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Structural Change and Non-Constant Biased Technical Change

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Abstract: Empirical evidence suggests that the differences in rates of technical progress across sectors are time-variant, implying that the bias in technological change is not constant. In this paper, we analyze the implications of this non-constant sectoral biased technical change for structural change and we assess whether this is an important factor behind structural transformations. To this end, we develop a multi-sectoral growth model where TFP growth rates across sectors are non-constant. We calibrate our model to match the development of the U.S. economy during the twentieth century. Our findings show that, by assuming non-constant biased technical change, a purely technological approach is able to replicate the sectoral transformations in the U.S. economy not only after but also prior to the World War II.

JEL Codes: O41, O47, O14, E29.

Keywords: Multi-sector growth model, Structural change, Sector biased technical change, Baumol effect.

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1. Introduction

Economic growth in developed economies is characterized by two prominent facts, namely, a constant capital-output ratio and changes in the relative sectoral shares in GDP as well as in the sectoral composition of the aggregate labor force. These are the so-called Kaldor-Kuznets stylized facts. The modern literature on structural change and economic growth encompasses these facts in the framework of multi-sectoral sectoral growth models (see Kongsamut, Rebelo and Xie, 2001; Meckl, 2002; Foellmi and Zweimuller, 2002; Ngai and Pissarides, 2007 and Acemoglu and Guerrieri, 2008).

A relevant explanation of structural change is sector biased technical change. According to this explanation, differences in rates of sectoral technical progress induce labor mobility from the progressive sectors (those with the highest productivity growth) to the stagnant sectors (those with the lowest productivity growth). This explanation goes back to Baumol (1967), while Ngai and Pissarides (2007) provide a modern formalization of the idea. They build a three-sector growth model where sectoral production functions differ only in total factor productivity (TFP) growth rates. Besides the assumptions of complementary goods and higher productivity growth in agriculture than in manufacturing and services, they assume that differences in TFP growth rates across sectors are constant. We refer to this assumption as constant biased technical change. As a consequence of these assumptions, this model replicates the main characteristic of the sectoral transformation: the fall of the agricultural sector and the rise of the service sector. However, the assumption of constant biased technical change is at odds with empirical evidence.

According to Dennis and Iscan (2007), and Alvarez-Cuadrado and Poshke (2011), technological progress at the sectoral level is not characterized by constant growth rates. Dennis and Iscan (2007) find that the differences in the respective rate of technological progress of the farm and non-farm sectors have been non-constant in the U.S. economy since the late 19th century. In particular, they show that sectoral technological progress is biased in favor of the non-farm sector at an early stage of development, which is followed by a shift in the bias of technological progress in favor of the farm sector. Alvarez-Cuadrado and Poshke (2012) find a similar pattern of sectoral technological progress across countries. They analyze available data for the relative prices of farm and non-farm goods for 11 advanced countries over the last two centuries. They find that changes in relative prices are related to changes in the bias of sectoral technological progress after controlling for the effects of international trade.

In this paper, we build a model based on Ngai and Pissarides's (2007) that introduces a non-constant bias of sectoral technological progress. Our aim is to analyze the economic implications of non-constant biased technical change for structural change. In order to address this aim, we assert that sectoral technological progress occurs via two channels. First, we assume that a constantly increasing stock of knowledge is generated exogenously, as in Ngai and Pissarides (2007). This channel captures the idea that technological progress can occur at the sector level based only on the available stock of knowledge in each sector. The second channel is technology adoption. We assume that a part of the sectoral technological progress is due to the adoption of new knowledge from the technological frontier. This frontier represents the maximum stock of new knowledge and ideas that is available in the economy and which can be adopted by the sectors. Adapting new techniques for the production process increases the stock of knowledge in each sector, which leads to an increase in sectoral technological progress. As in models of technology adoption, the distance or gap between the frontier and the sectoral technological level accounts for the stock of knowledge remaining to be adopted. This implies that relatively backward sectors, in the sense of having a higher gap relative to the frontier than others, tend to grow faster as long as there is a large stock of knowledge to be adopted.

To keep the model simple, we assert that the adoption rate of remaining knowledge occurs at an exogenous constant rate, which may differ between sectors. Under these assumptions, the growth rates of sectoral technological progress are not constant. We assume a functional form, in the spirit of the literature on technology adoption, for the sectoral productivity growth that is supported by data on agriculture, manufacturing and services TFP growth rates.

In the line with Ngai and Pissarides (2007), structural change in our model will be driven by differences in technological progress across sectors, whereas aggregated GDP, total expenditure consumption and capital grow at the same constant rate. Following Kongsamut et, al. (2001), we define this equilibrium path as a generalized balanced growth path (henceforth, GBGP). However, we show that the assumption of non-constant biased technical change introduces a relevant property in the patterns of structural change: sectoral composition can be degenerated or non-degenerated. The former characterizes an economy where the dominant sector is services and the weight of the remaining sectors is zero in the long run. The latter characterizes an economy where sectoral composition is asymptotically constant and with positive employment shares in all sectors.

We show that the nature of long-run sectoral composition depends on the sectoral ability to adopt knowledge. On the one hand, when all sectors can adopt technologies, sectoral composition is non-degenerated. This result arises because of our assumption regarding the adoption of knowledge from a common technological frontier. In this case, all sectoral TFP growth rates converge to the growth rate in the frontier and, consequently, asymptotic sectoral technology progress is unbiased. On the other hand, sectoral composition is degenerated if at least one sector producing only consumption goods cannot adopt new knowledge. We use this case to show that our model coincides with Ngai and Pissarides's model when no sector is able to adopt knowledge. Moreover, given the assumption of knowledge adoption, we show that the pace of structural change depends on technological backwardness in the agriculture sector. In particular, we show that a more marked degree of backwardness in agriculture causes labor to move rapidly from this sector to other sectors, thereby accelerating the pace of structural change.

In order to analyze how well a model based on non-constant biased technical change fits the features of the structural transformation, we conduct a numerical analysis of the model. To this end, we calibrate our model and a model based on the assumption of constant biased technical change to match the development process of the U.S. economy between 1870 and 2005. We use the second model as a benchmark for comparison. Based on these models, we simulate the time paths of the level of employment shares in agriculture, manufacturing and services. We compute the annual growth rates of the ratio between employment shares (RES, henceforth) in the agriculture and services sectors, and the RES between agriculture and manufacturing, where annual growth rates of these ratios represent the pace of industrialization in the economy. We then study the performance of both models in replicating these growth rates.

We evaluate the performance using two criteria. First, we examine the accuracy of both models when explaining the employment shares in agriculture, manufacturing and services over the period. To this end, we regress the actual values of employment shares on the simulated employment shares, and we then analyze how well these simulations fit the actual data on sectoral composition. As is standard in the literature, we report the root-mean-square error (RMSE), and the Akaike statistic for each regression, as measurements of accuracy. The second criterion is based on the value of the average annual growth rate of the RES obtained from numerical simulations. We compare the actual average annual growth rates of the RES with the growth rates obtained with our calibrated models. In particular, we compare the predicted with the actual average growth rates for three periods: 1870-1930, 1930-1950, and 1950-2005.

We focus on these periods because of the shifts in the sector biased technical change suggested by the data. According to Dennis and Iscan (2007), over these periods sectoral technical change shifts from being biased towards the non-farm sector to a bias in favor of the farm sector. These shifts in the bias of sectoral technical progress may accelerate the pace of industrialization in line with the technological explanation of structural change. By analyzing the performance of both models in predicting this change in the pace of industrialization over those periods, we can infer the importance of the shift in the bias of technical progress for explaining structural change.

Our numerical results show that a non-constant biased model fits the data better than a constant biased model. On the one hand, the numerical simulations based on our model fits the data better on the level of employment shares in agriculture, manufacturing and services than the benchmark model. This conclusion is robust to different values of the elasticity of substitution across goods.[1] In particular, we calibrate both models by

[1]The value of the elasticity of substitution has a significant role in the direction of structural

setting the value of the elasticity of substitution at 0.1, 0.5 and 0.9, and we obtain the simulated employment shares. Under these three scenarios, we highlight that the performance of the benchmark model increases as the utility function approaches a Leontief utility function. However, the accuracy is lower than that obtained with our model for the same values of the elasticity of substitution. That is, the model with non-constant technical bias provides a robust and better performance in replicating the structural change given the changes in the elasticity of substitution.

On the other hand, our model also provides a good fit with the data on the actual average growth growth rates of the RES. The numerical simulation based on our model replicates accurately the annual average growth rate of the RES before 1950. Indeed, our model explains 88 and 62 percent of the annual average growth of the RES between agriculture and manufacturing in the periods 1870-1930 and 1930-1950, respectively. In contrast, the benchmark model explains only 47 and 38 percent for the same periods. Interestingly, the accuracy of our model increases slightly when the elasticity of substitution increases to 0.90. In this case, our model explains 90 and 63 percent of the annual average growth rate of the RES for the same periods, whereas the accuracy of our benchmark model collapses to just 4 and 3 percent.

These numerical exercises show two interesting results. The first result is related to the discussion on how to model the process of sectoral transformation. Herrendorf et, al. (2014) show that if the aim is to obtain a good fit with the data using a consumption value-added specification, then the functional form of the utility that should be opted for is the Leontief utility function and assuming constant biased technical change. Our first result contributes to this discussion by showing that if the sectoral technological progress is modeled with exponential growth rates, then the Leontief utility function should be adopted in order to fit with the data, as Herrendorf et, al. (2014) point out. In contrast, if the utility function is assumed to differ from the Leontief specification, the performance of the model to fit the data will be poor under the assumption of constant biased technical change. Thus, our result suggests that a non-constant sectoral biased technological process is a necessary condition to model accurately structural change when a non-Leontief utility function is assumed.

The second result is related to the implications of non-exponential growth rates of sectoral TFP for structural change. When differences in rates of technological progress are time-variant, we show that a model in which sectoral production functions differ only in TFP growth is able to provide a good fit to the data on structural change not only after World War II (WWII), but also prior to it. This crucially depends on assuming a technological backwardness of the agricultural sector. If the initial backwardness in agriculture is higher than in manufacturing and services, then the agriculture

change (see Ngai and Pissarides, 2007; and Herrendorf, et, al. 2014). In particular, a low value of the elasticity of substitution across goods is required in order to replicate the rise of the services sector (see Boppart, 2014).

TFP growth rate increases inducing a non-constant decrease in the growth of relative prices of agricultural goods. This change in relative price implies that labor is rapidly pushed toward the stagnant sectors. In the case of constant technical biased change, growth rate of relative prices are constant, and therefore, it is also constant the pace at which labor moves from agriculture to other sectors. We show numerically that relaxing constant biased technical change, Baumol's effect can account for the process of industrialization in the early stage of development. In this regard, our results suggest that a purely technological approach to structural change is able to account for sectoral transformations in the U.S. economy prior to WWII.

The structure of the paper is as follows. In Section 2, we present empirical evidence of non-constant biased technological change. In Section 3, we build a model based on the assumption of non-constant biased technical change. In Section 4, we solve the model and characterize structural change. In Section 5, we present the main results of the numerical simulation. Finally, in Section 6, we present some concluding remarks and future lines of research, while the Appendix section contains the proofs of all the results of the paper.

2. The technology

We assert that sectoral technological progress occurs via two channels. The first channel is a stock of knowledge that increases at a constant rate and is generated exogenously. The second channel operates via the adoption of new knowledge from the technological frontier. This frontier encapsulates the maximum stock of new knowledge and ideas that are available for adoption by the sectors in the economy. These channels capture the idea that technological progress can occur both at the sector level, based on the available stock of knowledge in the sector, and based also on adoption from the technology frontier common to all sectors. In order to keep the model simple, we assume that adoption is costless. Furthermore, we assume that the stock of knowledge available in the technological frontier increases at an exogenous growth rate as follows

$$\frac{\dot{A}}{A} = \gamma, \tag{2.1}$$

where $\gamma > 0$ is the growth rate and A denotes the technology level in the frontier.[2] We then pose the law of motion of productivity in i sector as follows

$$\frac{\dot{A}_i}{A_i} = \phi_i + \omega_i \ln\left(\frac{A}{A_i}\right), \tag{2.2}$$

where $\omega_i > 0$ measures the rate of adoption, $\phi_i > 0$ measures the exogenous growth progress that takes place without adoption of knowledge, A_i is the level of TFP in the sector i and A/A_i measures the technological gap between sector

[2] In order to facilitate the notation we omit the time argument of all the variables.

i and the frontier. This gap accounts for the stock of knowledge remaining to be adopted.

To gain some intuition on the effect of the technological gap, let us suppose that $A > A_i$ and there is no exogenous technology growth in each sector, $\phi_i = 0$. In this case, sectoral technological progress depends only on the ability of each sector to adopt the remaining knowledge. Thus, differences in sectoral TFP growth rates or sectoral biased technical change will be determined by the magnitude of the technological gap and the rate at which technology is adopted across sectors. For simplicity, let us assume that all sectors can adopt knowledge from the frontier at the same rate, that is $\omega_i = \omega$ for all i. Therefore, sectoral biased technical change is due to differences in technological gaps. Those sectors with a lower stock of knowledge tend to grow faster than sectors that are closer to the frontier level. Although to the extent that backward sectors increase their TFP growth because of the adoption process, the growth rate decreases because fewer and fewer technologies from the frontier remain to be adopted. Eventually, both backward and advanced sectors converge to the frontier level, and this source of biased technological progress will vanish. On the contrary, the polar extreme case is when the source of biased technological progress lies on constant differences in exogenous growth rate, $\phi_i > 0$. If technology adoption is not possible, $\omega_i = 0$ for all i, sectoral biased technical change is due only to differences in ϕ_i across sectors, as in Ngai and Pissarides (2007). Hereinafter, we assume that $\omega_i \geq 0$, $\phi_i > 0$ for all i in order to analyze the implications of non-constant biased technical change for structural change. It is therefore convenient to derive the law of motion of technological gaps. Following Acemoglu (2008), we define the distance between sectors and the frontier as follows

$$v_i = \frac{A_i}{A}. \tag{2.3}$$

Taking the log-derivative of (2.3), and substituting (2.1) and (2.2), we obtain that the law of motion of technological gaps is

$$\frac{\dot{v}_i}{v_i} = \phi_i - \gamma - \omega_i \ln(v_i). \tag{2.4}$$

Once we solve (2.4),[3] it is easy to show that the long-run technological gap is

$$v_i^* = \begin{cases} 1 & if\ \phi_i = \gamma \\ \exp\left(\frac{\phi_i - \gamma}{\omega_i}\right) < 1, & if\ \phi_i < \gamma \\ \exp\left(\frac{\phi_i - \gamma}{\omega_i}\right) > 1, & if\ \phi_i > \gamma \end{cases}, \tag{2.5}$$

where v_i^* is the long-run gap in sector i. Note that there are three possible values that the technological gap can take in the long run. The first one, when the technological gap is equal to one, arises because exogenous sectoral technical progress is equal to the growth rate of the frontier, $\phi_i = \gamma$. This is

[3] See the Appendix B.

the case when the sectoral TFP level catches up the frontier level in the long run. The second value is less than one. This occurs when the exogenous sectoral technical progress is lower than γ at the frontier, $\phi_i < \gamma$. The third value occurs when exogenous sectoral technical progress is higher than γ at the frontier, $\phi_i > \gamma$. In this case, the sectoral TFP level is larger than the frontier level. Next, we show that this case is not possible given our estimation of the technology in (2.2).

To analyze whether the technology proposed can explain the time path of sectoral TFP, we estimate the parameters in equation (2.2) using sectoral data on productivity for the U.S. economy provided by the EUKLEMS project.[4] In particular, we estimate equation (2.2) using the growth rates of three broad sectors, namely agriculture, manufacturing and services.[5] We choose these three broad sectors since the analysis of structural change is commonly performed at this level.

Figure 1 shows both the level and growth rates of the technological gaps between the agriculture and services sectors, and between the agriculture and manufacturing sectors, as well as the trend in these series obtained with the Hodrick-Prescott filter. These gaps are measured by the ratios between the TFP in agriculture and services and the TFP in manufacturing. These ratios are defined as relative TFP. A superficial exploration of the plot shows that both relative TFPs have not been constant over the period 1970-2005 (see Figure 1; panel a, and b). Figure 1 points out that TFP in agriculture grew faster than TFP in manufacturing, meanwhile TFP in services grew at a lower rate than in manufacturing. These results are in line with those reported by Herrendorf et, al. (2014). Although a more careful inspection of data reveals that the trend of this relative sectoral TFP is not constant. In particular, both series show changes in trend around 1980 that have narrowed sectoral biased technical change. Despite variability in the growth rate of these series, shifts in the long-run trend of growth rates reveal the observed change in trend in relative TFP levels (see Figure 1, panels c and d).

[Insert Figure 1]

Table 1 shows the result of estimating the growth rates of TFP in equation (2.2) for the agriculture, manufacturing and services sectors. In order to estimate the parameters, we solve the differential equations in (2.2) under the assumption of exogenous growth of the technology frontier. Given the solution of (2.2) in Appendix B, we estimate the following system of equations

$$\ln A_i = \alpha_i + \beta_i t + e^{-\delta_i (t-n_i)}, \tag{2.6}$$

where

$$\alpha_i = \frac{\phi_i - \gamma}{\omega_i}; \ \beta_i = \gamma; \text{ and } \delta_i = \omega_i, \tag{2.7}$$

[4]This project has information about TFP growth across 74 sectors of the economy for the United States, Japan, and many countries in Europe for the period 1970-2005. For a summary overview of the methodology and construction of the EU KLEMS database, see O'Mahony, Mary and Marcel P. Timmer (2009).

[5]These broad sectors are defined as in Herrendorf et, al. (2014).

and n_i is the constant of integration. We estimate (2.6) imposing the constraints (2.7) by using non-linear squares.[6] The results are in Table 1. The point estimates for the rate of adoption, $\hat{\omega}_i$ and the exogenous growth progress $\hat{\phi}_i$ in agriculture and services are statistically different from zero. These results show that there exists a positive relation between the technological gap and the TFP growth in agriculture and services. In particular, the estimated rate of adoption $\hat{\omega}_i$ in agriculture is 0.026, whereas the estimated rate of adoption in services is, on average, 0.017. The point estimates for the rate of the exogenous growth progress $\hat{\phi}_i$ in agriculture and services are similar and statistically different from zero. Notably, the rate of adoption in the manufacturing sector is not statistically different from zero, and the estimated value of the exogenous growth progress $\hat{\phi}_m$ is close to the estimated growth rate of the technology frontier, $\hat{\gamma}$. Given the estimated parameters, we calculate the growth rate of the technological gap in agriculture and services.

[Insert Table 1]

These results suggest the existence of non-constant biased technical change in the U.S. economy across the agriculture, manufacturing and services sectors. We acknowledge that our results cover only a short period of time, nevertheless the reported results are in line with those reported by Dennis and Iscan (2009), who point out the existence of changes in relative TFP in farm and non-farm sectors over the period 1800-2000. Our results suggest that the bias in favor of technological progress in agriculture has declined over the period 1970-2005, converging to the growth rate of the manufacturing sector. This suggests again that biased technological progress is not constant.

In order to analyze the implications of non-constant biased technological progress on structural change, in the following section we build a three-sector growth model, based on the seminal work of Ngai and Pissarides (2007), which is characterized by non exponential sectoral TFP growth, as the empirical evidence indicates.

3. The model

We build a three-sector growth model in which the output in each sector is obtained from combining capital, K, and labor, L. We adopt the notation $a, s,$ and m to denote the agriculture, services, and manufacturing sectors, respectively. To facilitate the notation, we omit the time argument in all the variables. Following Ngai and Pissarides (2007), we assume that all sectors have the same capital intensity and produce an amount Y_i of commodity using the following production function:

$$Y_i = A_i \left(s_i K\right)^{\alpha} \left(u_i L\right)^{1-\alpha}, \text{ for } i = a, s, m, \tag{3.1}$$

[6] In Appendix B, we show the empirical strategy to estimate (2.4).

where s_i and u_i are the shares of capital and labor allocated in sector i, A_i is the sectoral total factor productivity (TFP), and $\alpha \in (0, 1)$ is the intensity of capital in this sector. Obviously, both capital and labor shares satisfy

$$s_a + s_s + s_m = 1, \tag{3.2}$$

and

$$u_a + u_s + u_m = 1. \tag{3.3}$$

We also assume that population is constant and we normalize it to one. We refer to C_a and C_s as the amount of agricultural and service goods devoted to consumption, so that the following equation is satisfied

$$Y_i = C_i \text{ for } i = a, s. \tag{3.4}$$

We assume that the commodity Y_m, namely the manufacturing good, can be either consumed or added to the stock of aggregate capital. Thus, the law of motion of the capital stock is given by

$$\dot{K} = Y_m - \delta K - C_m, \tag{3.5}$$

where C_m is the amount of good Y_m devoted to consumption, and $\delta \in [0, 1]$ is the depreciation rate of the capital stock.

The representative agent obtains utility from the consumption of agricultural, manufacturing and service commodities. In particular, we assume that the representative agent is characterized by the instantaneous utility function

$$U\left(\tilde{C}\right) = \ln\left(\tilde{C}\right), \tag{3.6}$$

where $\tilde{C}$ denotes a composite consumption good, which satisfies

$$\tilde{C} = \left(\eta_a C_a^{\frac{\epsilon-1}{\epsilon}} + \eta_s C_s^{\frac{\epsilon-1}{\epsilon}} + \eta_m C_m^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}, \tag{3.7}$$

where η_a, η_s, and η_m measure the relative preference for sectoral commodities, which are assumed to satisfy $\eta_a + \eta_s + \eta_m = 1$. The elasticity of substitution among commodities is denoted by the parameter $\epsilon > 0$.

4. The equilibrium

In this section, we obtain the system of differential equations characterizing the equilibrium. We use these equations to find the long-run equilibrium and to study how the introduction of technology adoption across sectors modifies the sectoral composition of the economy.

The representative agent maximizes the discounted sum of utilities

$$\int_0^\infty e^{-\rho t} U\left(\tilde{C}\right) dt, \tag{4.1}$$

subject to (3.2), (3.3), (3.4), and (3.5), where $\rho > 0$ is the subjective discount rate. In the Appendix, we obtain the following equations:

$$s_i = u_i, \text{ for } i = a, s, m; \tag{4.2}$$

and

$$p_a = \frac{A_m}{A_a}, \tag{4.3}$$

$$p_s = \frac{A_m}{A_s}, \tag{4.4}$$

where (4.2) is a set of static efficiency conditions for the allocation of factors, and (4.3) and (4.4) shows that relative prices, p_a and p_s, are functions of the ratio between the manufacturing sector productivity (the numeraire good) and agriculture and services, respectively.

To characterize the aggregate economy, we combine (3.4) and (4.3) to obtain the aggregate consumption expenditure, which is defined as $C = p_a C_a + p_s C_s + C_m$. As in Ngai and Pissarides (2007), we define the ratio of consumption expenditure on good i to consumption expenditure on the manufacturing good as

$$x_i \equiv \frac{p_i C_i}{C_m} = \left(\frac{\eta_i}{\eta_m}\right)^{\epsilon} p_i^{1-\epsilon}, \tag{4.5}$$

and using x_i, consumption expenditure can be rewritten as

$$C = C_m \left(1 + x_a + x_s\right). \tag{4.6}$$

Note that (4.5) only depends on relative prices. By combining (3.1), (4.2) and (4.3), we obtain the gross domestic product (GDP) as

$$Y = A_m K^{\alpha}. \tag{4.7}$$

Having obtained the equations that define the aggregate economy, we now characterize the sectoral employment share. In Appendix A, we show that substituting the market clearing condition (3.4) in (4.6) and taking into account the optimal capital shares and (4.9), the efficiency labor shares in the agriculture and services sector are

$$u_a = \left(\frac{x_a}{1 + x_a + x_s}\right) \frac{C}{Y}, \tag{4.8}$$

and

$$u_s = \left(\frac{x_s}{1 + x_a + x_s}\right) \frac{C}{Y}. \tag{4.9}$$

Equation (4.8) and (4.9) together with (3.3) define the sectoral composition of the economy.

We next obtain the system of differential equations that characterizes the equilibrium. In Appendix A, we obtain that the growth rate of consumption expenditure is

$$\frac{\dot{C}}{C} = \alpha A_m K^{\alpha-1} - (\rho + \delta), \tag{4.10}$$

and using (3.5) and (4.7), we can express the law of motion of the capital stock in terms of total consumption expenditure as follows

$$\frac{\dot{K}}{K} = A_m K^{\alpha-1} - \frac{C}{K} - \delta. \tag{4.11}$$

Equation (4.10) tells us that the growth rate of total consumption expenditure is independent of relative prices effects. This result is attributed to our preferences being represented by a logarithmic utility function.

In order to characterize the equilibrium path, we rewrite (4.10) and (4.11) by using the following transformed variables $z = KA_m^{\frac{1}{\alpha-1}}$ and $c = CA_m^{\frac{1}{\alpha-1}}$, where z and c denote, respectively, capital and total consumption expenditure in efficiency units. By taking log-derivatives of z and c, and using (4.10), (4.11) and (2.4), we rewrite the dynamic system as follows

$$\frac{\dot{z}}{z} = z^{\alpha-1} - \frac{c}{z} - \delta - \frac{\dot{v}_m + \gamma v_m}{(1-\alpha) v_m}, \tag{4.12}$$

$$\frac{\dot{c}}{c} = \alpha z^{\alpha-1} - \rho - \delta - \frac{\dot{v}_m + \gamma v_m}{(1-\alpha) v_m}, \tag{4.13}$$

and

$$\frac{\dot{v}_m}{v_m} = (\phi_m - 1)\gamma - \omega_m \ln v_m. \tag{4.14}$$

Following Ngai and Pissarides (2007), we define structural change as the change in the employment shares. By taking log-derivatives of (4.3), (4.5), (4.8), (4.9); and taking into account (2.4), we obtain that the growth rate of the employment share are

$$\frac{\dot{u}_a}{u_a} = \frac{\dot{\overline{C/Y}}}{C/Y} + (1-\epsilon)\left(\frac{\dot{v}_m}{v_m} - \frac{\dot{v}_a}{v_a}\right) - (1-\epsilon)\left[x_a\left(\frac{\dot{v}_m}{v_m} - \frac{\dot{v}_a}{v_a}\right) + x_s\left(\frac{\dot{v}_m}{v_m} - \frac{\dot{v}_s}{v_s}\right)\right], \tag{4.15}$$

and

$$\frac{\dot{u}_s}{u_s} = \frac{\dot{\overline{C/Y}}}{C/Y} + (1-\epsilon)\left(\frac{\dot{v}_m}{v_m} - \frac{\dot{v}_s}{v_s}\right) - (1-\epsilon)\left[x_s\left(\frac{\dot{v}_m}{v_m} - \frac{\dot{v}_s}{v_s}\right) + x_a\left(\frac{\dot{v}_m}{v_m} - \frac{\dot{v}_a}{v_a}\right)\right]. \tag{4.16}$$

The dynamic equilibrium is thus characterized by a set of paths $\{z, c, v_m\}$ such that, given $z(0)$ and $v_m(0)$, solves equations (4.12), (4.13) and (4.14), and satisfies the transversality condition

$$\lim_{t\to\infty} \mu_m \left(z v_m e^{\frac{\gamma}{1-\alpha}t}\right) = 0.$$

We define a balanced growth path (BGP, henceforth) equilibrium as an equilibrium path along which the efficiency units of capital, z, and total consumption expenditure, c, remain constant. The following result characterizes the steady-state equilibrium.

Proposition 4.1. *There exists an unique BGP, and the long-run values of the transformed variables are*

$$
\begin{aligned}
z^* &= \left(\frac{\alpha}{\gamma^* + \rho + \delta}\right)^{\frac{1}{1-\alpha}}, \\
c^* &= \left(\frac{(1-\alpha)(\delta + \gamma^*) + \rho}{\alpha}\right)\left(\frac{\alpha}{\gamma^* + \rho + \delta}\right)^{\frac{1}{1-\alpha}}, \\
v_m^* &= \exp\left(\frac{\phi_m - \gamma}{\omega_m}\right),
\end{aligned}
$$

where the long-run growth rate of GDP is

$$
\gamma^* = \begin{cases} \frac{\gamma}{1-\alpha} & \text{if } \omega_m > 0 \\ \frac{\phi_m}{1-\alpha} & \text{if } \omega_m = 0 \end{cases}.
$$

Note that the GDP growth rate is higher if there is technology adoption in the manufacturing sector. Given the assumption of equal capital intensity across sectors, aggregate TFP is equal to the TFP in manufacturing. When technology adoption occurs in the manufacturing sector, the growth rate of technological progress in this sector increases and, consequently, so does the GDP growth rate. On the other hand, when there is not technology adoption in the manufacturing sector, the GDP growth rate increases proportionally at the rate ϕ_m. The following propositions and definitions characterize the equilibrium path and the structural transformations in our economy.

Proposition 4.2. *The BGP is saddle-path stable.*

As Ngai and Pissarides (2007) pointed out, equations (4.12) and (4.13) are similar to the two differential equations in the one-sector Ramsey economy. Our model shows similar transitional dynamics to those of the Ramsey model if we assume that $\omega_m = 0$ or $A_m = A$ in the initial period. Obviously, in this case, the transitional dynamics are governed only by equation (4.12) and (4.13). In contrast, if $\omega_m > 0$ and $A_m \neq A$, the transitional dynamics are characterized by equations (4.12), (4.13), and (4.14); and the equilibrium dynamics are different from those obtained in Ngai and Pissarides model. And yet, in both cases, the patterns of structural change are not necessarily the same as those reported by Ngai and Pissarides (2007). For instance, when technology adoption occurs both in the agriculture and services sectors, the growth rates of sectoral TFP are not constant. This implies that the bias of sectoral technical progress is time varying and, therefore, the rate of reallocation of labor out of agriculture is not constant. This affects the pace and the patterns of structural transformation in the transitional and dynamics.

As is usual in this literature, we analyze the structural change that arises when aggregate variables are in the BGP i.e. the model satisfies the

Kaldor facts. We, therefore, focus on characterizing the structural change that arises when the economy is in the BGP, and technology adoption occurs in manufacturing. At this point, we highlight that this assumption only has implications for the stationary solutions (z^*, c^*, v_m^*) outlined in the previous propositions, and not for the structural changes that we characterize next. The following definition and propositions characterize the process of structural change of the economy along the equilibrium path and the sectoral composition in the long run.

Definition 4.3. *Sectoral composition is degenerated if the asymptotic employment share of at least one sector is zero. Otherwise, sectoral composition is non-degenerated.*

Proposition 4.4. *Necessary and sufficient conditions for the existence of a non-degenerated sectoral composition are $\omega_a \neq 0$ and $\omega_s \neq 0$. Otherwise, the sectoral composition is degenerated.*

Proposition 4.5. *In the BGP with non-degenerated sectoral composition, employment shares in the agriculture and services sectors are asymptotically:*

$$
\begin{aligned}
u_a^* &= \frac{(1-\hat{\sigma})}{1+\left(\frac{\eta_m}{\eta_a}\right)^{\epsilon}\left(\frac{v_a^*}{v_m^*}\right)^{1-\epsilon}+\left(\frac{\eta_s}{\eta_m}\right)^{\epsilon}\left(\frac{v_m^*}{v_s^*}\right)^{1-\epsilon}}, \\
u_s^* &= \frac{(1-\hat{\sigma})}{1+\left(\frac{\eta_a}{\eta_m}\right)^{\epsilon}\left(\frac{v_m^*}{v_a^*}\right)^{1-\epsilon}+\left(\frac{\eta_m}{\eta_s}\right)^{\epsilon}\left(\frac{v_s^*}{v_m^*}\right)^{1-\epsilon}},
\end{aligned}
$$

and

$$u_m^* = 1 - u_a^* - u_s^* > 0.$$

In the asymptotic BGP with degenerated sectoral composition, employment shares in the agriculture and services sectors are $u_m^* = \hat{\sigma}$, $u_s^* = 1-\hat{\sigma}$, and $u_a^* = 0$; where

$$\hat{\sigma} = \alpha \frac{\delta + \gamma^*}{\delta + \rho + \gamma^*}$$

is the savings rate along the aggregate balanced growth path, and v_i^*, i=a,s,m are defined in (2.5) .

Proposition (4.4) shows the conditions under which the economy converges to a non-degenerated sectoral composition. These conditions require that sectors producing consumption goods adopt knowledge from the technological frontier. When these conditions are fulfilled, the rates of TFP growth in the agriculture and services sectors converge to the growth rate of the frontier. This implies that (4.15) and (4.16) are equal to zero in the long run, when both technology gaps in agricultural technology and services reach their stationary values. In contrast, a degenerated sectoral composition arises when one of the sectors that produces consumption goods (agriculture or services) is not able to adopt any knowledge from the frontier. It is important to note that these results are independent of whether technology adoption occurs in the manufacturing sector or not. That is, in our model, sectoral composition is determined by the technological characteristics of sectors producing only consumption goods. The extreme case of a degenerated economy occurs when no sector adopts knowledge. In that case, the implications of our model are the same as those in Ngai and Pissarides model (2007).

The results shown in Proposition (4.4) characterize the sectoral composition in the long run, whereas one of the main features of economic development is the structural transformations in the short run. To characterize this structural change, we focus on studying changes in the ratio between agriculture and services, and the ratio between agriculture and manufacturing employment shares (RES, henceforth), as a measure of the relative importance of agriculture in the economy. The annual relative variation in these ratios indicates the changes in the number of farm workers per worker engaged in non-agricultural activities. Thus, the growth rates of the RES show the pace of industrialization. Kuznets (1973) emphasized that a rapid decline in the RES (a higher growth rate) is one of the main features of structural transformations across countries. The following proposition characterizes the growth rate of the RES in our economy.

From using (4.15) and (4.16), the growth rate of the RES between agriculture and service is:

$$\frac{\dot{u}_a}{u_a} - \frac{\dot{u}_s}{u_s} = \underbrace{(1-\epsilon)\left(\phi_s - \phi_a\right)}_{\textit{Constant biased effect}} - \underbrace{(1-\epsilon)\left(\omega_s \ln v_s - \omega_a \ln v_a\right)}_{\textit{Backwardness effect}}.$$

This equation shows that the RES growth rates are functions of technological gaps between agriculture and other sectors, if technology adoption is possible in at least one of these sectors.[7] In this case, the RES growth rates depend on two components. The first component, the constant biased effect, is equal to the constant differences in the rate of exogenous technological progress between the service and agriculture sectors. The second component, the backwardness effect, depends on the difference in the distances of each sector to the technology frontier. To understand the effect of each component on the RES growth rate, let us assume that adoption of knowledge in agriculture and services is not possible. In this case, the constant biased effect determines the magnitude and direction of the RES growth rate. To replicate the observed structural change, a decreasing relative employment share in agriculture, a model with no adoption will require that $\phi_s < \phi_a$. In contrast, when knowledge adoption in both sectors is possible and we assume that there is no biased effect, the RES growth rate is determined by the backwardness effect. In this case, the RES growth rates are not constant because the technological gaps vary over time. To gain some intuition about

[7]For the sake of clarity, we present only the growth rate of the RES between agriculture and services. Despite the fact that the main feature of sectoral change is the polarization in the distribution of labor between agriculture and services, we also report numerically the growth rate of the RES between agriculture and manufacturing.

the effect of the backwardness effect on RES growth rate, we assume that $\phi_s = \phi_a$. If the technological gap in agriculture is larger than the gap in the services, then the RES growth rate rises (the larger the magnitude of the difference). Otherwise, the change in the RES growth rate is lower, if agriculture is closer than the services sector to the frontier (the lower the magnitude of the difference). That is, the second component measures the effect of backwardness in the agriculture sector on structural transformation. To analyze whether the effects of non-constant biased technical change is an important factor accounting for structural transformations, we conduct a numerical analysis of our model.

5. Numerical analysis

In this section we analyze the accuracy of the model for replicating the patterns of structural change observed in the United States during the period 1870-2005. Structural change is characterized by using both the employment shares in agriculture, manufacturing and services, and also by using the annual growth rates of the RES between agriculture and services, and between agriculture and manufacturing. To this end, we calibrate both the model based on non-constant biased technical change and the model built on the constant biased technical assumption to match the development process of the US economy in the period 1870-2005. We use both models to simulate the time path of the levels of sectoral employment shares, and we use them to calculate the growth rate of the RES. We then study the performance of both models in replicating these features of the structural change by taking into account two different criteria.

We first compare the accuracy of the non-constant biased model and the constant biased model's (our benchmark model) predictions on sectoral employment allocation by regressing actual employment shares in agriculture, and services on simulated data. We analyze how well these simulations fit actual data by reporting the root mean square error (RMSE), and the Akaike statistic for each regression. The second criterion is based on the value of the average annual growth rate of the RES obtained from numerical simulations. We compare the actual average annual growth rates of the RES with those growth rates obtained with our calibrated models. In particular, we compare the actual average growth rates of the RES for the periods 1870-1930; 1930-1950 and 1950-2005; and we then compare them to those predicted by our model and the benchmark. We focus on these periods because of the shifts in the sector biased technical change suggested by the data.

According to Dennis and Iscan (2007), over these periods sectoral technical change shifted from being biased towards the non-farm sector to being in favor of the farm sector. These changes affect the actual average growth rates of the RES and, therefore, comparing the performance of both models in predicting these changes provides a measure of the feasibility of the assumptions on which they are based for replicating the structural change in the U.S. economy. Based on these two criteria, we determine which model is more suitable for replicating the main patterns in the data. In what follows, we describe the strategy for calibrating both models and we present the main results.

5.1. Calibration strategy

To calibrate both non-constant and constant biased models, we first set the values of the parameters that are common in both frameworks. These parameters are $\alpha, \gamma, \rho, \delta, \eta_a, \eta_s, \epsilon$. From the The Economic Report of the President (2007), we set the value of $\alpha = 0.315$ to match the average labor income share for the period 1959-2005. We set the value of γ, ρ and δ so that they match the value of the average rate of GDP growth, the average capital-out ratio for the period of 1929-1998, and the interest rate. According to Ngai and Pissarides (2004), the average rate of GDP growth is around 2 percent, and the value of the capital-out ratio is 3. We set the interest rate equal to 5.2% in the steady-state as in Alonso-Carrera and Raurich (2010). Thus, we obtain that $\rho = 0.03$, $\delta = 0.05$ and the growth rate of the technology frontier $\gamma = 0.0137$. In the literature, there is not a specific estimation for the value of ϵ. Its value ranges from 0.002 to 0.89, depending on the calibration strategy and the estimation procedures applied (see Boppart, 2014). We perform three numerical simulations of both models by setting the value of ϵ equal to 0.1, 0.5, and 0.90 in order to cover the range of values reported in the literature. These values let us examine how our results change in response to shifts in the value of the elasticity of substitution. Obviously, these changes affect the values of η_a, η_s, v_a, and v_s. Therefore, in the case of the non-constant biased technical change, we set the value of the parameters η_a, and η_s, so that they match the expenditure consumption share in the agriculture and services sectors, at 2005 for given values of ϵ. Simultaneously, for each value of ϵ, we set initial values for technological gaps v_a, and v_s so that they match the employment labor shares in agriculture and services in 1870, respectively, and we normalize $v_m = 1$.[8] In the case of the constant biased technical change model, we follow Ngai and Pissarides's procedure. We set the values of η_i and the initial values of the sectoral TFPs to match the values of sectoral employment shares in 1870 (see Ngai and Pissarides, 2004).

Finally, we set the values of $\omega_a, \omega_s, \omega_{,m}, \phi_a, \phi_s, \phi_m$ as follows. In the case of our benchmark model, constant biased technical change implies that $\omega_a = \omega_s = \omega_m = 0$, so that we then need to set the values of ϕ_a, ϕ_s, and ϕ_m so that they match TFP growth rates in agriculture, manufacturing and services according to equation (2.2). Ngai and Pissarides (2004) set the value for TFP growth in agriculture, manufacturing and services at 2.4%, 1.4% and 0.4% for the period 1870-2000. Accordingly, we set $\phi_a = 0.024$, $\phi_s = 0.04$, and $\phi_m = 0.014$. In the case of non-constant bias, both the rate of adoptions $(\omega_a, \omega_s, \omega_m)$ and the exogenous growth rates (ϕ_a, ϕ_s, ϕ_m) are obtained by using

[8] This assumption implies that the manufacturing sector have reached the technological frontier. In this way, we do not need to impose a value for ω_m.

the growth rate of relative prices (4.3). As our aim is to analyze a long period of sectoral transformation, we estimate the value of these parameters using the information available on relative prices, rather than the estimated parameter values in Section 2. Limited availability of data for sectoral TFP growth would mean our having to use estimated values of these parameters for a short period (1970-2005). We exploit the fact that, given our assumptions, relative prices are linked to sectoral productivity, and hence to their growth rates by using equations (2.2) and (4.3). Thus, we overcome this data limitation by using time series of relative prices for the period 1929-2005.[9] The econometric procedure to estimate these parameters is shown in the Appendix B and results are in Table 2.[10] Table 3 summarizes the parameter values for the simulation of the two models.

[Insert Table 2 and Table 3]

5.2. Sectoral employment shares

Figure 2 shows the goodness of fit of our simulation based on both models. As can be seen with the naked eye, both models reproduce the main patterns of sectoral change: the decline of the agriculture sector and the rise of the services sector. However, the accuracy of such predictions differs between the models. At first glance, it is evident that constant-biased model predictions change as the degree of elasticity varies. In particular, the predictions based on this model differ greatly from the actual values of labor shares in agriculture and services as elasticity increases. By contrast, the robustness of predictions based on a non-constant biased model is notorious to changes in this parameter. To analyze the degree of accuracy of the simulations further, we report three measurements of accuracy.

[Insert Figure 2]

Table 4 reports three measures that allow us to compare the accuracy of the models, and the robustness of these simulations to variations in the elasticity and the rate of adoption. Specifically, Table 4 reports the the root-mean-square error (RMSE), and the Akaike information criterion (AIC).[11] We calculate these accuracy measures by regressing actual labor shares in

[9]Relative prices for 1929-1970 are from the Historical Statistics of the United States: Colonial Times to 1970, Part 1 and 2. The implicit price deflator for services in series E17, and the wholesale price index for industrial commodities and farm products in series E23-25, E42, E52-E53. Relative prices for 1970-2005 are from: Economic Report of the President, 2013. Price index for industrial commodities and farm products in table B-67. Price indexes for services, table B-62.

[10]Our simulated series are based on point estimated parameters ($\hat{\omega}_a, \hat{\omega}_s, \hat{\omega}_{,m}, \hat{\phi}_a, \hat{\phi}_s, \hat{\phi}_m$). We also show the simulation series based on the confidence intervals that allow us to measure the model's robustness to a variation in the rates of adoption.

[11]RMSE is the standard deviation of the differences between observed and predicted values values. Finally, the AIC provides a measure for comparing models. AIC allows us to determine the probability that a model is the best model to replicate the data given the set of information and alternative models.

agriculture, manufacturing and services on those shares predicted by our non-constant biased model and the benchmark.[12]

[Insert Table 4]

Table 4 shows that both models are able to explain the dynamics of sectoral change. However, there are quantitative differences in their performance. On the one hand, reading from left to right, Table 4 shows the differences in accuracy across the models based on these statistics. The simulations based on the non-constant biased model provide a better fit than those based on the benchmark model. In the case of agriculture, for instance, changes in the RMSE are minimal in our model compared with those obtained by the benchmark model for three different values in the elasticity. In particular, the accuracy of the benchmark model decreases as the value of the elasticity increases, whereas the simulation based on the non-constant biased model is robust to replicate observed data. In the case of services, Table 4 shows similar results as in previous case, except in the case of low elasticity ($\epsilon = 0.1$). In this case, the benchmark model reports a lower RMSE value than shown by our model. The results show that both models are able to explain qualitatively the structural change in the U.S. economy. However, they also show that the performance of the benchmark model decreases as the elasticity increases.

5.3. The growth rate of the RES

Tables 5 and 6 show the actual average growth rate of the RES between agriculture and services, and between agriculture and manufacturing, respectively, for three periods: 1870-1930; 1930-1950; and 1950-2005. Tables 5 and 6 also report the average growth rate of the RES that are calculated based on the simulation of non-constant biased and benchmark models for different values of the elasticity. Thus, Tables 5 and 6 allow us to compare the robustness of the models to replicate the structural change for variations of this parameter. From these tables, we can observe two interesting results.

[Insert Table 5 and Table 6]

First, we highlight the accuracy of the non-constant biased model for simulating the relative annual changes in the RES. In general, over the entire period considered, the non-constant biased model can account for most of the growth in the RES between agriculture and services, and between agriculture and manufacturing, whereas the benchmark model replicates poorly the observed growth rates. In particular, the non-constant biased model replicates the actual average annual growth rate in the RES in

[12] Alternately, we use the filtered data of the labor shares in agriculture and services to make the regressions. Actual data were filtered by using the Hodrick-Prescott method to reduce fluctuations in actual data due to the business cycle. This approach does not change the results in the main text.

the early stage of development. For the periods 1870-1930 and 1930-1950, our model replicates 88 and 62 percent of the relative change in the RES between agriculture and services, and between agriculture and manufacturing, respectively. By contrast, the benchmark model replicates 42 and 25 percent, respectively. That is, the non-constant biased model improves the explanation for the relative change in the RES by around two times compared to the prediction from a model based on constant biased technical change.

Second, we highlight the robustness of the non-constant biased model respect to the benchmark model. Tables 5 and 6 also show the sensitivity of results to variations in the elasticity. On the one hand, our model can explain a large part of the labor reallocation in the post-war U.S. economy, regardless of the value of the elasticity. Herrendorf, Rogerson, and Valentinyi (2009) calibrate utility function parameters to be consistent with the sectoral transformation and consumption data in the post-war U.S. economy under the assumption of constant biased technical change. They find that a Leontief utility specification ($\epsilon = 0$) is necessary to provide a good fit for both the value-added sectoral consumption and the sectoral labor shares data. Given our results, a non-constant sectoral technical progress can explain the sectoral transformation without imposing a Leontief utility function. We interpret these results as a measure of the importance of the technological explanation for the structural change in the United States during the post-war period.

Moreover, the literature points out that both technological and demand factors affect structural change throughout the development process in the United States (see Dennis and Iscan, 2007; Buera and Kaboski, 2009; and Herrendorf et, al., 2014). These papers highlight that a technological factor, such as the constant biased technical change, plays a major role in explaining the sectoral shift observed after WWII, whereas the income effect is the dominant factor in accounting for the structural transformation prior to 1950. Our findings show that if we move away from the assumption of constant biased technical change, a purely technological explanation could account for the sectoral transformations in the U.S. economy prior to WWII.

6. Concluding remarks

In this paper, we present a multi-sectoral sectoral growth model based on Ngai and Pissarides' model. In their model, sectoral technological progress is assumed to be a constant process. This implies that differences in TFP growth rates across sectors are constant over time. According to the literature, however, this assumption is at odds with empirical evidence.

We relax the constant biased technical assumption by asserting that sectoral TFP growth rates change due to technology adoption. We assert that a sector benefits from adopting new technologies or ideas (knowledge) available at the technological frontier. This process prompts their sectoral technological progress and induces non-constant sectoral TFP growth. Based on our proposed model, we analyze the implications for structural change when sectoral technological progress is not constant.

We find that predicted patterns of sectoral labor allocation across sectors are affected by the non-constant biased technical progress in two major ways. First, we find that labor allocation over time and sectoral composition in the long run are determined by a sector's ability to adopt knowledge. We show that if technology adoption occurs in every sector, then sectoral composition is constant in the long run, while the dynamic path of employment share is affected by the rate of technology adoption. Second, in our model, the pace of industrialization depends on the relative technology level in each sector. In contrast with a constant biased model, the growth rate of the RES depends on the technological gap between the agriculture, services and manufacturing sectors and to the frontier. We show that as long as the technological gap in the agriculture sector remains large, the pace of industrialization increases.

We analyze numerically the importance of non-constant biased technical change in explaining the structural change observed in the US economy in the period 1870-2005. We show that the patterns of sectoral labor allocation and the pace of industrialization are better explained by a model based on non-constant TFP growth than by a model based on constant biased technical change.

Our findings show that if we move away from the assumption of constant biased technical change, a purely technological explanation could account for part of the sectoral transformations in the U.S. economy prior to WWII. In our model, the relative backwardness of the agricultural sector at an early stage of development fosters the rate at which labor moves from this sector to the rest of the economy. We interpret this result as a suggestion for reconsidering the role of the technology in explaining structural transformation. In this regard, our results suggest that economic factors that promote technology adoption would foster the pace of industrialization and structural change. In this regard, a natural extension of our paper is to analyze in-depth those factors that promote sectoral technology progress, such as technological adoption, human capital, and R&D as possible future lines of research on the determinants of structural change.

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Appendix

A. Equilibrium properties

Solution to the representative consumer optimization problem.

The Hamiltonian function associated with the maximization of (4.1) subject to (3.2), (3.3), (3.4), and (3.5) is

$$\mathcal{H} = \ln \tilde{C} + \sum_{i=a,s} \mu_i \left(Y_i - C_i \right) + \mu_m \left(Y_m - \delta K - C_m \right),$$

where μ_a, μ_s and μ_m are the co-state variables corresponding to the constraints (3.4) and (3.5), respectively. The first order conditions are

$$\eta_a \tilde{C}^{\frac{1-\epsilon}{\epsilon}} C_a^{-\frac{1}{\epsilon}} = \mu_a, \quad \text{(A.1)}$$

$$\eta_2 \tilde{C}^{\frac{1-\epsilon}{\epsilon}} C_s^{-\frac{1}{\epsilon}} = \mu_s, \quad \text{(A.2)}$$

$$\eta_3 \tilde{C}^{\frac{1-\epsilon}{\epsilon}} C_m^{-\frac{1}{\epsilon}} = \mu_m, \quad \text{(A.3)}$$

$$(1-\alpha)\,\mu_a \frac{Y_a}{l_a} = (1-\alpha)\,\mu_m \frac{Y_m}{l_m}, \quad \text{(A.4)}$$

$$(1-\alpha)\,\mu_s \frac{Y_s}{l_s} = (1-\alpha)\,\mu_m \frac{Y_m}{l_m}, \quad \text{(A.5)}$$

$$\alpha \mu_a \frac{Y_a}{\upsilon_a} = \alpha \mu_m \frac{Y_m}{\upsilon_m}, \quad \text{(A.6)}$$

$$\alpha \mu_s \frac{Y_s}{\upsilon_s} = \alpha \mu_m \frac{Y_m}{\upsilon_m}, \quad \text{(A.7)}$$

and

$$-\dot{\mu}_m + \mu_m \rho = \mu_a \alpha \frac{Y_a}{K} + \mu_s \alpha \frac{Y_s}{K} + \mu_m \left(\alpha \frac{Y_m}{K} - \delta \right). \quad \text{(A.8)}$$

The sectoral allocation of capital and relative prices

From combining $(A.4)$ and $(A.6)$, we obtain

$$\upsilon_a = \upsilon_m \frac{l_a}{l_m},$$

and combining $(A.5)$ and $(A.7)$, we obtain

$$\upsilon_s = \upsilon_m \frac{l_s}{l_m}.$$

We substitute υ_a^* and υ_s^* in (3.2), and taking into account (3.3), the optimal capital share in the manufacturing sector is

$$\upsilon_m = l_m, \quad \text{(A.9)}$$

which implies

$$\upsilon_a = l_a, \tag{A.10}$$
$$\upsilon_s = l_s. \tag{A.11}$$

By assuming that the manufacturing good is the numerarie and dividing equations $(A.4)$ by $(A.5)$, and combining (3.3), $(A.9)$, $(A.10)$ and $(A.11)$, we obtain the relative prices

$$p_a \equiv \frac{\mu_a}{\mu_m} = \frac{Y_m l_a}{Y_a l_m} = \frac{A_m}{A_a}, \tag{A.12}$$
$$p_s \equiv \frac{\mu_s}{\mu_m} = \frac{Y_m l_s}{Y_s l_m} = \frac{A_m}{A_s}. \tag{A.13}$$

Note that the relative prices are the ratio between the co-state variables.

The GDP is obtained by substitution of (4.2) and (4.3) in $(A.1)$. Firstly, we substitute (4.2) in (3.1) to obtain

$$Y_i = A_i K^{\alpha} l_i, \tag{A.14}$$

and GDP

$$Y = p_a Y_a + p_s Y_s + Y_m. \tag{A.1}$$

By combining with (4.3), $(A.14)$ and $(A.1)$, we obtain

$$Y = A_m \; K^{\alpha} \left(l_a + l_m + l_m \right),$$

and, given (3.3), we obtain that

$$Y = A_m \; K^{\alpha}.$$

The Euler equation

From (4.5), we obtain C_a and C_s as functions of C_m and the relative prices

$$C_i = \left(\frac{\eta_i}{\eta_m} \right)^{\epsilon} p_i^{-\epsilon} C_m \text{ for } i = a, s.$$

We then substitute these equation in (3.7) and combining with $(A.3)$, we obtain

$$\left(1 + x_a + x_s\right) C_m = \mu_m^{-1}. \tag{A.2}$$

Substituting $(A.2)$ in (4.6) we obtain

$$C = \mu_m^{-1},$$

where total expenditure is a function of the co-state variable corresponding to the constraint (3.5). By substituting (4.2) in (3.1), and combining with $(A.8)$, we obtain the growth rate of the co-state variables μ_m as follows

$$-\hat{\mu}_m = \alpha A_m K^{\alpha - 1} - (\rho + \delta). \tag{A.3}$$

We then log-differentiate $C = \mu_m^{-1}$ and combine with $(A.3)$, we obtain (4.10) .
Proof of Proposition 4.1. If $\omega_m > 0$, then the dynamic system is

$$\begin{aligned}
\hat{z} &= z^{\alpha-1} - \frac{c}{z} - \delta - \frac{\gamma\phi_m - \omega_m \ln u_m}{(1-\alpha)}, \\
\hat{c} &= \alpha z^{\alpha-1} - (\rho+\delta) - \frac{\gamma\phi_m - \omega_m \ln u_m}{(1-\alpha)}, \\
\hat{v}_m &= (\phi_m - 1)\gamma - \omega_m \ln v_m,
\end{aligned}$$

From equation $\hat{v}_m$, it follows that there is a unique steady value such that

$$v_m = \exp\left(-\frac{(1-\phi_m)\gamma}{\omega_m}\right),$$

and substituting in $\hat{z}$ and $\hat{c}$, we obtain

$$\begin{aligned}
\hat{z} &= z^{\alpha-1} - \frac{c}{z} - \delta - \frac{\gamma}{(1-\alpha)}, \\
\hat{c} &= \alpha z^{\alpha-1} - (\rho+\delta) - \frac{\gamma}{(1-\alpha)}.
\end{aligned}$$

The steady state, it must be satisfied that $\hat{z} = \hat{c} = 0$, implying that

$$\begin{aligned}
0 &= z^{\alpha-1} - \frac{c}{z} - \delta - \frac{\gamma}{(1-\alpha)}, \\
0 &= \alpha z^{\alpha-1} - (\rho+\delta) - \frac{\gamma}{(1-\alpha)}.
\end{aligned}$$

Solving the system for z and c, we obtain

$$\begin{aligned}
z^* &= \left(\alpha\frac{(1-\alpha)}{\gamma + (1-\alpha)(\rho+\delta)}\right)^{\frac{1}{1-\alpha}}, \\
c^* &= \frac{(\gamma + \rho + (1-\alpha)\delta)}{\alpha}\left(\frac{\alpha(1-\alpha)}{\gamma + (1-\alpha)(\delta+\rho)}\right)^{\frac{1}{1-\alpha}}.
\end{aligned}$$

If $\varpi_m = 0$, then the dynamic system at the steady state is

$$\begin{aligned}
\hat{z} &= z^{\alpha-1} - \frac{c}{z} - \delta - \frac{\phi_m\gamma}{(1-\alpha)}, \\
\hat{c} &= \alpha z^{\alpha-1} - (\rho+\delta) - \frac{\phi_m\gamma}{(1-\alpha)}.
\end{aligned}$$

At the steady state, we obtain

$$\begin{aligned}
z^* &= \left(\alpha\frac{(1-\alpha)}{\phi_m\gamma + (1-\alpha)(\rho+\delta)}\right)^{\frac{1}{1-\alpha}}, \\
c^* &= \frac{(\phi_m\gamma + \rho + (1-\alpha)\delta)}{\alpha}\left(\frac{\alpha(1-\alpha)}{\phi_m\gamma + (1-\alpha)(\delta+\rho)}\right)^{\frac{1}{1-\alpha}}.
\end{aligned}$$

Proof of Proposition 4.2. If $\omega_m > 0$, there are two state variables and one variable control. Using (4.12), (4.13), and (4.14), we obtain the following Jacobian matrix evaluated at the steady state

$$J = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{pmatrix},$$

where

$$\begin{array}{lll} a_{11} \equiv \frac{\partial \hat{z}}{\partial z} = \rho, & a_{12} \equiv \frac{\partial \hat{z}}{\partial c} = -1, & a_{13} \equiv \frac{\partial \hat{z}}{\partial v_m} = \frac{\omega_m}{1-\alpha}\frac{z^*}{v_m^*}, \\ a_{21} \equiv \frac{\partial \hat{c}}{\partial z} = \alpha\left(\alpha-1\right) z^{*\alpha-2}c^*, & a_{23} \equiv \frac{\omega_m}{1-\alpha}\frac{c^*}{v_m^*}, & a_{33} \equiv \frac{\partial \hat{v}_m}{\partial v_m} = -\omega_m. \end{array}$$

It is immediate to see that the eigenvalues are $\lambda_1 = -\omega_m$, and the two roots λ_2 and λ_3 are the solution of the following equation

$$Q\left(\lambda\right) = \lambda^2 - \lambda\left(\rho\right) + a_{21} = 0,$$

where the solutions are

$$\lambda_2, \lambda_3 = \frac{\rho \pm \sqrt{\rho^2 - 4a_{21}}}{2}.$$

Insofar as $a_{21} < 0$ and $\rho > 0$, it follows that one of the roots, for example λ_2 is always negative and the other one, λ_3, is positive. So, λ_1, $\lambda_2 < 0$ and $\lambda_3 > 0$. This result implies that there is a two-dimensional stable manifold in (z, c, v_3) space.

On the other hand, If $\omega_m = 0$, there is one state variable and one control variable. Using (4.12), (4.13), we obtain the following Jacobian matrix evaluated at the steady state

$$J = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix},$$

where

$$\begin{array}{ll} b_{11} \equiv \frac{\partial \hat{z}}{\partial z} = \rho, & b_{12} \equiv \frac{\partial \hat{z}}{\partial c} = -1, \\ b_{21} \equiv \frac{\partial \hat{c}}{\partial z} = \alpha\left(\alpha-1\right) z^{*\alpha-2}c^*, & b_{22} \equiv \frac{\partial \hat{c}}{\partial c} = 0. \end{array}$$

As

$$\det J = \lambda_1\lambda_2 = -\left(b_{21}\right)\left(b_{12}\right) < 0,$$

the eigenvalues of the system are real numbers of opposite signs, and the steady state is saddle path stable.

B. Estimation of the technology

Solution of differential equation

We then pose the law of motion of productivity in the i sector as follows

$$\frac{\dot{A}_i}{A_i} = \phi_i + \omega_i \ln\left(\frac{1}{v_i}\right),$$

where we define the inverse of the distance across sectors and the frontier as follows

$$v_i = \frac{A_i}{A}. \tag{B.1}$$

By taking the log-derivative of $(B.1)$, we obtain that the law of motion of technological gaps is

$$\dot{v}_i = (\phi_i - \gamma)\, v_i - \omega_i \ln(v_i)\, v_i. \tag{B.2}$$

We rewrite $(B.2)$ as follows

$$\frac{dv_i}{(\phi_i - \gamma)\, v_i - \omega_i \ln(v_i)\, v_i} = dt, \tag{B.3}$$

then $(B.3)$ can be integrated after a single substitution. Let

$$m = \phi_i - \gamma - \omega_i \ln(v_i),$$

where

$$\frac{dm}{dv_i} = \frac{-\omega_i}{v_i} \rightarrow \frac{dv_i}{v_i} = \frac{dm}{-\omega_i}.$$

Substituting in$(B.3)$, integrating and solving the integral equation,

$$\frac{1}{-\omega_i}\int \frac{1}{m_i} dm_i = \int dt,$$

we obtain

$$m_i = e^{-\omega_i(t+c)},$$

We substitute back into the m_i to obtain

$$v_i = \exp\left(\frac{\phi_i - \gamma}{\omega_i} + e^{\omega_i(c-t)}\right), \tag{B.4}$$

and substituting $(B.4)$ in $(B.1)$,we finally obtain

$$A_i = \exp\left(\frac{\phi_i - \gamma}{\omega_i} + e^{\omega_i(c-t)}\right) A. \tag{B.5}$$

Estimation Procedures: Using EUKLEMS (1970-2005)

From the definition of $(B.5)$, we can estimate ω_i and ϕ_i using the data on the TFP growth rates of the agriculture, manufacturing, and services sectors from the EUKLEMS database that covers 1970-2005. To this end, we estimate the following equation which derives from $(B.5)$ and our assumption that the technology frontier grows at a constant rate in equation (2.1). Taking logs in $(B.5)$ we obtain

$$\ln A_i = \frac{\phi_i - \gamma}{\omega_i} + \ln A_0 + e^{\omega_i(c-t)} + \gamma t,$$

and normalizing the initial stock in the frontier to one, $A_0 = 1$, we can estimate the following system of equations

$$\begin{aligned} \ln A_a &= \alpha_a + \beta_a t + e^{-\delta_a(t-c_a)}, \\ \ln A_m &= \alpha_m + \beta_m t + e^{-\delta_m(t-c_m)}, \\ \ln A_s &= \alpha_s + \beta_s t + e^{-\delta_s(t-c_s)}, \end{aligned} \tag{B.6}$$

where

$$\alpha_i = \frac{\phi_i - \gamma}{\omega_i}; \ \beta_i = \gamma; \text{ and } \delta_i = \omega_i.$$

We estimate the parameters in $(B.6)$ constrained to $\beta_i = \gamma$ for all sector. We use non-linear squares to estimate $(B.6)$. We report the results in Table 1.

Estimation Procedures: Relative prices (1929-2005)

In order to have an estimation of ω_i and ϕ_i prior to 1970, we estimate the parameters in (2.2) by using relative prices as long as these are related to the dynamic of relative productivity in (4.3). The major problem in estimating adoption rates arises from the empirical specification of our law of motions, which depends on an unobservable factor (the technological frontier). In order to use relative prices as a proxies of sectoral TFP, here, we assume that the manufacturing sector is a proxy for the frontier.[13] Given this assumption, we know from Kruguer (2008) that the annual manufacturing TFP growth rate for the period 1870-2000 is around 0.014. Therefore,

$$\frac{\dot{A}_m}{A_m} \equiv \phi_m = 0.014.$$

From (4.3), we know that

$$\frac{\dot{p}_a}{p_a} = \frac{\dot{A}_m}{A_m} - \frac{\dot{A}_a}{A_a} = \phi_m - \phi_a - \omega_a \ln(p_a), \tag{B.7}$$

and

$$\frac{\dot{p}_s}{p_s} = \frac{\dot{A}_m}{A_m} - \frac{\dot{A}_s}{A_s} = \phi_m - \phi_s - \omega_s \ln(p_s). \tag{B.8}$$

We solve $(B.7)$ and $(B.8)$ to obtain

[13] In terms of our model, we can assume that the manufacturing sector is in the frontier. That is, TFP in manufacturing has reached the technology frontier.

$$\ln p_a = \frac{\phi_m - \phi_a}{\omega_a} + e^{-\omega_a(C_a+t)}, \tag{B.9}$$

and

$$\ln p_s = \frac{\phi_m - \phi_s}{\omega_s} + e^{-\omega_s(C_s+t)} \tag{B.10}$$

where C_a and C_s are constants of integration. We use nonlinear seemingly unrelated regression to estimate C_a and C_s; ϕ_a and ϕ_s; ω_a, and ω_s from (B.9) and (B.10) by fitting the following system of equations:

$$\ln p_i = \alpha_i + e^{\beta_i(C_i+t)} \text{ for } i = a, s,$$

where

$$\alpha_i = \frac{\phi_m - \phi_i}{\omega_i}; \text{ and } \beta_i = -\omega_i,$$

and the constant C_a and C_s subject to the constraint $\phi_m = 0.0140$. Tables 2 reports the estimated values of ω_a, and ω_s and the values of ϕ_a and ϕ_s, which are obtained by nonlinear combinations of the estimated parameters $\hat{\alpha}_i$ and $\hat{\beta}_i$ using nlcom (nonlinear combination) command in STATA.

Figures

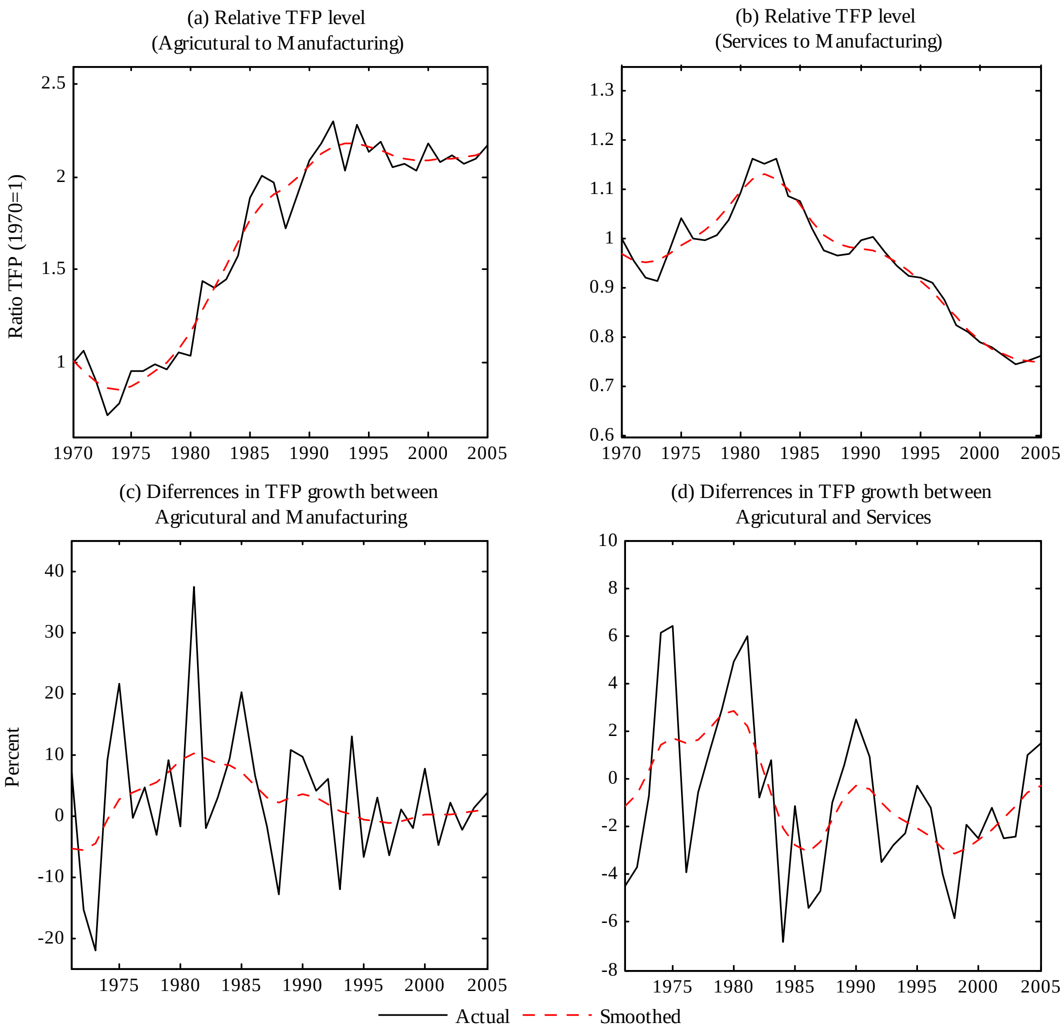

Figure 1: Relative Sectoral TFP (Levels and Growth). Panel (a) plots the ratio of TFP levels between agriculture and manufacturing sectors. Panel (b) plots the ratio of TFP levels between agriculture and services sectors. Panel (c) an (d) plot the actual and smoothed growth rates of these TFP ratios, respectively. We use the Hodrick-Prescott to filter actual data to obtain a trend component. We set the smooth parameter, lambda, equal to 6.25

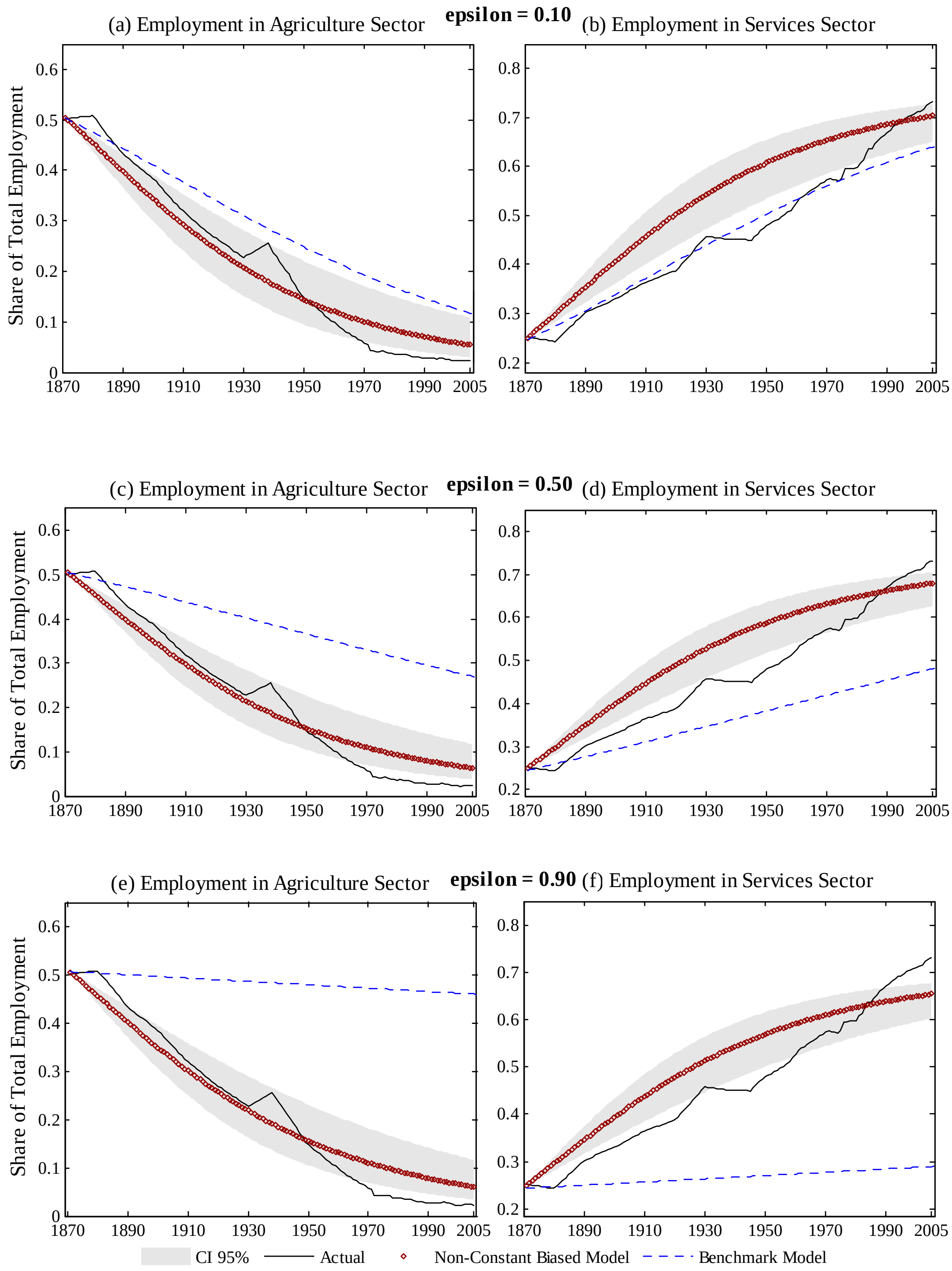

Figure 2: Patterns of Structural Change. Figure 2 shows the simulated patters of labor shares obtained by assuming three different values of the elasticity of substitution. Thus, this figure shows the robustness of both models to changes in the elasticity. Here, Figure 2 also plots the predicted labor shares in the 95 percent confidence intervals values for the point estimates (shaded area).

Tables

Table 1:
Estimation of adoption rates EUKLEMS 1970-2005.

	Agriculture	Manufacturing	Services
ω	0.026***	-0.001	0.017***
	(-0.002)	(0.002)	(0.003)
φ	0.034***	0.011***	0.039***
	(0.002)	(0.002)	(0.007)
γ	0.012***	0.012***	0.012***
	(0.002)	(0.002)	(0.002)
R^2	0.93	0.63	0.76

Standard errors in parentheses
* p<0.05, ** p<0.01, *** p<0.001

Table 2: Estimation of adoption rates. Relative Agricultural and Services Prices

	Agriculture (mean)	Services (mean)	Agriculture (a)	Agriculture (b)	Services (a)	Services (b)
ω	0.005***	0.013***	0.004	0.006	0.005	0.021
	(0.001)	(0.004)				
φ	0.019***	0.025***	0.017	0.019	0.024	0.026
	(0.001)	(0.0005)				
R^2	0.42	0.84				

Standard errors in parentheses. (a) and (b) are the lower and upper values in the confidence interval.
* p<0.05, ** p<0.01, *** p<0.001

Table 3: Calibration

Parameters	Values	Targets	data
α	0.315	Labor income share	0.685
γ	0.0137	GDP growth rate (1870-2005)	0.02
ρ	0.03	Capital-output ratio	3
δ	0.05	Interest rate	0.052
ε	0.5	By assumption	
η_a	0.01	Consumption expenditure share in food	0.03
η_s	0.9	Consumption expenditure share in services	0.75
v_a	0.0013	Labor share in agriculture (1870)	50%
v_s	0.1803	Labor share in services (1870)	25%
v_m	1	Normalized	
ω_a	0.004	Estimation from relative agricultural price	
ω_s	0.01	Estimation from relative services price	
ω_m	0	By assumption	
φ_a	0.019	Estimation from relative agricultural price	
φ_s	0.025	Estimation from relative services price	
φ_m	1	Normalized	

Note: We perform three numerical simulations for three different values of the elasticity, and accordingly, the initial values of technological gap were set. Reported values in Table 3 are set by assuming ε=0.5. For the values of the elasticity equal to 0.1, and 0.9, we set the initial gaps in agriculture sector equal to 0.000005, and 0.001; and the initial gaps in the services sector are 0.053 and 0.000012, respectively. Finally, the values of ηa are 0.00001 and 0.0082 and the values of ηs are 0.9999 and 0.77 for ε=0.1 and ε=0.9, respectively.

Table 4: Accuracy Measures of Simulated Structural Change.

Case	Accuracy measure	Agriculture		Manufacturing		Services	
		(a)	(b)	(a)	(b)	(a)	(b)
$\varepsilon = 0.10$	RMSE	**0.032**	0.071	**0.044**	0.044	0.046	**0.030**
	AIC	**-546**	-328	**-459**	-457	-446	**-559**
$\varepsilon = 0.50$	RMSE	**0.032**	0.123	**0.030**	0.039	**0.048**	0.054
	AIC	**-546**	-183	**-566**	-494	**-438**	-406
$\varepsilon = 0.90$	RMSE	**0.035**	0.157	**0.037**	0.039	**0.050**	0.119
	AIC	**-518**	-116	**-504**	-493	**-426**	-191
Observations		135		135		135	

We calculate these accuracy-measures by regressing actual labor shares in agriculture, manufacturing and services on those predicted shares by our non-constant biased model and the benchmark for the period 1870-2005. Here, we report the results for three values of the elasticity of substitution. Column (a) reports the statistical measures for the non-constant biased model. Column (b) reports the statistical measures for the benchmark model.

Table 5: Average growth rate: RES between Agriculture and Services

		(a)	(b)	(c)	(d)	(e)
$\varepsilon = 0.10$	1870-1930	-2.32	-2.78	-1.78	1.20	0.77
	1930-1950	-2.30	-2.35	-1.78	1.02	0.77
	1950-2005	-3.98	-2.03	-1.78	0.51	0.45
$\varepsilon = 0.50$	1870-1930	-2.32	-2.67	-0.96	1.15	0.41
	1930-1950	-2.30	-2.21	-0.96	0.96	0.42
	1950-2005	-3.98	-1.86	-0.96	0.47	0.24
$\varepsilon = 0.90$	1870-1930	-2.32	-2.60	-0.19	1.12	0.08
	1930-1950	-2.30	-2.18	-0.19	0.95	0.08
	1950-2005	-3.98	-1.97	-0.19	0.49	0.05

Column (a) reports the actual average annual growth rates. Columns (b) and (c) report the predicted average growth rates based on the non-constant and the benchmark model, respectively. Columns (d) and (e) report the fraction of actual growth rate that is replicated by the model (b) and (c), respectively.

Table 6: Average growth rate: RES between Agriculture and Manufacturing

		(a)	(b)	(c)	(d)	(e)
$\varepsilon = 0.10$	1870-1930	-1.74	-1.53	-0.82	0.88	0.47
	1930-1950	-2.87	-1.77	-1.09	0.62	0.38
	1950-2005	-2.44	-1.72	-1.31	0.71	0.54
$\varepsilon = 0.50$	1870-1930	-1.74	-1.52	-0.39	0.88	0.22
	1930-1950	-2.87	-1.70	-0.47	0.59	0.16
	1950-2005	-2.44	-1.59	-0.54	0.65	0.22
$\varepsilon = 0.90$	1870-1930	-1.74	-1.57	-0.07	0.90	0.04
	1930-1950	-2.87	-1.78	-0.07	0.62	0.03
	1950-2005	-2.44	-1.79	-0.08	0.73	0.03

Column (a) reports the actual average annual growth rates. Columns (b) and (c) report the predicted average growth rates based on the non-constant and the benchmark model, respectively. Columns (d) and (e) report the fraction of actual growth rate that is replicated by the model (b) and (c), respectively.