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A new class of double phase variable exponent problems and a Nehari manifold approach

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During the last decade the so-called double phase operator has drawn attention from researchers. Originally it was introduced by Zhikov in the context of homogenization and elasticity theory (see [7]) and as an example for the Lavrentiev phenomenon (see [6]). It regained popularity after some novel regularity results for local minimizers of the corresponding functional (see [1, 2, 3]).

In this talk we introduce a new class of quasilinear elliptic equations driven by the so-called double phase operator with variable exponents. This part of the talk is based on [4], where we prove useful properties of the corresponding Musielak-Orlicz Sobolev spaces (an equivalent norm, uniform convexity, Radon-Riesz property with respect to the modular, density of smooth functions) and also properties of this new double phase operator (continuity, strict monotonicity, (S+)-property). In contrast to the previously known constant exponent case we are able to weaken the assumptions on the data.

After that we consider a problem with superlinear right-hand side. This last part of the talk is based on [5], in which under very general assumptions on the nonlinearity we prove a multiplicity result for such problems, whereby we show the existence of a positive solution, a negative one and a solution with changing sign. The sign-changing solution is obtained via the Nehari manifold approach and, in addition, we can also give information on its nodal domains. Furthermore, we derive a priori estimates on the solutions in the L^∞-norm under the very general setting used above.

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