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Time of ruin in a risk model with generalized Erlang(n) interclaim times and a constant dividend barrier

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Abstract

In this paper we analyze the time of ruin in a risk process with the interclaim times being Erlang(n) distributed and a constant dividend barrier. We obtain an integro-differential equation for the Laplace Transform of the time of ruin. Explicit solutions for the moments of the time of ruin are presented when the individual claim amounts have a distribution with rational Laplace transform. Finally, some numerical results and a compare son with the classical risk model, with interclaim times following an exponential distribution, are given

Resumen:

En este artículo analizamos el momento de ruina en un proceso del riesgo donde el tiempo de ocurrencia entre los siniestros se distribuye según una Erlang(n) y con una barrera de dividendos constate. Obtenemos una ecuación integro diferencial para la Transformada de Laplace del momento de ruina.

Presentamos soluciones explicitas para el momento de ruina cuando la cuantía individual de un siniestro cumple que la Transformada de Laplace de su función distribución es racional. Finalmente, se muestran resultados numéricos y una comparación con el modelo clásico (con tiempos de interocurrencia exponencial)

Keywords: Risk theory, Generalized Erlang (n) distribution, constant dividend barrier, time of ruin, Laplace Transform

JEL Classification: G22

1 Introduction

In the classical model of risk theory, the insurer's surplus process at a given time t, $R(t)$, is given by

$$R(t) = u + c \cdot t - \sum_{i=1}^{N(t)} Z_i \qquad , \; t \in [0, \infty)$$

with $u = R(0)$ being the insurer's initial surplus. $N(t)$, the number of claims occurred until time t, follows a Poisson process with parameter λ, and Z_i is the amount of the *i-th* claim and has density function $f(z)$. The instantaneous premium rate, c, is $c = E[N] \cdot E[Z] \cdot (1 + \rho)$, where ρ, called the security loading, is a positive constant.

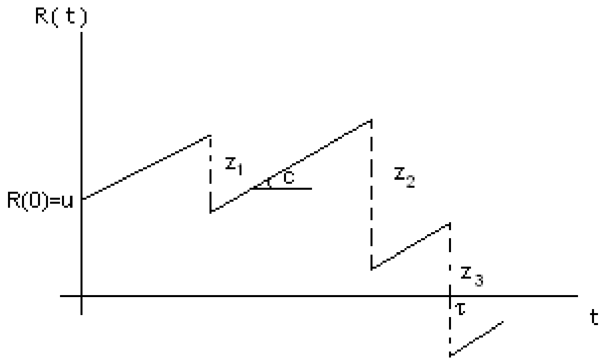

Figure 1: Sample path for the classical risk process

Define τ to be the time of ruin so that $\tau = \inf \{t : R(t) < 0\}$, with $\tau = \infty$ if $R(t) \geq 0$ for all $t > 0$. We denote the ultimate ruin probability from initial surplus u by $\psi(u)$, so that $\psi(u) = P[\tau < \infty]$. The time to ruin in the classical risk model is considered in Gerber and Shiu (1998), Lin and Willmot (2000), Dickson and Waters (2002), Drekic and Willmot (2003), or Ren (2005) where a perturbed model is analyzed.

In this paper the Poisson number process of the classical risk model is replaced with a more general renewal risk process with inter-occurrence times of a General Erlang(n) type. The time of ruin in an Erlang risk process is considered in Albrecher et al. (2005), Dickson and Hipp (2001) and Dickson et al. (2003) and Li and Garrido (2004) where an integro-differential equation for the Gerber-Shiu function is derived.

In what follows we shall use the modified model with a constant dividend barrier b , $0 \leq u \leq b$, so that when the surplus reaches the level b, premium income is paid out as dividend to shareholders and the modified surplus process remains at level b until the occurrence of the next claim. In this model the probability of ruin is equal to one, so we can assure that $\tau < \infty$.

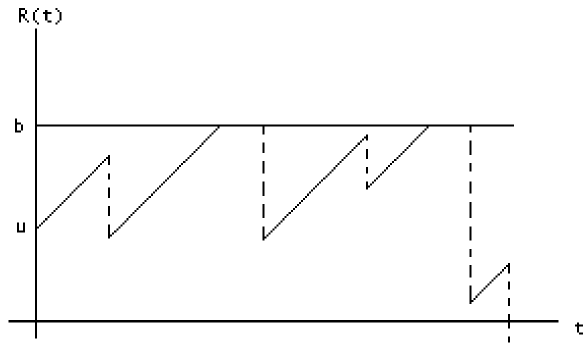

Figure 2: Modified model with a constant dividend barrier

The layout of this paper is as follows. In the next section we obtain the integro-differential equation for the Laplace transform of the time of ruin, $\phi(u)$, in a model modified with a constant dividend barrier in a Sparre Andersen model with generalized Erlang(n) interclaim times. Using rational Laplace Transforms, we solve the corresponding differential equation.

Section 3 presents the results for the particular case when the interclaim time follows an Erlang$(2, \lambda)$ process. In 3.1 we obtain the boundary condition when the individual claim amount is distributed as an exponential distribution, and in 3.2 when the individual claim amount follows an Erlang$(2,\beta)$ distribution.

In section 4 some ideas about the interpretation of the Laplace transform are given, and by differentiating $\phi(u)$ we will obtain the n-moments of the time of ruin and some numerical results are presented. Finally a comparison with the classical risk model is presented.

2 Laplace Transform of the time of ruin

We start by obtaining the integro-differential equation for the Laplace Transform of the time of ruin. We define,

$$\phi(u) = E\left[e^{-\delta\tau}\right].$$

We assume that the interocurrence times $T_i, i = 1, 2, ..$ follow a generalized Erlang(n) distributed, i.e. each T_i is a sum of n independent exponential random variables with possibly different parameters $\lambda_1, ..., \lambda_n$ each causing a "sub-claim" of size 0 and at the time of the nth subclaim a claim with distribution F occurs. We consider n states of the risk process (starting at time 0 in state 1) , where every subclaim causes a transition to the next state and in the time of occurrence of the nth subclaim the risk process jumps into state 1 again (see Albrecher at al. (2005)).

Let $\phi^j(u)$ denote the Laplace transform of the time of ruin if the risk process is in state $j = 1, ..., n$.

Theorem 1 *The integro-differential equation for $\phi(u)$ is,*

$$\left(\prod_{j=1}^{n}\left(\delta+\lambda_j-c\frac{\partial\cdot}{\partial u}\right)\right)\phi(u)-\prod_{j=1}^{n}\lambda_j\int_0^u\phi(u-z)\,dF(z)-\prod_{j=1}^{n}\lambda_j\left[1-F(u)\right]=0, \tag{1}$$

with boundary condition,

$$\prod_{j=1}^{k-1}\left(\frac{\delta+\lambda_j-c\frac{\partial\cdot}{\partial u}}{\lambda_j}\right)\left.\frac{\partial\phi(u)}{\partial u}\right|_{u=b}=0 \qquad \textit{for } k=1,...,n \tag{2}$$

Proof. For $0\leq u<b$, by differential argument

$$\begin{aligned}\phi^j(u) = {} & (1-\lambda_j dt)\,\phi^j(u+cdt)\,e^{-\delta dt}+ \\ & \lambda_j dt\phi^{j+1}(u+cdt)\,e^{-\delta dt}+\theta(dt)\end{aligned} \qquad \text{for } j=1,...,n-1 \tag{3}$$

beeing $\theta(dt)$ the probability of more than one claim occurs in dt.

$$\begin{aligned}\phi^n(u) = {} & (1-\lambda_n dt)\,\phi^n(u+cdt)\,e^{-\delta dt}+ \\ & \lambda_n dt\int_0^{u+cdt}\phi^1(u+cdt-z)\,dF(z)\,e^{-\delta dt}+ \\ & \lambda_n dte^{-\delta dt}\int_{u+cdt}^{\infty}dF(z)+\theta(dt)\end{aligned} \qquad \text{for } j=n \tag{4}$$

Then by linear approximation, dividing by dt, and taking $dt\to 0$, from (3) and (4) we obtain,

$$c\frac{\partial\phi^j(u)}{\partial u}-(\delta+\lambda_j)\,\phi^j(u)+\lambda_j\phi^{j+1}(u)=0, \qquad \text{for } j=1,...,n-1. \tag{5}$$

$$\begin{aligned} & c\tfrac{\partial\phi^n(u)}{\partial u}-(\delta+\lambda_n)\,\phi^n(u)+ \\ \lambda_n\int_0^u & \phi^1(u-z)\,dF(z)+\lambda_n[1-F(u)]=0.\end{aligned} \qquad \text{for } j=n \tag{6}$$

From (5),

$$\phi^{j+1}(u)=\left(\frac{(\delta+\lambda_j)-c\frac{\partial\cdot}{\partial u}}{\lambda_j}\right)\phi^j(u), \qquad j=1,...,n-1,$$

and following a recursive process,

$$\phi^{n}(u) = \left(\prod_{j=1}^{n-1} \frac{(\delta + \lambda_j) - c\frac{\partial \cdot}{\partial u}}{\lambda_j} \right) \phi^{1}(u). \tag{7}$$

We can write (6) as,

$$\left((\delta + \lambda_n) - c\frac{\partial \cdot}{\partial u} \right) \phi^{n}(u) - \lambda_n \int_0^u \phi^{1}(u-z)\, dF(z) - \lambda_n [1 - F(u)] = 0. \tag{8}$$

Substituting (7) in (8), (1) is obtained.

For $u = b$ using an analogous process for $j = 1, ..., n-1$

$$\phi^{j}(b) = (1 - \lambda_j dt)\, \phi^{j}(b)\, e^{-\delta dt} + \lambda_j dt \phi^{j+1}(b)\, e^{-\delta dt} + \theta(dt)$$

we obtain

$$\lambda_j \phi^{j+1}(b) - (\delta + \lambda_j)\, \phi^{j}(b) = 0$$

that comparing with (5),

$$\left. \frac{\partial \phi^{j}(u)}{\partial u} \right|_{u=b} = 0 \tag{9}$$

And for $j = n$,

$$\phi^{n}(b) = (1 - \lambda_n dt)\, \phi^{n}(b)\, e^{-\delta dt} + \lambda_n dt \int_0^b \phi^{1}(b-z)\, dF(z)\, e^{-\delta dt} + \\ \lambda_n dt e^{-\delta dt} \int_b^\infty dF(z) + \theta(dt)$$

then,

$$\lambda_n \int_0^b \phi(b-z)\, dF(z) + \lambda_n \int_b^\infty dF(z) - (\delta + \lambda_n)\, \phi^{n}(b) = 0$$

and comparing with (6)

$$\left. \frac{\partial \phi^{n}(u)}{\partial u} \right|_{u=b} = 0 \tag{10}$$

From (9), (10) and (7) the boundary condition (2) is obtained. ■

Using an alternative method, Li and Garrido (2004) obtained an equivalent integro-differential equation and boundary conditions for the Gerber-Shiu function.

Applying Laplace transforms to (1) we obtain,

$$\vartheta(s)\widetilde{\phi}(s)+L_{n-1}(s)-\prod_{j=1}^{n}\lambda_j\left(\widetilde{\phi}(s)\widetilde{f}(s)-\frac{1-\widetilde{f}(s)}{s}\right)=0, \tag{11}$$

being $\widetilde{\phi}(s)$ the Laplace transform of $\phi(u)$ and $\widetilde{f}(s)$ the Laplace transform of the claim density function $f(z)$.

$L_{n-1}(s)$ represents the $n-1$ degree polynomial, whose coefficients involve the quantities $\left.\frac{\partial\phi^{j)}(u)}{\partial u}\right|_{u=0}$, $j=0,...,n-1$, and $\vartheta(s)$ is the n degree polynomial $\vartheta(s)=\prod_{j=1}^{n}(\delta+\lambda_j-cs)$.

From (11) we obtain $\widetilde{\phi}(s)$,

$$\widetilde{\phi}(s)=\frac{\prod_{j=1}^{n}(\lambda_j)\left(\frac{1-\widetilde{f}(s)}{s}\right)-L_{n-1}(s)}{\vartheta(s)-\prod_{j=1}^{n}\lambda_j\widetilde{f}(s)}. \tag{12}$$

Example 2 *Assuming the classical model of risk theory with interocurrence claims Erlang*$(1,\lambda)$*, expression* (12) *is,*

$$\widetilde{\phi}(s)=\frac{\lambda\left(\frac{1-\widetilde{f}(s)}{s}\right)-c\phi(0)}{(\delta+\lambda-cs)-\lambda\widetilde{f}(s)}. \tag{13}$$

Let us now restrict the further analysis to the case of claim size distribution with rational Laplace transform

$$\widetilde{f}(s)=\frac{Q_{r-1}(s)}{P_r(s)},$$

where $P_r(s)$ and $Q_{r-1}(s)$ denote polynomials of degree r and (at most) $r-1$ respectively with no common zeros. Moreover $P_r(s)$ has leading coefficient 1, no roots in the positive half plane and $P_r(0) = Q_{r-1}(0)$ .For this claim size distribution, the denominator of (12) has $n+r$ zeroes, which are $s_1, ..., s_{n+r}.$, and we assume that the roots are real and distinct. From the above we conclude that r of these zeroes are located in the negative halfplane. Then using partial fractions, it is obtained,

$$\phi(u) = \sum_{i=1}^{n+r} \alpha_i(b) e^{s_i u}. \tag{14}$$

Note that $\alpha_i(b)$ are functions of b, but for brevity we write α_i. Now, we need to find $n+r$ equations satisfied by $\alpha_i, i = 1, ..., n+r$. The first n are obtained from (2) .

3 Laplace transform of Time of ruin in Erlang$(2, \lambda)$ risk model

Assuming the case of a Sparre Andersen model with Erlang$(2, \lambda)$ interclaim times ($n = 2$ and $\lambda_1 = \lambda_2 = \lambda$) the integro-differential equation for $\phi(u)$ is

$$\left(\delta + \lambda - c\frac{\partial\cdot}{\partial u}\right)^2 \phi(u) - \lambda^2 \int_0^u \phi(u-z)\, dF(z) - \lambda^2 [1 - F(u)] = 0, \tag{15}$$

From (12), it is easy to obtain

$$\widetilde{\phi}(s) = \frac{\lambda^2 \left(\frac{1-\widetilde{f}(s)}{s}\right) + c^2\phi'(0) + (c^2 s - 2c(\delta+\lambda))\phi(0)}{(\delta+\lambda-cs)^2 - \lambda^2 \widetilde{f}(s)}, \tag{16}$$

and assuming a claim distribution with rational Laplace transform, it is obtained

$$\phi(u) = \sum_{i=1}^{2+r} \alpha_i(b) e^{s_i u}. \tag{17}$$

3.1 Claim distribution $z \sim \exp(\gamma)$

If we assume that $f(z) = \gamma e^{-\gamma z}$, then $\widetilde{f}(s) = \frac{\gamma}{\gamma+s}$. We obtain the roots $s_i, i = 1, 2, 3$, from

$$(\delta + \lambda - cs)^2 (\gamma + s) - \lambda^2 \gamma = 0. \tag{18}$$

Then the solution of $\phi(u)$ is,

$$\phi(u) = \sum_{i=1}^{3} \alpha_i e^{s_i u}. \tag{19}$$

The first two equations obtained from (2) are $\sum_{i=1}^{3} s_i \alpha_i e^{s_i b} = 0$ and $\sum_{i=1}^{3} s_i^2 \alpha_i e^{s_i b} = 0$.

To find the third equation, we substitute (19) and $f(z) = \gamma e^{-\gamma z}$ in the integro-differential equation (15), and resolving the integral,

$$\sum_{i=1}^{3} \alpha_i e^{s_i u} \left[(\lambda + \delta)^2 - 2c(\lambda + \delta) s_i + c^2 s_i^2 - \frac{\lambda^2 \gamma}{(s_i + \gamma)} \right] + \lambda^2 \gamma \sum_{i=1}^{3} \frac{\alpha_i e^{-\gamma u}}{(s_i + \gamma)} - \lambda^2 e^{-\gamma u} = 0.$$

From (18) the coefficient of $e^{s_i u}$ is equal to zero, then we obtain the third equation, $\sum_{i=1}^{3} \frac{\alpha_i}{(s_i+\gamma)} = \frac{1}{\gamma}$.

We thus have a system of three equations from which we can easily solve for $\alpha_i, i = 1, 2, 3$ using Mathematica.

3.2 Claim distribution $z \sim Erlang\,(2, \beta)$

Consider a risk process in which claims occur as an Erlang$(2, \beta)$ distribution, $f(z) = \beta^2 z e^{-\beta z}$ with Laplace transform $\widetilde{f}(s) = \frac{\beta^2}{(\beta+s)^2}$. Then the fourth roots are obtained from

$$(\delta + \lambda - cs)^2 (\beta + s)^2 - \lambda^2 \beta^2 = 0 \tag{20}$$

The solution in this case is,

$$\phi(u) = \sum_{i=1}^{4} \alpha_i e^{s_i u}. \tag{21}$$

We need to find four equations to calculate the unknowns $\alpha_i, i = 1, 2, 3, 4$.

Two equations are obtained from (2): $\sum_{i=1}^{4} s_i \alpha_i e^{s_i b} = 0$ and $\sum_{i=1}^{4} s_i^2 \alpha_i e^{s_i b} = 0$.

From (15), (21) and knowing that $f(z) = \beta^2 z e^{-\beta z}$, we find the other two, $\sum_{i=1}^{4} \frac{\alpha_i}{(s_i + \beta)^2} = \frac{1}{\beta^2}$ and $\sum_{i=1}^{4} \frac{\alpha_i}{(s_i + \beta)} = \frac{1}{\beta}$.

4 Getting information from Laplace transform of the time of ruin

From $\phi(u)$ we can get two kind of different informations about the time of ruin.

First $\phi(u) = E\left[e^{-\delta\tau}\right]$ can be interpreted as the expected present value of one monetary unit that was paid at the time of ruin. Then the parameter of the Laplace transform δ can be interpreted as the interest rate used to obtain the present value.

But also $\phi(u)$ allows us to find the the n-moments of the random variable τ, i.e. $E[\tau^n]$, that can be obtained differentiating $\phi(u)$ with respect to δ,

$$\frac{\partial^n \phi(u)}{\partial \delta^n} = \frac{\partial^n}{\partial \delta^n} E\left[e^{-\delta\tau}\right] = E\left[(-\tau)^n e^{-\delta\tau}\right].$$

and evaluating these at $\delta = 0$,

$$E[\tau^n] = (-1)^n \left.\frac{\partial \phi^n(u)}{\partial \delta^n}\right|_{\delta=0}.$$

In what follows, some numerical results are obtained.

If we assume that $f(z) = \gamma e^{-\gamma z}$, for $\gamma = 1, \lambda = 1, b = 10$ and $c = 0.6$, the results for $E[\tau], \sigma[\tau]$ and the variation coefficient of τ defined by $cv[\tau] = \frac{100.\sigma[\tau]}{E[\tau]}$ are summarized in Table 1,

	$E[\tau]$	$\sigma[\tau]$	$cv[\tau]$
$u = 0$	93.9577	217.63	231.625
$u = 1$	157.031	265.267	168.926
$u = 2$	205.805	288.281	140.075
$u = 3$	243.077	299.847	123.355
$u = 4$	271.099	305.565	112.714
$u = 5$	291.68	308.253	105.682
$u = 6$	306.277	309.411	101.023
$u = 7$	316.06	309.845	98.0336
$u = 8$	321.974	309.973	96.2727
$u = 9$	324.794	309.996	95.4437
$u = 10$	325.372	309.997	95.2744

Table 1: $E[\tau], \sigma[\tau]$ and $cv[\tau]$
for $f(z) = e^{-z}, \lambda = 1, b = 10$ and $c = 0.6$

Graphically,

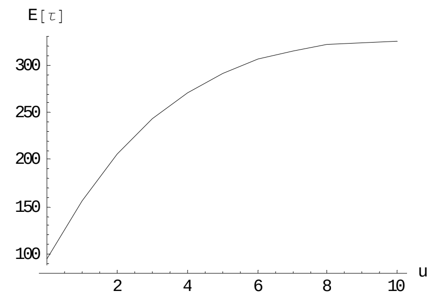

Figure 3: $E[\tau]$ assuming $T_i \sim Erlang(2,1)$ and $Z \sim \exp(1)$

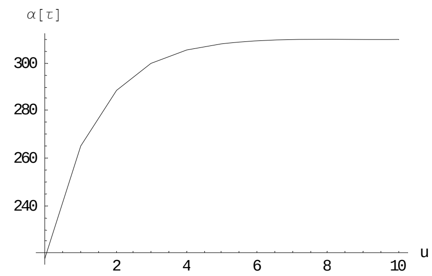

Figure 4: $\alpha[\tau]$ assuming $T_i \sim Erlang(2,1)$ and $Z \sim \exp(1$

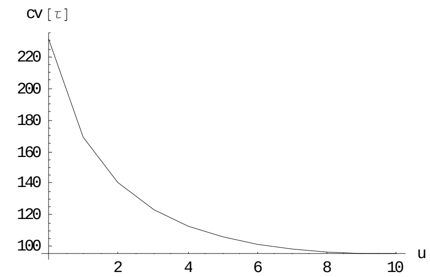

Figure 5: $cv\,[\tau]$ assuming $T_i \sim Erlang(2,1)$ and $Z \sim \exp(1)$

4.1 Comparison with the classical model

In this section we are going to compare the time of ruin with a constant barrier in an Erlang risk model with the corresponding in the classical risk model.

In the classical risk model, claims occur as a Poisson process. Note that in a Poisson process with parameter λ, $T_i, i \geq 1$ has an exponential distribution with mean $\frac{1}{\lambda}$, i.e. with an Erlang$(1, \lambda)$ distribution. So the time of ruin in the classical risk model is included in Theorem 1 as a particular case. From (1), for $n = 1$, we get the integro-differential equation for the Laplace transform of the time of ruin in the classical risk model and the boundary condition,

$$(\delta + \lambda)\,\phi(u) - c\phi'(u) - \lambda \int_0^u \phi(u - z)\,dF(z) - \lambda\,[1 - F(u)] = 0 \qquad (22)$$

$$\left.\frac{\partial \phi(u)}{\partial u}\right|_{u=b} = 0.$$

This equation can be obtained too from equation (2.5) in Lin et al (2003) for the Gerber-Shiu function in the classical risk model. (See too Dickson and Waters (2004))

To solve this model, from (13), assuming that $f(z) = \gamma e^{-\gamma z}$, the roots are obtained from $-c^2 s^2 + (\delta + \lambda - c\gamma)\,s + \delta\gamma = 0$. Then $\phi(u) = \sum_{i=1}^{2} \alpha_i e^{s_i u}$. To calculate α_i, substituting $\phi(u) = \sum_{i=1}^{2} \alpha_i e^{s_i u}$ and $f(z) = \gamma e^{-\gamma z}$ in (22) we get $\sum_{i=1}^{2} \frac{\alpha_i}{(s_i + \gamma)} = \frac{1}{\gamma}$.

In order to analyze the influence that the distribution of the interclaim times has in the time of ruin, the two models have to be comparable, i.e. $E[N_t]$ when $t \to \infty$ must be assimptotically the same in the Erlangian model and in the classical model.

Following Cox (1962), in an ordinary renewal process the renewal function, $E[N_t]$, is related with T_i

$$\widetilde{L}(s) = \frac{1}{s}\frac{\widetilde{g}_{T_i}(s)}{1-\widetilde{g}_{T_i}(s)}, \text{ for } \widetilde{g}_{T_i}(s) < 1$$

being $\widetilde{L}(s)$ the Laplace transform of $E[N_t]$ and $\widetilde{g}_{T_i}(s)$ the Laplace transform of the density function of the interrocurrence times.

For an Erlangian model, i.e. $T_i \sim Erlang(n,\lambda)$ with $\widetilde{g}_{T_i}(s) = \left(\frac{\lambda}{\lambda+s}\right)^n$, then

$$\widetilde{L}(s) = \frac{\lambda^n}{s\left((\lambda+s)^n - \lambda^n\right)}. \tag{23}$$

From (23), if $T_i \sim Erlang(2,\lambda), E[N_t]$ is easily obtained inverting $\widetilde{L}(s) = \dfrac{\lambda^2}{s\left((\lambda+s)^2-\lambda^2\right)}$, then $E[N_t] = \dfrac{1}{E[T_i]}t - \dfrac{1}{4} + \dfrac{1}{4}e^{-\frac{4}{E[T_i]}t}$.

From (23), if $T_i \sim Erlang(1,\lambda)$, i.e. for the classical risk model with $E[T_i] = \dfrac{1}{\lambda}$, $\widetilde{L}(s) = \dfrac{\lambda}{s^2}$ and inverting, we have $E[N_t] = \dfrac{t}{E[T_i]}$.

So, if $E[T_i]$ is the same, the renewal function $E[N_t]$ has the same behavior when $t \to \infty$. In general in an ordinary renewal process $\lim\limits_{t\to\infty} \frac{E[N_t]}{t} = \frac{1}{E[T_i]}$ (see Parzen (1972))

It´s easy to see also that the security loading included in the premium is different in the two models, and that if $E[T_i]$ is the same, asymptotically the security loading in the two models coincides: It´s known that the total premium income until time t is $ct = E[Z]\,E[N_t]\,(1+\rho_t)$, then $\rho_t = \dfrac{ct}{E[Z]\,E[N_t]} - 1$, and assuming $E[Z] = 1$, for the Erlangian model

$$\rho_t = \frac{ct}{\dfrac{1}{E[T_i]}t - \dfrac{1}{4} + \dfrac{1}{4}e^{-\frac{4}{E[T_i]}t}} - 1$$

and for the classical model

$$\rho_t = cE\left[T_i\right] - 1$$

Then, in order to compare the time of ruin in the $Erlang(2,1)$ model with $E\left[T_i\right] = 2$ with the classical model, where the interocurrence time follows an exponential distribution, the parameter of the exponential must be 0.5, i.e. $T_i \sim \exp(0.5)$ with $E\left[T_i\right] = 2$.

The difference between $E\left[\tau\right]$ in the Erlang(2,1) model and in the classical model with $T_i \sim Exp(0.5)$ is represented in Figure 6. In the Figures 7 and Figure 8 the difference for $\sigma\left[\tau\right]$ and the variation coefficient of τ is represented,

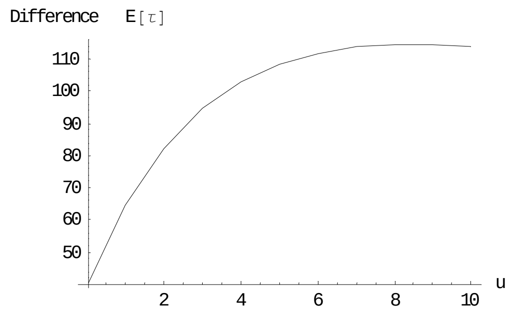

Figure 6: $E_{Erlang}\left[\tau\right] - E_{Exp}\left[\tau\right]$

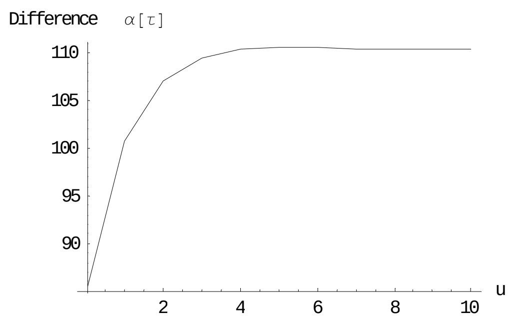

Figure 7: $\sigma_{Erlang}\left[\tau\right] - \sigma_{Exp}\left[\tau\right]$

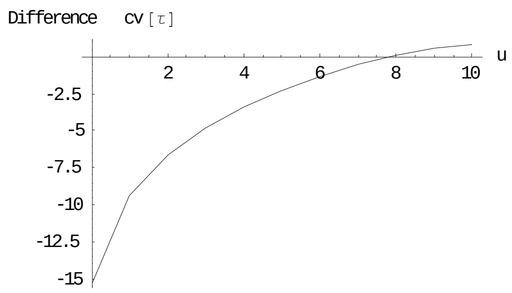

Figure 8: $cv_{Erlang}[\tau] - cv_{Exp}[\tau]$

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