Asymptotic Distribution Free Interval Estimation

for an Intraclass Correlation Coefficient with Applications to Longitudinal Data

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Abstract. Confidence intervals for the intraclass correlation coefficient (ICC) have been proposed under the assumption of multivariate normality. We propose confidence intervals which do not require distributional assumptions. We performed a simulation study to assess the coverage rates of normal theory (NT) and asymptotically distribution free (ADF) intervals. We found that the ADF intervals performed better than the NT intervals when kurtosis was greater than 4. When violations of distributional assumptions were not too severe, both the intervals performed about the same. The point estimate of the ICC was robust to distributional violations. We provide R code for computing the ADF confidence intervals for the ICC.

Keywords: intraclass correlation, confidence intervals, autocorrelation, distributional assumptions, model assumptions

The intraclass correlation coefficient (ICC) is commonly used in a variety of applications, including familial resemblance and twin studies, reliability theory, and as a measure of the degree of correlation among repeated measurements within an individual. The ICC is also commonly used in multilevel or hierarchical data (e.g., students in classrooms). Here, the focus is on the use of the ICC in longitudinal research studies. In the context of longitudinal data the ICC is the proportion of the total variance in the outcome that is attributable to between individual variation.

For t-repeated measures on a random sample of N respondents from the population of interest, the ICC can be computed under a variety of model assumptions (for a review, see Kistner & Mueller, 2004). The simplest model that can be considered for longitudinal data is the two-way mixed effects analysis of variance (ANOVA) model. Letting Y_{ij} denote the response of individual i at time j the model assumes that

$$Y_{ij} = \mu_j + \beta_i + \varepsilon_{ij} \tag{1}$$

where μ_j is the population mean (i.e., fixed effect) of all individuals at time j, β_i are the between individual effects (i.e., random effect), and ε_{ij} are the residual effects. Notice that there is no interaction between the fixed time effects and the random individual effects.

In matrix form, Equation (1) can be written as

$$\mathbf{Y}_i = \boldsymbol{\mu} + \beta_i \mathbf{1} + \boldsymbol{\varepsilon}_i, \tag{2}$$

where $\mathbf{Y}_i$, $\mathbf{1}$, $\boldsymbol{\mu}$, and $\boldsymbol{\varepsilon}_i$ are $t \times 1$ vectors. The model then assumes that

$$\begin{pmatrix} \beta_i \\ \boldsymbol{\varepsilon}_i \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ \mathbf{0} \end{pmatrix}, \begin{pmatrix} \sigma_\beta^2 & \mathbf{0} \\ \mathbf{0} & \sigma_\varepsilon^2 \mathbf{I} \end{pmatrix} \right), \tag{3}$$

where $\mathbf{I}$ is a $t \times t$ identity matrix, σ_β^2 is the variance of the between individual effects, and σ_ε^2 is the variance of the residuals. As a result, under the model, the covariance matrix $\boldsymbol{\Sigma}$ of the response variables has the form

$$\Sigma = \sigma_\beta^2 \mathbf{1}\mathbf{1}' + \sigma_\varepsilon^2 \mathbf{I} \tag{4}$$

This covariance structure is generally referred to as compound symmetry. In this structure, the variances for all time points, σ_t^2, are equal to $\sigma_\beta^2 + \sigma_\varepsilon^2$ and the covariances between all time points, $\sigma_{tt'}$, are equal to σ_β^2, implying a common correlation, ρ, among all time points. Under the model, the common correlation, referred to as the ICC, is

$$\rho = \frac{\sigma_{tt'}}{\sigma_t^2} = \frac{\sigma_\beta^2}{\sigma_\beta^2 + \sigma_\varepsilon^2} \tag{5}$$

and Equation (4) can be written as a function of the intraclass correlation coefficient as

$$\Sigma = \sigma^2 \left[\rho \mathbf{1}\mathbf{1}' - (1-\rho)\mathbf{I} \right] \tag{6}$$

where $\sigma^2 = \sigma_t^2 = \sigma_\beta^2 + \sigma_\varepsilon^2$ is the common variance of the response variables (i.e., it does not depend on t).

Also, when the population covariance structure is compound symmetric, Equation (5) may be written in matrix form as

$$\rho = \frac{\mathbf{1}'\boldsymbol{\Sigma}\mathbf{1} - \operatorname{tr}(\boldsymbol{\Sigma})}{(t-1)\operatorname{tr}(\boldsymbol{\Sigma})}. \tag{7}$$

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Notice that the correlation coefficient given in Equation (5) – or equivalently in Equation (7) – is the proportion of the total variance of a measurement that is attributable to individuals (i.e., the proportion of variance explained by the grouping structure in the population). Thus, in the terminology of McGraw and Wong (1996), Equation (7) is a *consistency* type of ICC for *single* measurements. A consistency-type of ICC for average measurements (i.e., the proportion of variance of the average across the *t* measurements that is attributable to between individual variation) is

$$\alpha = \left(\frac{t}{t-1}\right)\left(1 - \frac{\mathrm{tr}(\mathbf{\Sigma})}{\mathbf{1'\Sigma 1}}\right). \quad (8)$$

We use the symbol α in Equation (8) because this is the formula for coefficient alpha (van Zyl, Neudecker, & Nel, 2000). Thus, coefficient alpha is a special type of ICC. The relationship between both ICCs – the one for single measurements given in Equation (7) and the one for average measurements given in Equation (8) – is

$$\alpha = \frac{t\rho}{1 + (t-1)\rho}. \quad (9)$$

Using the results of an ANOVA table for a two-way mixed effects ANOVA model, an ordinary least squares (OLS) estimate of ρ may be easily obtained as

$$\hat{\rho} = \frac{MS_B - MS_E}{MS_B + (t-1)MS_E}, \quad (10)$$

where MS_B is the between individual mean squares and MS_E is the within individual mean squares from the model. Also, the OLS estimate of α is obtained as

$$\hat{\alpha} = \frac{MS_B - MS_E}{MS_B}. \quad (11)$$

Furthermore, when the measurements, $\mathbf{Y}$, are multivariate normal, Equations (10) and (11) are also the maximum likelihood (ML) estimators of the ICC in Equation (7) and Equation (8), which can be alternatively estimated as

$$\hat{\rho} = \frac{\mathbf{1'S1} - \mathrm{tr}(\mathbf{S})}{(t-1)\ \mathrm{tr}(\mathbf{S})}, \quad (12)$$

and

$$\hat{\alpha} = \left(\frac{t}{t-1}\right)\left(1 - \frac{\mathrm{tr}(\mathbf{S})}{\mathbf{1'S1}}\right), \quad (13)$$

respectively, where $\mathbf{S}$ is the $t \times t$ sample covariance matrix among the responses, and $\mathbf{1}$ is $t \times 1$ vector of ones.

As with any point estimate, the sample ICCs given by equations (12) and (13) are subject to variability around the true parameter value, particularly in small samples. Thus, a better appraisal of these ICCs is obtained using interval estimators. Several methods for computing confidence intervals for ICCs under the assumption that the distribution of the measurements is multivariate normal have been proposed (see Donner, 1986; and Donner & Wells, 1986 for a review of various procedures). Also, there is an extensive literature on additional specific procedures for computing confidence intervals under normality assumptions for coefficient alpha (for a review, see Duhachek & Iacobucci, 2004).

Yuan and Bentler (2002) showed that the normal theory (NT) confidence intervals for coefficient alpha are robust to violations of normality assumptions under certain conditions. However, these conditions cannot be verified in practice. Asymptotic confidence intervals that do not require an assumption of multivariate normality were proposed by Yuan, Guarnaccia, and Hayslip (2003) in the context of an application. Recently, Maydeu-Olivares, Coffman, and Hartmann (2007) have shown by means of simulation studies that the Yuan et al. (2003) asymptotic distribution free (ADF) confidence intervals for coefficient alpha are preferable to NT intervals when sample size is larger than 100. For sample sizes larger than 100, the ADF intervals accurately portray the variability of sample coefficient alpha, even in situations of high kurtosis or skewness. In contrast, NT intervals for coefficient alpha underestimate the variability of sample coefficient alpha (and generally underestimate the population alpha) when the absolute value of skewness is larger than 1 or when kurtosis is larger than 4.[1]

In many situations, measurements are not continuous, although the distribution may be approximately normal. In this situation, the NT confidence intervals may maintain nominal coverage rates. However, behavioral data is often not normally distributed (Micceri, 1989) and there are situations in which the measurements may demonstrate considerable skewness or kurtosis. In these situations, we hypothesize that the NT confidence intervals for the ICC will not be robust to violations of multivariate normality. Therefore, we propose an ADF interval estimator for the ICC.

Purpose of the Current Study

This paper focuses on the single-measure consistency-type ICC given in Equation (12). The purpose of the current study is to derive ADF confidence intervals for this ICC. Using simulations, we compare its performance against the performance of confidence intervals (CIs) based on normality assumptions (i.e., NT CIs), under a variety of conditions of skewness and kurtosis of the measurements.

Asymptotic Distribution Free Confidence Intervals for the Single-measure Consistency-type ICC

Let $\mathbf{s}$ = vecs($\mathbf{S}$) be a vector of nonduplicated (i.e., unique) elements of $\mathbf{S}$; similarly let $\boldsymbol{\sigma}$ = vecs($\mathbf{\Sigma}$), where both $\mathbf{s}$ and

[1] The skewness and kurtosis of a standard normal distribution are 0 and 3, respectively.

$\boldsymbol{\sigma}$ are of dimension $q = t(t+1)/2$, the number of unique variances and covariances. Then the vector $\sqrt{N}\mathbf{s}$ is asymptotically normally distributed with mean $\boldsymbol{\sigma}$ and covariance matrix $\boldsymbol{\Gamma}$ of dimensions $q \times q$. Because the ICC is a function of $\mathbf{s}$, the ICC is asymptotically normally distributed with mean ρ and variance

$$\varphi^2 = \frac{1}{N}\boldsymbol{\delta}'\boldsymbol{\Gamma}\boldsymbol{\delta} \tag{14}$$

where $\delta' = \dfrac{\partial\rho}{\partial\sigma'}$ is a $1 \times q$ vector of derivatives of ρ with respect to $\boldsymbol{\sigma}$. The elements of $\boldsymbol{\delta}$ are:

$$\frac{\partial\rho}{\partial\sigma_{ij}} = \begin{cases} -\dfrac{\mathbf{1}'\boldsymbol{\Sigma}\mathbf{1} - \operatorname{tr}(\boldsymbol{\Sigma})}{(t-1)\operatorname{tr}(\boldsymbol{\Sigma})^2} & \text{if} \quad i = j \\ \dfrac{2}{(t-1)\operatorname{tr}(\boldsymbol{\Sigma})} & \text{if} \quad i \neq j \end{cases} \tag{15}$$

Thus, in large samples an x% confidence interval for ρ can be obtained as

$$\left(\hat{\rho} - z_{x/2}\hat{\boldsymbol{\varphi}}; \hat{\rho} + z_{x/2}\hat{\boldsymbol{\varphi}}\right), \tag{16}$$

where $\hat{\boldsymbol{\varphi}}$ is the square root of the estimated asymptotic variance given in Equation (14), and $z_{x/2}$ is the $(1 - x/2)$% quantile of a standard normal distribution. Thus, for instance, $z_{x/2} = 1.96$ for a 95% confidence interval for ρ.

The $\boldsymbol{\Gamma}$ matrix differs depending on the distributional assumptions being made. Henceforth, we use $\boldsymbol{\Gamma}_{\text{NT}}$ to refer to the $\boldsymbol{\Gamma}$ matrix under normality assumptions and $\boldsymbol{\Gamma}_{\text{ADF}}$ to refer to the $\boldsymbol{\Gamma}$ matrix under ADF assumptions.[1]

Under normality assumptions,

$$\hat{\boldsymbol{\Gamma}}_{\text{NT}} = 2\mathbf{K}'(\mathbf{S} \otimes \mathbf{S})\mathbf{K}, \tag{17}$$

where $\mathbf{K}$ is the $t^2 \times q$ transition matrix for a symmetric matrix (Magnus & Neudecker, 1999) and ⊗ is the Kronecker product.

To estimate $\boldsymbol{\Gamma}_{\text{ADF}}$, let $\mathbf{y}_i$ be the $t \times 1$ vector of data for observation i, and $\bar{\mathbf{y}}$ be the $t \times 1$ vector of sample means. Also, let

$$\mathbf{s}_i = \operatorname{vecs}\left[(\mathbf{y}_i - \bar{\mathbf{y}})(\mathbf{y}_i - \bar{\mathbf{y}})'\right]$$

be a $q \times 1$ vector of squared deviations from the mean. Then, $\boldsymbol{\Gamma}_{\text{ADF}}$ can be estimated (Satorra & Bentler, 1994) as

$$\hat{\boldsymbol{\Gamma}}_{\text{ADF}} = \frac{1}{N-1}\sum_{i=1}^{N}(\mathbf{s}_i - \mathbf{s})(\mathbf{s}_i - \mathbf{s})'. \tag{18}$$

Using this equation, it may be shown following Maydeu-Olivares et al. (2007) that an estimate of the asymptotic variance under ADF assumptions can be obtained directly as

$$\hat{\varphi}^2_{\text{ADF}} = \frac{1}{N}\hat{\boldsymbol{\delta}}'\hat{\boldsymbol{\Gamma}}_{\text{ADF}}\hat{\boldsymbol{\delta}} = \frac{1}{N(N-1)}\sum_{i=1}^{N}\left(\hat{\boldsymbol{\delta}}'(\mathbf{s}_i - \mathbf{s})\right)^2 \tag{19}$$

Simulation Study

In the simulation study, data are generated using a properly specified covariance structure, but measurements vary in their degree of skewness and kurtosis. Thus, in this simulation study we investigate the robustness of the NT and ADF CIs for the ICC under investigation under violations of NT distributional assumptions when model assumptions are satisfied.

Method

We examined three different sample sizes (N = 100, 200, and 400), three different values for ρ (.2, .5, and .8), and the following combinations for skewness and kurtosis: (0, 2.50), (0, 3.88), (.41, 1.17), (.98, 2.80), (1.96, 4.84), (2.67, 8.11). We generated four time points according to a compound symmetric covariance structure and used r = 1000 replications for all 54 conditions. For each replication, we computed the sample ICC, the NT and ADF standard errors, and the NT and ADF 95% confidence intervals.

The procedure used to generate the data was similar to that of Muthén and Kaplan (1985, 1992) and is as follows:

1. Choose a population correlation matrix, **P**, and a set of thresholds.
2. Generate multivariate normal data with mean zero and the covariance matrix **P**.
3. Categorize the data using the set of thresholds.
4. Compute the true population covariance matrix, **Σ**, after categorization.
5. Compute the true population ICC using **Σ**.

The same set of thresholds was used for all variables. The thresholds were chosen so as to give the previously mentioned values of skewness and kurtosis.

For each condition, we computed the mean estimate of the sample ICC as

$$\overline{ICC} = \frac{\sum_{i=1}^{r} ICC_i}{r}, \tag{20}$$

the absolute bias as $\overline{ICC} - ICC$, and the relative bias as $(\overline{ICC} - ICC)/ICC$. We computed the coverage rates for both the NT and ADF 95% confidence intervals as the proportion of intervals that contained the population ICC.

Results

The ADF and NT coverage rates are presented in Figure 1 and Figure 2, respectively. The coverage rates should be as close to the nominal level of .95 as possible. Cov-

[1] In addition to the usual regularity conditions, the only additional assumption required for ADF estimation is that the eighth order moments of the distribution of the data are finite (Browne, 1984).

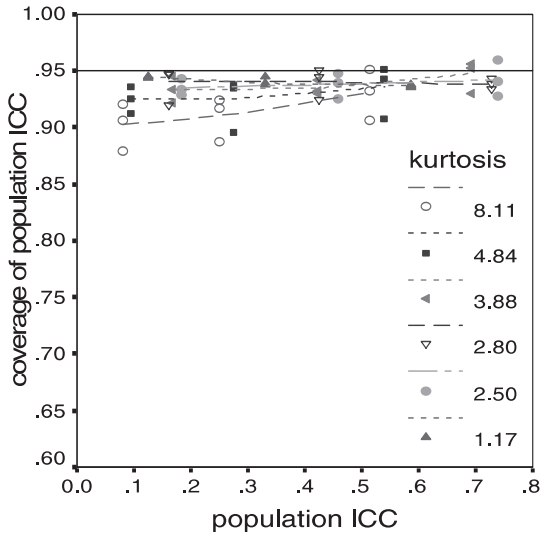

Figure 1. Coverage rates for asymptotically distribution free 95% confidence intervals by kurtosis under violation of distributional assumptions.

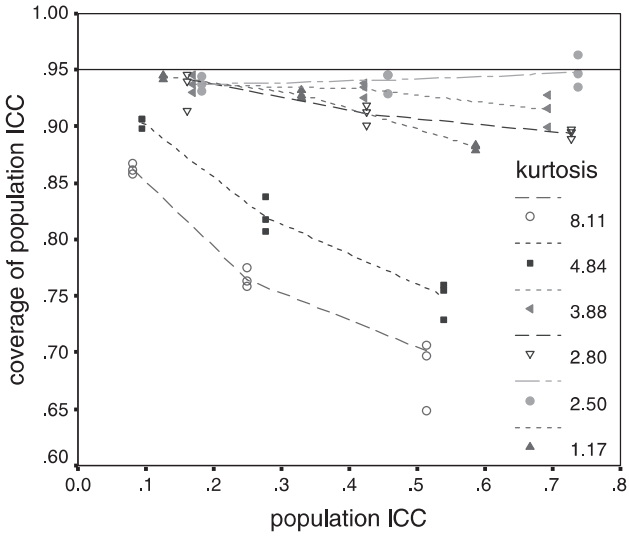

Figure 2. Coverage rates for normal theory 95% confidence intervals by kurtosis under violation of distributional assumptions.

erage greater than the nominal level indicates that the estimated confidence intervals are too wide and coverage less than the nominal level indicates that the confidence intervals are too narrow. For the former, the variability of the sample ICC is overestimated and for the latter the variability of the sample ICC is underestimated. In general, the coverage rates are less than the nominal level. For all but the two largest kurtosis conditions, the NT and ADF coverage rates are similar. For the two largest kurtosis conditions, the ADF coverage rates are much closer to the nominal level than the NT coverage rates. As kurtosis increases and the true population ICC increases, the NT coverage rates are poorer (see Figure 2). In contrast, the ADF coverage rates are poorer as the true population ICC decreases (see Figure 1).

Table 1 shows the coverage rates for both the NT and the ADF confidence intervals by kurtosis and sample size. Not surprisingly, as sample size increases, coverage rates improve. More importantly, in nearly all conditions, the ADF confidence intervals perform better than or as well as the NT confidence intervals. As may be expected, the NT coverage rates are best when kurtosis is approximately that of a normal distribution. For kurtosis lower than that of a standard normal distribution, the ADF coverage rates are better.

Table 1. Coverage rates for normal theory (NT) and asymptotically distribution free (ADF) confidence intervals by kurtosis and sample size

Kurtosis		Sample size		
		100	200	400
1.17	ADF	**.94**	**.94**	**.94**
	NT	.92	.92	.92
2.50	ADF	.93	**.94**	**.95**
	NT	**.94**	**.94**	**.95**
2.80	ADF	**.93**	**.95**	**.95**
	NT	.90	.92	.92
3.88	ADF	**.93**	**.94**	**.95**
	NT	.92	.93	.93
4.84	ADF	**.91**	**.94**	**.94**
	NT	.81	.83	.83
8.11	ADF	**.89**	**.92**	**.93**
	NT	.76	.78	.78

Note. **Bold** values indicate which estimator maintains better coverage rates.

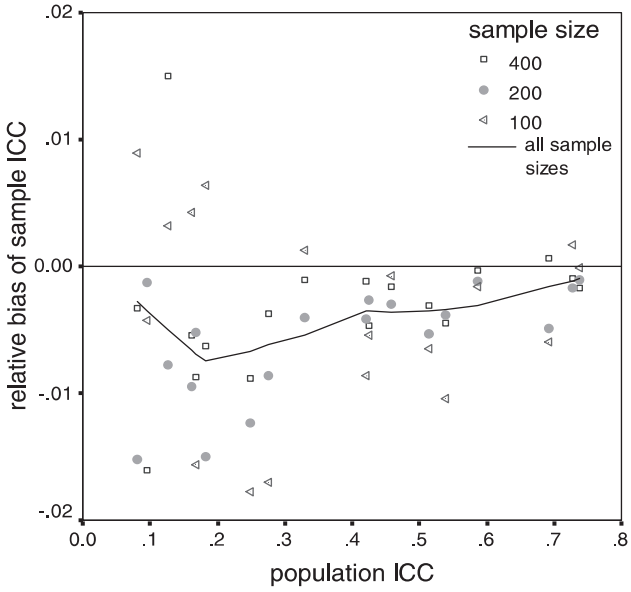

Figure 3. Relative bias of the sample intraclass correlation coefficient (ICC) point estimate by sample size under violation of distributional assumptions.

The point estimate of the sample ICC is slightly underestimated. However, for most conditions the underestimation is less than 1%. As the true population ICC increases, the slight bias disappears. Figure 3 shows the relative bias of the point estimate for the ICC by sample size.

Discussion

The results showed that the bias of NT confidence intervals under violation of distributional assumptions increased as kurtosis increased. In particular, bias was most apparent when the kurtosis was greater than 4. The results further showed that the ADF confidence intervals maintained better coverage under the more extreme values of skewness and kurtosis than did the NT confidence intervals. Violations of distributional assumptions affect the standard errors, and therefore the confidence intervals, of the parameter estimate. Our results showed that under the more severe violations of distributional assumptions, the NT standard errors, and therefore the NT confidence intervals, were affected. However, when the violation of distributional assumptions was not too severe, the ADF and NT procedures performed similarly.

Although our study has focused on longitudinal data, both the NT and ADF confidence intervals may be computed in other situations in which ICCs commonly arise, such as twin studies, or data with an hierarchical structure, such as students nested within classrooms. The ADF confidence intervals may also be computed for the ICC when it is used as a measure of interrater reliability (see Shrout, 1998; and McGraw & Wong, 1996 for further details on confidence intervals for the ICC when used as a measure of interrater reliability).

The assumption of compound symmetry is not likely to hold, particularly for longitudinal data. For longitudinal data, it is more likely that the correlation between two adjacent time points decreases over time. There are many covariance structures which commonly arise with longitudinal data (e.g., autoregressive) and the results we present are a starting point for further research on the effects of violations of model assumptions. Kistner and Muller (2004) developed an exact method for obtaining confidence intervals for the ICC that does not require the assumption of compound symmetry. In addition, they developed an approximation to the exact method based on the *F* distribution that is sufficiently accurate and computationally simpler. Both their exact method and their approximation rely on the assumption of multivariate normally distributed data. The robustness of these methods to violations of the multivariate normality assumptions has not been examined. In addition, we did not examine violations of distributional assumptions and model assumptions together.

In conclusion, we would suggest reporting confidence intervals for the ICC. To facilitate this practice we have included R code in the Appendix to compute the ADF confidence intervals. Further, we suggest examining the distribution of the sample data and if there is indication that the assumption of multivariate normality may be severely violated, we suggest computing the ADF confidence intervals.

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Appendix

R Code for Computing ADF CIs for ICC: Instructions

In order to compute the ADF CIs, the raw data matrix needs to be read into R in the usual manner. The name of the data matrix should replace "dat," which is **bolded** in the code below; or you should change the name of your data matrix to "dat." The number of subjects (subj) also needs to be specified.

```
samp.cov <- var(dat)

subj <- nrow(dat)
nvars <- nrow(samp.cov)
trS <- sum(diag(samp.cov))
ones <- matrix(c(1, 1, 1, 1), 4, 1)
summat <- t(ones)%*%samp.cov%*%ones

#Compute ADF ICC CIs
#Derivatives
delta <- matrix(0,nvars,nvars)
for (i in 1:nvars) {
  for (j in 1:i) {
    if(i==j) {
      delta[i,j] <- (-nvars/(nvars-1))%*%((summat-trS)/summat^2)
    }
    else {
      delta[i,j] <- (2*nvars/(nvars-1))%*%(trS/summat^2)
    }
    j <- j + 1
  }
  i <- i + 1
}

dvecs <- delta[row(delta)>=col(delta)]
svecs <- samp.cov[row(samp.cov)>=col(samp.cov)]
tmp3 <- 0.0
dev <- matrix(0,1,nvars)
sqdev <- matrix(0,nvars,nvars)
ybar <- apply(dat,2,mean)

for (i in 1:subj) {
  dev <- dat[i,]-ybar
  sqdev <- dev%*%t(dev)
  sivecs <- sqdev[row(sqdev)>=col(sqdev)]
  tmp2 <- (dvecs%*%(sivecs-svecs))^2
  tmp3 <- tmp3 + tmp2
  i <- i + 1
}

icc <- (nvars/(nvars-1))%*%(1-trS/summat)
ADFphi2 <- (1/(subj*(subj-1)))*tmp3
ADFse <- sqrt(ADFphi2)
ADFCIL <- icc - ADFse%*%1.96
ADFCIU <- icc + ADFse%*%1.96
```

