INVARIANT MEASURES IN HOLOMORPHIC DYNAMICS

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The following result was proved by Lyubich in 1983, and independently by Freire, Lopez, and Manñé, and generalizes an earlier result for polynomials due to Brolin:

Theorem 1 (Lyubich, 1983)**.** *Let $f : \widehat{\mathbb{C}} \to \widehat{\mathbb{C}}$ be a rational function of degree $d \geq 2$. Then there exists a unique invariant ergodic probability measure μ of maximal entropy* $\log(d)$*. The support of the measure equals the Julia set $J(f)$.*

Lyubich also gives an explicit recursive algorithm for constructing the measure μ: start with a Dirac mass δ_w at an arbitrary point $w \notin \mathcal{E}(f)$, and repeatedly pull back this measure:

$$\mu_n = \frac{1}{d^n} \sum_{z : f^n(z) = w} \delta_z,$$

where possible multiplicities of preimages are taken into account. Then the measures μ_n converge weakly to the equilibrium measure μ.

This construction provides an insightful understanding of the dynamics of f and the fundamental role of the measure μ, but it does not offer a very clear description of the measure μ nor of the action of f on the Julia set.

In this course, we will discuss the proof of the result by Lyubich, and discuss several alternative ways of defining the measure μ, some only valid for subclasses or specific examples of rational functions. We will consider these alternative definitions for two reasons: they may give a more explicit characterization of the measure μ, and their construction may be more easily adapted to the iteration of other classes of holomorphic maps, such as entire functions and holomorphic maps in several complex variables.