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An interior regularity result for the MEMS problem

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In this talk we present an interior regularity result for the class of stable solutions to a semilinear elliptic equation with a singular nonlinearity. More precisely, given a bounded open set $\Omega \subset \mathbb{R}^n$, we consider the problem

$$\begin{cases} -\Delta u = f(u) & \text{in } \Omega \\ 0 < u < 1 & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where the nonlinearity $f \in C^1((0,1))$ is assumed to be positive, nondecreasing, and to satisfy $f(1) = +\infty$ and $\int_0^1 f(t)dt = +\infty$. The model nonlinearities for this problem are the powers, i.e., $f(t) = (1-t)^{-p}$ for $p > 1$. For $p = 2$ and $n = 2$ this equation models the deflection of a dielectric elastic membrane in a microelectromechanical system (MEMS).

A solution u to problem (1) is said to be stable if the following inequality holds:

$$\int f'(u)\xi^2 \leq \int |\nabla \xi|^2, \quad \forall \xi \in C_c^\infty(\Omega). \tag{2}$$

Under a Crandall-Rabinowitz type assumption on f, we are able to prove that $u < 1$ in Ω up to the optimal dimension $n = 6$, and as a consequence, u is smooth in Ω (in contrast to singular solutions, which attain the value 1 somewhere in Ω).

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