Non-autonomous affine dynamics and partition functions

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Abstract

Partition functions encode the statistical properties of lattice and graph models in statistical mechanics and can be viewed as analytic functions of their physical parameters. Zero-free regions of their complex analytic continuation are tied both to correlation decay and to the existence of efficient approximation algorithms, a connection that has recently attracted interest in computational complexity. In the hard-core model on graphs of maximum degree at most d, correlation decay, zero-freeness, and efficient algorithms occur in the same parameter regime, suggesting a more general relation between these phenomena.

A result of Guus Regts shows that, for families of bounded-degree graphs closed under taking induced subgraphs, zero-free regions imply strong spatial mixing (SSM), a robust notion of exponential decay of correlations. In current work with Han Peters and Guus Regts, we prove a complementary statement: the strictly stronger property of very strong spatial mixing (VSSM) at a given parameter forces the existence of a zero-free neighbourhood.

Our proof relies on translating zeros of partition functions to the event that a certain probability ratio equals -1, and recursively expressing this ratio as a function of the corresponding ratios on smaller subgraphs. On trees, these "ratio maps" can be written as compositions of Möbius transformations

$$f_n(z) = \frac{\lambda_n}{1+z},$$

for a sequence $(\lambda_n)_n$ of positive reals determined by boundary conditions. VSSM implies that, for extremal boundary conditions (either all leaves "in" or "out"), the associated sequences $(\lambda_n)_n$ get exponentially close. Viewing the compositions of the f_n as a non-autonomous dynamical system, we construct a sequence of non-autonomous changes of coordinates that conjugate these maps to affine maps. Studying these sequences of affine maps and coordinate changes then yields uniform contraction for the tree ratio maps. Critically, this contraction persists for initial values in a complex neighbourhood, ensuring that no orbit can reach -1.

In this talk I will give an overview of known relations between zero-free regions, decay of correlations, and algorithms, with an emphasis on the role of rational iteration and normal families in establishing and relating decay of correlations and zero-free regions.