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Endogenous growth with capital in R&D production functions

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Abstract: In this paper we claim that capital is as important in the production of ideas as in the production of final goods. Hence, we introduce capital in the production of knowledge and discuss the associated problems arising from the public good nature of knowledge. We show that although population growth can affect economic growth, it is not necessary for growth to arise. We derive both the social planner and the decentralized economy growth rates and show the optimal subsidy that decentralizes it. We also show numerically that the effects of population growth on the market growth rate, the optimal growth rate and the optimal subsidy are small. Besides, we find that physical capital is more important for the production of knowledge than for the production of goods.

JEL Codes: O30, O40, O41.

Keywords: knowledge, public good, growth.

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"*The historical record makes clear that science depends on technology in that it depends on the instruments and tools that are needed for science to advance*".
Joel Mokyr (2013)

1 Capital as an input of knowledge when this is a public good

It seems clear from Joel Mokyr that capital is as important in the production of ideas as in the production of final goods. Otherwise, what would applied economists and econometricians have become without computers? In fact, without better and better microscopes the medicine would not be what it has become. Besides, would physics be the same without the particle accelerator? Of course, capital producing ideas embodies particular knowledge and ideas, but it is not so different than capital producing goods, which also embodies particular knowledge and ideas. In the following, we introduce capital in the production of knowledge and discuss the associated problems arising from the public good nature of knowledge.

Consider an economy with two technologies: one for final goods Y and one for knowledge A. Final goods are private goods while knowledge is a public good. This has a very important implication: final goods can be written in per capita terms, but knowledge cannot, since all of us benefit from it in the same way. And this fact could originate the emergence of some type of scale effects: the size of population matters in the determination of knowledge and, in turn, per capita income.

First, let us consider the final goods sector. Total population L works either producing goods L_Y or producing knowledge L_A. Since both labor inputs are a fraction of total labor, define $L_A = sL$ and $L_Y = (1-s)L$, where s is the fraction of labor employed in the knowledge sector. The technology of a final goods firm that chooses the amount of capital K and labor L_Y can be summarized as

$$Y = (AL_Y)^{\alpha} K^{1-\alpha},$$

where $\alpha \in (0,1)$, so that there are constant returns to scale at the firm level. By defining the variables per efficiency units of labor, $\widetilde{y} = Y/AL$ and $\widetilde{k} = K/AL$, this technology can be rewritten as

$$\widetilde{y} = (1-s)^{\alpha} \widetilde{k}^{1-\alpha}.$$

Hence, a balanced growth path (BGP) exists if $\widetilde{k}$ is constant, that is, if K grows at the same rate than AL, i.e. $\dot{K}/K = \dot{A}/A + \dot{L}/L$. This means that per capita income grows at the same rate than knowledge.

Let us now consider the knowledge sector. From Romer (1990) and Jones (1995, 1999), it is commonly thought that when knowledge is a public good then endogenous growth suffers from some type of scale effects.[1] But, it is really true

[1] Jones (1999) cites a huge amount of articles with this characteristic.

that a nonrival and nonexcludable knowledge implies growth with scale effects? Next, we will argue that this implication is exclusively the result of the chosen technologies for the production of knowledge. In particular, we will show that the linearity in the researchers input for firms is the key for the emergence of scale effects. And that the introduction of capital as an input in the production of knowledge can get the economy out of the scale effects. Thus, we will have a little relation between growth of per capita income and the size of the economy, as suggested by Backus, Kehoe and Kehoe (1992).

The increase in knowledge $\dot{A}$ arises from the production of new designs in the R&D sector, since although the firm that discovers a new design is the only that can produce it, the new design is publicly observable. In Romer (1990), the technology of a R&D firm that produces $\dot{A}$ and chooses the amount of labor L_A is

$$\dot{A} = \delta L_A A,$$

where δ is the success probability that any researcher has of inventing a new design in any period. Note that the technology is linear in the researchers input for firms, so that there are constant returns to scale at the firm level. Also note that the new knowledge $\dot{A}$ discovered by one particular firm constitutes a common knowledge for the rest of the firms. By rewriting this equation, we have

$$\frac{\dot{A}}{A} = \delta s L,$$

so that constant population is needed for a BGP to exist. Note that the higher the number of researchers, the higher the growth rate. Moreover, for the same fraction of labor employed in the R&D sector, the economy with the highest population has the highest growth rate. Thus, the economy suffers from scale effects.

In Jones (1995), the technology of a R&D firm is

$$\dot{A} = \delta L_A A^{\phi} l_A^{\lambda-1},$$

where ϕ represents external returns and can have any sign, l_A represents an externality due to duplication in R&D such that in equilibrium $l_A = L_A$, and $0 < \lambda \leq 1$. Note that the technology is also linear in the researchers input for firms, so that there are constant returns to scale at the firm level. By rewriting this equation, in equilibrium we have

$$\frac{\dot{A}}{A} = \frac{\delta s^{\lambda} L^{\lambda}}{A^{1-\phi}},$$

so that $L^{\lambda} A^{\phi-1}$ has to be constant for a BGP to exist. If population grows at the rate n, this is true whenever

$$\frac{\dot{A}}{A} = \frac{\lambda n}{1-\phi},$$

so that when population is constant we have no growth at all. Further, growth is independent of the R&D success probability and the number of researchers.

Moreover, the economy with the highest population growth rate has the highest growth rate. Thus, the economy also suffers from scale effects.

The two previous R&D linear technologies could explain R&D during the past centuries or thousands of years. But at least from the 17th century (see Mokyr, 2013, for past and present examples) almost all R&D is not only done with researchers, but also with physical capital as laboratories or microscopes. Thus, the linearity in the researchers input for firms is an interesting case of study, but it does not seem to be the present case. Consider now the following technology of a R&D firm that chooses the amount of labor L_A and capital K_A:

$$\dot{A} = \delta \left(AL_A\right)^{\gamma} K_A^{1-\gamma},$$

where $\gamma \in (0,1)$. Note that there are constant returns to scale at the firm level. Redefining the capital used in the final goods production by K_Y, defining $K_A = uK$ and $K_Y = (1-u)K$, where K is total capital and u is the fraction of capital employed in the R&D sector, this technology can be rewritten as

$$\frac{\dot{A}}{A} = \delta s^{\gamma} u^{1-\gamma} \widetilde{k}^{1-\gamma} L,$$

so that constant population is needed for a BGP to exist. Therefore, the introduction of capital in the R&D technology does not necessaryly get the economy out of the scale effects. For this to happen, we need to introduce some type of externality. Thus, consider the following technology of a R&D firm:

$$\dot{A} = \delta \left(AL_A\right)^{\gamma} K_A^{1-\gamma} \left(\varepsilon L\right)^{-1},$$

where $\varepsilon > 0$. The externality εL can be interpreted as a diffusion externality such that the higher ε, the lower the diffusion of the innovation among population (access of customers to the innovation, access of scientifics to other scientific results, etc.).[2] Note that the linear case where $\gamma = 1$ does not coincide with Romer's and Jones' R&D technologies because of different externalities.[3] We can now rewrite the two technologies governing the economy as

$$\widetilde{y} = (1-s)^{\alpha} (1-u)^{1-\alpha} \widetilde{k}^{1-\alpha}, \tag{1}$$

and

$$\frac{\dot{A}}{A} = \delta \varepsilon^{-1} s^{\gamma} u^{1-\gamma} \widetilde{k}^{1-\gamma}, \tag{2}$$

so that now the unique requirement for a BGP to exist is $\widetilde{k}$ been constant.

Three facts have to be stressed. First, the exact externality in the R&D technology is crucial for the results, which is alike in spirit to the one required

[2] Since knowledge is a public good, the externality can also represent a coordination cost such that the higher ε, the higher the cost of coordinating people (see Becker and Murphy, 1992). An example is the Human Genome Project. Note that since $L_A = sL$, then $(\varepsilon L_A)^{-1}$ does not seem from an economic point of view a suitable coordination cost because then the higher the number of researchers the lower the growth rate.

[3] In this case we have $\dot{A}/A = \delta \varepsilon^{-1} s$, a kind of thechnology dismissed by Jones (1995). This is the reason for which we assume $\gamma \in (0,1)$.

in Romer (1986). Thus, any other kind of externality should be counterbalanced in a suitable way in order to have a BGP. Second, population growth can affect economic growth, but it is not neccessary for growth to arise. And third, growth depends on the R&D success probability and the number of researchers, what seems natural to think about. We analyze these facts in the next section through the social planner problem. After, we show how important are both capital and population in the production of ideas and, hence, growth through a calibration of the market economy. Finally, we find the optimal subsidy that makes the market growth rate to coincide with the social planner one in order to realize the existing economic inefficiency.

2 Social planner: population and growth

Consider an isoelastic utility function. Rewritting equations (1) and (2) in per capita variables, the social planner problem is[4]

$$Max \quad \int_0^\infty e^{-(\rho-n)t}\left(\frac{c^{1-\sigma}-1}{1-\sigma}\right)dt \tag{3}$$

$$s.t. \quad \dot{k} = y - c - nk = (1-s)^{\alpha}(1-u)^{1-\alpha}A^{\alpha}k^{1-\alpha} - c - nk, \tag{4}$$

$$\dot{A} = \delta\varepsilon^{-1}s^{\gamma}u^{1-\gamma}A^{\gamma}k^{1-\gamma}, \tag{5}$$

where ρ is the discount time factor, $\sigma > 0$, and y, c and k are production, consumption and capital per capita, respectively. The first order conditions with respect to c, s, u, k and A can be written, respectively, as

$$c^{-\sigma} - \beta = 0, \tag{6}$$

$$-\alpha y(1-s)^{-1}\beta + \gamma s^{-1}\dot{A}\xi = 0, \tag{7}$$

$$-(1-\alpha)y(1-u)^{-1}\beta + (1-\gamma)u^{-1}\dot{A}\xi = 0, \tag{8}$$

$$(1-\alpha)yk^{-1}\beta - n\beta + (1-\gamma)k^{-1}\dot{A}\xi = -\dot{\beta} + (\rho-n)\beta, \tag{9}$$

$$\gamma A^{-1}\dot{A}\xi + \alpha A^{-1}y\beta = -\dot{\xi} + (\rho-n)\xi, \tag{10}$$

where β and ξ are the multipliers associated to equations (4) and (5), respectively.

In a BGP, s and u are constant and all the per capita variables and knowledge grow at the same rate g. From equations (7) and (8) we have that the relationship between capital and labor used between sectors satisfies

$$\frac{1-s}{s} = \frac{1-u}{u}\frac{1-\gamma}{\gamma}\frac{\alpha}{1-\alpha}. \tag{11}$$

[4] We maintain the same notation of the previous section.

Differentiating equations (6) and (7) with respect to time, and noting that in a BGP $\ddot{A}/\dot{A} = \dot{A}/A = g$, we have

$$-\sigma g = \frac{\dot{\beta}}{\beta} = \frac{\dot{\xi}}{\xi}. \tag{12}$$

Substituting for β in equation (10) from equation (8), and using equations (11) and (12) we obtain

$$s = \frac{\gamma g}{\sigma g + (\rho - n)}. \tag{13}$$

Note that $sign\,(\partial g/\partial s) > 0$, and using equation (11) we have $sign\,(\partial g/\partial u) > 0$, too. Substituting for ξ in equation (9) from equation (7), using equations (11) and (12), substituting for yk^{-1} from equation (1) and after for Ak^{-1} from equation (5), and finally using equations (11) and (13) yields

$$(\sigma g + \rho)\,(\sigma g + \rho - n)^{\frac{\alpha}{1-\gamma}} = (1-\alpha)\left(\frac{\delta\gamma}{\varepsilon}\right)^{\frac{\alpha}{1-\gamma}}\left(\frac{1-\gamma}{\gamma}\frac{\alpha}{1-\alpha}\right)^{\alpha}. \tag{14}$$

In view of this equation, it is clear that there is only one BGP.

Proposition 1 *If $n = 0$ then the economy grows in the social planner BGP at the rate*

$$g = \frac{\left[(1-\alpha)\left(\frac{\delta\gamma}{\varepsilon}\right)^{\frac{\alpha}{1-\gamma}}\left(\frac{1-\gamma}{\gamma}\frac{\alpha}{1-\alpha}\right)^{\alpha}\right]^{\frac{1-\gamma}{1-\gamma+\alpha}} - \rho}{\sigma}. \tag{15}$$

Moreover, the social planner growth rate is increasing in the population growth rate, i.e. $\partial g/\partial n > 0$.

The effect of population growth on income growth differs from the Ramsey-Cass-Koopmans model, where population growth has no effect on the steady state. In principle, the higher the population growth, the higher the amount of final goods that have to be dedicated for capital maintenance and, then, the lower the labor dedicated to research. In the Ramsey-Cass-Koopmans model, the planner's weight on future increases with population in such a way that offsets capital maintenance. In our economy, this increase in the preference over the future makes the social planner to directly increase the resources devoted to R&D.

3 Decentralized economy: the importance of capital and population on growth

Next, we present the decentralized economy and make a calibration exercise to show how important are both capital and population in the production of knowledge. Since in the next sextion we analyze optimal subsidies, we allow for them. There are three sectors in this economy. A competitive research sector uses labor and intermediate goods to produce new designs. A monopolistically competitive intermediate goods sector uses these designs and foregone output to produce inputs for the research sector and a final goods sector. Apart from the intermediate goods, the competitive final goods sector uses labor to produce final output, which can be either consumed or saved. Thus, there are two basic inputs, capital and labor, which productivity is affected by the state of technology. Capital is measured in units of consumption goods. There is a government that subsidizes intermediate goods production through a lump-sum tax. Since there is a monopolistically sector, the decentralized equilibrium is not efficient.

Final goods firms: Final output Y is produced through intermediate goods and labor. The firm's problem is

$$Max \quad L_Y^{\alpha}\left(\int_0^A {x_Y^i}^{1-\alpha} di\right) - w_Y L_Y - \int_0^A p_i x_Y^i di,$$

where the production function is à la Dixit-Stiglitz, x_Y^i is the quantity of the intermediate good i used to produce final goods, A measures the number of available designs of intermediate goods in the economy, w_Y is the wage paid in this sector per unit of labor, and p_i is the price of the intermediate good i. The optimal conditions are

$$w_Y = \alpha L_Y^{\alpha-1}\left(\int_0^A {x_Y^i}^{1-\alpha} di\right) = \frac{\alpha Y}{L_Y}, \tag{16}$$

$$p_i = (1-\alpha) L_Y^{\alpha} {x_Y^i}^{-\alpha}. \tag{17}$$

R&D firms: The technology of a R&D firm that produces the amount of new designs $\dot{A}$ is

$$\dot{A} = \delta L_A^{\gamma}\left(\int_0^A {x_A^i}^{1-\gamma} di\right)(\varepsilon L)^{-1}, \tag{18}$$

where x_A^i is the quantity of the intermediate good i used to produce new designs. The firm's problem is

$$Max \quad p_A \delta L_A^{\gamma}\left(\int_0^A {x_A^i}^{1-\gamma} di\right)(\varepsilon L)^{-1} - w_A L_A - \int_0^A p_i x_A^i di,$$

where p_A is the price of a design, and w_A is the wage paid in this sector per unit of labor. The optimal conditions are

$$w_A = \gamma p_A \delta L_A^{\gamma-1}\left(\int_0^A {x_A^i}^{1-\gamma} di\right)(\varepsilon L)^{-1} = \frac{\gamma p_A \dot{A}}{L_A}, \tag{19}$$

$$p_i = (1-\gamma)\, p_A \delta L_A^{\gamma} {x_A^i}^{-\gamma} (\varepsilon L)^{-1}. \tag{20}$$

Intermediate goods firms: A producer of an intermediate good purchases a design created in the R&D sector, which confers monopoly power over that particular good. As in Romer (1990), a putty-putty technology is considered, where the producer needs 1 unit of final good to produce 1 unit of intermediate good. The problem faced by each firm i is to maximize profits $\pi_i = (p_i + z)\left(x_Y^i + x_A^i\right) - r\left(x_Y^i + x_A^i\right)$, subject to its inverse demand functions, equations (17) and (20), where r is the interest rate, and z is a subsidy on the production of intermediate goods given by the government. Moreover, since discrimination is not allowed, the price of the intermediate good has to be the same for all the buyers, so that

$$(1-\alpha)L_Y^{\alpha}{x_Y^i}^{-\alpha} = (1-\gamma)\, p_A \delta L_A^{\gamma}{x_A^i}^{-\gamma}\,(\varepsilon L)^{-1}. \qquad (21)$$

Using the constraints, the firm i's problem becomes

$$Max \quad \pi_i = (p_i + z - r)\left(\frac{(1-\alpha)^{\frac{1}{\alpha}} L_Y}{p_i^{\frac{1}{\alpha}}} + \frac{\left[(1-\gamma)\, p_A \delta\,(\varepsilon L)^{-1}\right]^{\frac{1}{\gamma}} L_A}{p_i^{\frac{1}{\gamma}}}\right).$$

Using equations (17) and (20), the optimal condition can be written as

$$p_i = (r-z)\left[\frac{\gamma x_Y^i + \alpha x_A^i}{\gamma(1-\alpha)x_Y^i + \alpha\,(1-\gamma)\,x_A^i}\right], \qquad (22)$$

from where profits are

$$\pi_i = \left(x_Y^i + x_A^i\right) p_i \left(\frac{\alpha\gamma x_Y^i + \alpha\gamma x_A^i}{\gamma x_Y^i + \alpha x_A^i}\right). \qquad (23)$$

Households: A dynasty maximizes (3) subject to

$$\dot{a} = [s w_A + (1-s) w_Y] + (r-n)\, a - c - \frac{T}{L},$$

where T is a lump-sum tax, and a is per capita assets. The Euler condition is

$$\frac{\dot{c}}{c} = \frac{(r-\rho)}{\sigma}. \qquad (24)$$

Government: The government subsidizes intermediate production and taxes households, such that

$$T = z\left[\int_0^A \left(x_A^i + x_Y^i\right) di\right]. \qquad (25)$$

Market clearing conditions: Equilibrium in the labor market implies that wages must be the same regardless of the firm. Thus, equations (16) and (19) imply

$$\alpha L_Y^{\alpha-1}\left(\int_0^A {x_Y^i}^{1-\alpha} di\right) = \frac{\alpha Y}{L_Y} = \frac{\gamma p_A \dot{A}}{L_A} = \gamma p_A \delta L_A^{\gamma-1}\left(\int_0^A {x_A^i}^{1-\gamma} di\right)(\varepsilon L)^{-1}. \qquad (26)$$

Since it takes 1 unit of final good to produce 1 unit of intermediate good, capital is related to the number of intermediate goods. Therefore, total usage of capital in each sector is

$$K_Y = \int_0^A x_Y^i di \quad \text{and} \quad K_A = \int_0^A x_A^i di. \tag{27}$$

Assets in this economy are capital and patents. Therefore,

$$a = k + \frac{p_A A}{L},$$

which implies that

$$\dot{a} = \dot{k} + \frac{\dot{p}_A A}{L} + \frac{p_A \dot{A}}{L} - \frac{p_A A n}{L}.$$

The price of a new design reflects the incentives of the producers of intermediate goods to acquire it. Following Grossman and Helpman (1991), we can express that, at every moment in time, the instantaneous excess of revenue over the marginal cost must be just sufficient to cover the interest cost on the initial investment in a design. Or, in other words, the price of a design is equal to the present value of the net revenue that a monopolist can extract. In our case, that means $\pi_i + \dot{p}_A = r p_A$, which combined with equation (23) implies

$$r = \left(\frac{p_i}{p_A}\right)\left(x_Y^i + x_A^i\right)\left(\frac{\alpha\gamma x_Y^i + \alpha\gamma x_A^i}{\gamma x_Y^i + \alpha x_A^i}\right) + \frac{\dot{p}_A}{p_A}. \tag{28}$$

Symmetric equilibrium and BGP: In a symmetric equilibrium all the intermediate goods firms produce the same quantity through the same amount of inputs, so that $x_Y^i = x_Y \ \forall i$, $\int_0^A x_Y^i di = A x_Y$ and $\int_0^A {x_Y^i}^{1-\alpha} di = A {x_Y}^{1-\alpha}$, and $x_A^i = x_A \ \forall i$, $\int_0^A x_A^i di = A x_A$ and $\int_0^A {x_A^i}^{1-\gamma} di = A {x_A}^{1-\gamma}$. In a BGP the fractions of labor and capital used in each sector are constant, so that s and u are constant. Therefore, equations in (27) can be rewritten as

$$x_Y = \frac{(1-u)K}{A} \quad \text{and} \quad x_A = \frac{uK}{A}. \tag{29}$$

Combining equations (18), (21) and the final goods production function gives

$$\frac{(1-\alpha)\, Y}{A x_Y} = \frac{(1-\gamma)\, p_A \dot{A}}{A x_A}, \tag{30}$$

which, using equations (26) and (29), becomes

$$\frac{1-s}{s} = \frac{1-u}{u}\frac{1-\gamma}{\gamma}\frac{\alpha}{1-\alpha}. \tag{31}$$

Note that this equation coincides with that of the social planner, equation (11). This does not mean that, in order to achieve the social planner growth rate in the next section, the market values of s and u have to coincide with those of the social planner, since they have to correct the market power.

From the final goods production function and equation (29), in a BGP we have $\dot{Y}/Y = \dot{A}/A + n$. Using this fact, differentiating equation (26) and noting that $\ddot{A}/\dot{A} = \dot{A}/A$ yields $\dot{p}_A/p_A = n$. Using this equation and equation (29), equation (28) becomes

$$(r-n) = \left(\frac{p_i}{p_A}\right)\left(\frac{K}{A}\right)\left[\frac{\alpha\gamma(1-u)+\alpha\gamma u}{\gamma(1-u)+\alpha u}\right].$$

Using equations (22), (29) and (26) yields

$$\frac{p_i}{p_A} = \left[\frac{\gamma(1-u)+\alpha u}{\gamma(1-\alpha)(1-u)+\alpha(1-\gamma)u}\right]\left[\frac{\gamma}{\alpha}\frac{\dot{A}}{Y}\frac{(1-s)}{s}(r-z)\right].$$

And combining these two last equations gives

$$(r-n) = \left[\frac{\alpha\gamma(1-u)+\alpha\gamma u}{\gamma(1-\alpha)(1-u)+\alpha(1-\gamma)u}\right]\left[\frac{\gamma}{\alpha}\frac{K}{A}\frac{\dot{A}}{Y}\frac{(1-s)}{s}(r-z)\right]. \quad (32)$$

Substituting in equation (22) one p_i from equation (17) and the other p_i from equation (20), summing up for i, using the final goods production function and equations (18) and (29) and after (30) gives

$$(r-z)K[\gamma(1-u)+\alpha u] = (1-\alpha)Y\left[\gamma(1-\alpha)+\alpha(1-\gamma)\frac{u}{1-u}\right]. \quad (33)$$

Equations (18) and (29) yields

$$g = \left(\frac{\delta}{\varepsilon}\right)s^{\gamma}u^{1-\gamma}\left(\frac{K}{AL}\right)^{1-\gamma}. \quad (34)$$

Combining the final goods production function with equation (29) gives

$$\frac{y}{k} = (1-s)^{\alpha}(1-u)^{1-\alpha}\left(\frac{A}{k}\right)^{\alpha}. \quad (35)$$

Combining equations (24), (32) and (33) yields

$$(\sigma g+\rho-n) = \left[\frac{\alpha\gamma(1-u)+\alpha\gamma u}{\gamma(1-u)+\alpha u}\right]\left[\frac{(1-s)}{s}\frac{(1-\alpha)}{\alpha}\frac{\gamma g}{(1-u)}\right]. \quad (36)$$

And, finally, combining equations (24), (31), (33), (34) and (35) gives

$$(\sigma g+\rho) = (1-\alpha)\left[\frac{\gamma(1-\alpha)(1-u)+\alpha(1-\gamma)u}{\gamma(1-u)+\alpha u}\right]\left[\frac{1-\gamma}{\gamma}\frac{\alpha}{1-\alpha}\right]^{\alpha}\left(\frac{\delta}{\varepsilon}\frac{s}{g}\right)^{\frac{\alpha}{1-\gamma}}+z. \quad (37)$$

Equations (31), (36) and (37) implicitly gives the growth rate of the economy.

Calibration: We illustrate the importance of capital and population on growth through a numerical exercise which strategy is the following: first, we calibrate certain parameters of the decentralized BGP taking into account a benchmark economy without subsidies. This allows us to show the importance of capital in the production of knowledge. Second, we change population growth to show how it affects income growth. And, third, we make a robustness analysis.

We fix the value of the parameters as follows. $\sigma = 2$ so that the intertemporal elasticity of substitution is 0.5. The population growth rate n is 1%. The interest rate r equals 5.2% and $\rho = 0.012$ so that the growth rate is 2%. In order to find the values of α and γ, we assume that the labor income share in the national income is 65% while the assets income share is 35%. Thus, and following Echevarria (1997), we have that

$$w_A L_A + w_Y L_Y = 0.65 GDP,$$

which can be rewritten as

$$\frac{w_A L_A}{p_A \dot{A}} \frac{p_A \dot{A}}{GDP} + \frac{w_Y L_Y}{Y} \frac{Y}{GDP} = 0.65.$$

From Jones and Williams (2000) we have that R&D spending to GDP $p_A \dot{A}/GDP =$ 3.1%, so that $Y/GDP = 96.9\%$ (note that intermediate goods are included in both productions).[5] Thus, and substituting equations (16) and (19), the previous equation becomes

$$\gamma 0.031 + \alpha 0.969 = 0.65. \tag{38}$$

Now, since the assets income share in the national income is 35%, we have

$$rK + A\pi = 0.35 GDP,$$

which can be rewritten as

$$\frac{rK}{Y} \frac{Y}{GDP} + \frac{A\pi}{GDP} = 0.35. \tag{39}$$

In order to recover the share of profits in the national income, McGrattan and Prescott (2005a), in studying intangible capital, obtain that dividends are the 11.5%, i.e. $A\pi/GDP = 0.115$. But, also from McGrattan and Prescott (2005b), we can conclude that the value of equities is around the 100% of GDP, i.e. $Ap_A = GDP$. From the non-arbitrage condition in the assets market we have that $Ap_A = A\pi/(r-n)$. Thus, from these two equations we have $A\pi/GDP = r - n = 0.042$. Since we have two different values, we take the mean of these two values, 0.0785, noting that, since dividends and equities include the payment or value of both physical and intangible capital, this value should be a maximum value. Thus, equation (39) becomes

$$\frac{rK}{Y} 0.969 + 0.0785 = 0.35.$$

[5] See footnote 15 of their paper, where they think that this number is a lower bound.

Then, substituting rK/Y into equation (33) with $z = 0$ gives

$$0.28\left[\gamma(1-u)+\alpha u\right] = (1-\alpha)\left[\gamma\left(1-\alpha\right)+\alpha\left(1-\gamma\right)\frac{u}{1-u}\right]. \qquad (40)$$

Finally, from equations (31), (36), (38) and (40) we recover $\alpha = 0.67$, $\gamma = 0.042$, $u = 11.37\%$ and $s = 0.28\%$. Equation (37) gives the discovering probability net of coordination costs $\delta\varepsilon^{-1} = 0.019$. The summary of the calibration analysis is in the two middle columns of Table 1. There are three important results to be noted from this calibration exercise. First, the R&D sector is much more capital intensive than the final goods sector, which remarks the actual big importance of capital in the production of ideas.[6] Second, the 0.28% of labor and the 11.37% of capital would be employed in the R&D sector.[7] And third, we have $p = 0.069$, so that the mark-up m for the intermediate goods firms is $m = (p-r)/p = 24.95\%$.[8]

Table 1. Calibration

	R&D spending to GDP 3.1%			**Dividends to GDP 7.85%**		
Dividends	4.2%	7.85%	11.5%	–	–	–
R&D spending	–	–	–	1.5%	3.1%	6.2%
$\boldsymbol{\gamma}$	0.002	0.042	0.125	0.066	0.042	0.006
$\boldsymbol{\alpha}$	0.67	0.67	0.67	0.66	0.67	0.69
u	3.05%	11.37%	16.28%	13.33%	11.37%	5%
s	0.003%	0.28%	1.37%	0.56%	0.28%	0.014%
m	6.05%	24.95%	39.07%	30.05%	24.95%	10.47%

Table 2 shows the effect of population growth on income growth. We have to solve equations (31), (36) and (37), since the interest rate is now endogenous. Note that since the discount rate is $(\rho - n) > 0$, we have an upper bound for n. A higher growth rate is accompanied by a lower mark-up at the same time that the labor and capital proportions employed in the R&D sector increase in order to rise the growth rate. Nevertheless, although the magnitude of the growth changes seems small, we can conclude that the effect of population growth on the growth rate comes basically from an increase in the capital devoted to R&D.

As a robustness analysis, we have repeated the same exercise first, when the R&D spending is the 3.1% of the GDP and the dividends are the 4.2% and 11.5% of the GDP (following McGrattan and Prescott, 2005a and 2005b), and second, when the dividends are the 7.85% of the GDP and the R&D spending is the 1.5% and 6.2% of the GDP (half and double of that of Jones and Williams, 2000). The results of the calibration are in Table 1. As we can observe, when

[6] This is in contrast with the human capital literature, where it is commonly assumed that the final goods sector is more capital intensive than the human capital sector.

[7] Recall that in our economy there are neither knowledge spillovers nor duplication externalities, as in Jones and Williams (2000).

[8] This implies a gross mark-up (the ratio of price to marginal cost) of 1.327, which belongs to the empirical estimates of 0.052 to 1.4.

dividends increase the R&D sector becomes more labor intensive and, then, the labor and capital devoted to this sector dramatically rise. Contrarily, when R&D spending increases, the R&D sector becomes less labor intensive and, then, the labor and capital devoted to this sector decrease. In both cases, labor intensiveness remains equal in the final goods sector. Thus, we can conclude that the more labor intensive is the R&D sector, the higher the labor employed in this sector, the higher the dividends to GDP, and the lower the R&D spending to GDP.

Table 2. Changes in population growth

	n = 0	**n = 0.002**	**n = 0.005**	**n = 0.01**	**n = 0.011**
g	1.81%	1.85%	1.9%	2%	2.02%
u	9.81%	10.1%	10.54%	11.37%	11.54%
s	0.236%	0.244%	0.257%	0.279%	0.284%
r	4.75%	4.82%	4.94%	5.2%	5.17%
m	27.34%	26.86%	26.14%	24.95%	24.71%

4 Optimal subsidies to intermediate goods production

Next, we find the optimal subsidy to intermediate goods production that makes the market growth rate to coincide with the social planner one. This allows us to realize the economic inefficiency due to the monopolistically sector. Note that this is only a BGP analysis and, therefore, this is not an optimal fiscal policy problem. Thus, we should calculate z introducing into equations (31), (36) and (37) the growth rate given by equation (14). Since we have calibrated the decentralized economy without subsidies in the previous section, the strategy now is the following: first, we use the calibrated parameters to calculate the social planner BGP in the benchmark economy of 3.1% of R&D spending to GDP and 7.85% of dividends to GDP. Second, we use the social planner growth rate to recover the optimal subsidy. Third, we change the population growth rate to show how it affects the optimal growth rate and the optimal subsidy. And fourth, we make a robustness analysis.

Applying the parameters of the benchmark economy to the social planner BGP, we obtain a growth rate of 6%. Moreover, the 2.14% of labor and the 49.32% of capital would be employed by the social planner in the R&D sector in order to have an almost three times larger growth rate. These seven times labor and four times capital with respect to the market economy is due to the fact that the social planner assigns the ressources in a marginal (competitive) way solucionating the surplus appropiability (monopolistic competition) problem.

Substituting the social planner growth rate in the decentralized BGP, we obtain $u = 11.73\%$, $s = 0.30\%$, $z = 0.107$, $r = 13.18\%$ and $p = 0.033$, so that the mark-up for intermediate goods firms is now $m = (p + z - r) / (p + z) = 5.9\%$. Thus, although one may think that the subsidy is 3.22 times the intermediate goods price, the effect of the subsidy is to lower the mark-up in order to increase production. The rationale is that the subsidy moves the marginal income out at the same time that the marginal cost increases, what makes the mark-up to decrease.[9]

From Proposition 2 we know the positive effect of population growth on the social planner growth rate. Table 3 shows the effect of population growth on the growth rate and the optimal subsidy. A higher growth rate is accompanied by a lower mark-up but the subsidy to price remains almost invariant, although the labor and capital proportions employed in the R&D sector slightly increase in order to rise the growth rate. Nevertheless, since the magnitude of the changes seems small, we can conclude that the effect of population growth on the growth rate is not so important.

Table 3. Efficient growth and optimal subsidy

	n = 0	**n = 0.002**	**n = 0.005**	**z=0** **n = 0.01**	**n = 0.01**	**n = 0.011**
g	5.78%	5.82%	5.89%	**2%**	6%	6.01%
u	11.15%	11.26%	11.44%	**11.37%**	11.73%	11.79%
s	0.28%	0.28%	0.29%	**0.28%**	0.30%	0.30%
z/p	3.17	3.18	3.19	**0**	3.22	3.23
r	12.76%	12.85%	12.97%	**5.2%**	13.18%	13.23%
m	6.18%	6.12%	6.04%	**24.95%**	5.9%	5.87%

We do the same robustness analysis of the previous section. In Table 4 we show the efficient growth rates and the optimal subsidies associated to the calibration of Table 1. A lower γ implies a higher inefficiency and, then, a higher social planner growth rate and a higher subsidy. The rationale is that a lower γ means that the public good has a lower weight in the production of R&D,

Table 4. Efficient growth in different scenarios

	R&D spending to GDP 3.1%			**Dividends to GDP 7.85%**		
Dividends	4.2%	7.85%	11.5%	–	–	–
R&D spending	–	–	–	1.5%	3.1%	6.2%
γ	0.002	0.042	0.125	0.066	0.042	0.006
g	9.58%	6%	5.55%	5.60%	6%	8.30%
u	3.61%	11.73%	16.64%	13.57%	11.73%	4.84%
s	0.0055%	0.30%	1.40%	0.57%	0.30%	0.012%
z/p	8.21	3.22	2.46	2.7	3.22	7.65
r	20.36%	13.18%	12.30%	12.4%	13.18%	17.80%
m	0.80%	5.9%	11.18%	8%	5.9%	1.14%

[9] Note that an alternative interpretation could be that, due to the production linearity, the subsidy reduces the marginal cost of the intermediate goods firms, so that the marginal cost becomes $(r - z) = 0.0246$ and the mark-up is $(p - r + z)/p = 0.23$. But this interpretation would imply a lower marginal cost and the same marginal income, which would imply a higher mark-up.

what implies that the market undervalues even more the R&D. Thus, a higher subsidy on intermediate goods is needed.

5 Conclusions

We have introduced capital as an input in the production of knowledge and discussed the associated problems arising from the public good nature of knowledge. We have shown that although population growth can affect economic growth, it is not necessary for growth to arise. We have derived both the social planner and the decentralized economy growth rates and showed the optimal subsidy that decentralizes it. We have shown numerically that the effects of population growth on the market growth rate, the optimal growth rate and the optimal subsidy are small. Besides, we have found that physical capital is more important for the production of knowledge than for the production of goods.

Clearly, future analysis should focus on the technology for the production of knowledge or R&D technology and its microfoundations. A good example is García-Rodríguez and Sánchez-Losada (2014).

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