Can the model explain the positive relationship between the number of registered businesses per capita ("business density") and income observed in e.g. the World Bank Group Entrepreneurship Survey (WBGES) data? Suppose that businesses above a certain size threshold find it optimal to register. This would be observed if the benefits of doing so increase more quickly with size than registration costs. (Indeed, empirical work long identified the informal sector with small firms with for instance less than 20 workers; see e.g. Rauch (1991).) Although the effect of $M$ on $a_L$ and $a_H$ implies that the proportion of large firms increases with income in the model (see also Figure 9(c)), the fraction of the population running a firm above a certain size does not necessarily do so because of the accompanying fall in the entrepreneurship rate. In the U.S. calibration used here, the first effect dominates throughout and the population fraction running firms with more than 20 workers increases monotonically with per capita income. The model thus is qualitatively consistent with business density across countries as measured in the WBGES data. When introducing
5.4 Alternative explanations fit less well

Skill-biased change in entrepreneurial technology thus is consistent with patterns in the firm size distribution across countries. This section shows that alternative factors that also look promising at first sight are not consistent with all facts. In particular, while there are several theories that can fit with observed patterns in the entrepreneurship rate and in average firm size, they typically are not consistent with the patterns in dispersion and skewness. This section briefly illustrates the promise and disappointment of regulation, financial frictions, and returns to scale as candidate explanations.

Differences in regulation across countries definitely are a factor that affects the firm size distribution, see e.g. Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). However, while factors related to regulation can explain wedges between firm productivity and size, it is not easy to come up with rules and regulations that explain the set of facts shown in Section 2. For instance, entry regulation, which is more burdensome in poorer countries (Djankov et al. 2002), would imply fewer entrepreneurs and larger firms in poorer countries. The explanation performs better if small firms can escape entry regulation: in this case, entry costs create an “informal” sector, reduce the share of large firms, and may reduce average firm size in poorer countries. However, this effect should mainly affect firms around the threshold where entry regulation becomes effective, and should not affect firms in the right tail of the size distribution. It should thus have only a weak effect on overall dispersion and skewness of the firm size distribution.

Financial frictions could also be thought to stunt the growth of productive firms in low-income countries. If financial frictions are stronger in poorer countries, they may limit the size of high-productivity firms there, implying smaller firms and less dispersed and skewed firm size. However, for this channel to generate the dispersion patterns observed in the data, it is crucial that financial frictions apply in particular to the most productive firms in a country. This is in contrast to the recent literature on the topic, which stresses that entrepreneurs react to financial frictions by accumulating wealth and may eventually outgrow financing constraints (see in particular Cagetti and De Nardi (2006) and Buera, Kaboski and Shin (forthcoming)). Moreover, this is a partial equilibrium effect. Antunes, Cavalcanti and Villamil (2008) show quantitatively that with an endogenous interest rate, the change in dispersion is ambiguous and quantitatively small in the empirically plausible range of financial frictions. While financial frictions can be important in affecting the allocation of resources among firms and, as a result, aggregate productivity, they are thus unlikely to be higher business registration costs in poor countries, as documented by Djankov, La Porta, Lopez-de-Silanes and Shleifer (2002), the relationship becomes even stronger.
the driving force behind the patterns shown in Section 2.

Finally, the relationship between scale and ability could be modelled differently. In this paper, skill-biased change in entrepreneurial technology is modelled as a relationship between the cost per differentiated activity supervised and ability. This modelling strategy allows taking into account that more able entrepreneurs may have larger span of control than less able ones, while preserving easy aggregation, balanced growth and constant factor income shares. This would not be the case in a setting with explicit heterogeneity in the span of control parameter $\gamma$ as proposed e.g. by Parker and Vissing-Jørgensen (2010) in a different context. In such a setting, an upward trend in $\gamma$ would imply a more skewed distribution of profits and could therefore deliver results consistent with the patterns reported in Section 2. However, it would also have the implication that the profit share of output tends to fall with per capita income. This prediction is hard to assess, but most likely runs counter to the constancy of labor income shares both across countries and within country histories, as documented by Gollin (2002).

6 Conclusion

How and why does the firm size distribution differ across countries? This paper documents that features of the firm size distribution are strongly associated with income per capita. Richer countries have fewer entrepreneurs and fewer small firms. The average, dispersion and skewness of firm size are all larger in richer countries. We should thus not expect the United States to have the same entrepreneurship rate as Brazil.

A simple model of skill-biased change in entrepreneurial technology calibrated to U.S. data can account very well for these patterns. It also shows a convenient way of integrating some results from the micro literature on entrepreneurs and skills into a macroeconomic model. The model provides a good fit although it relies exclusively on technology as the driver of cross-country differences and abstracts from other factors such as regulation or financial constraints. Linking these to the mechanism explored here may make for exciting future work.

\footnote{In addition, when including the ratio of domestic private credit to GDP (from the IMF International Financial Statistics for 2005) as a measure of financial development in the regressions depicted in Figure 1, estimates for the impact of output per capita are hardly affected, while the coefficient on credit is not significant.}
Figure 9: Entrepreneurship and the firm size distribution versus output per capita: model (curved line) and data (OLS fit)

Notes: Data sources as in Figure 1.
References


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