
VI Congreso de Jóvenes Investigadores

Real Sociedad Matemática Española

León, Febrero de 2023

# Global regularity of stable solutions to semilinear equations

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In this talk, I will announce some recent results on the global regularity of stable solutions to semilinear equations. These are a follow-up to the interior estimates in [2].

Given a bounded domain $\Omega \subset \mathbb{R}^n$ of class $C^{1,1}$, we consider stable solutions to $a_{ij}(x)u_{ij}+b_i(x)u_i+f(u)=0$ in $\Omega$ vanishing on the boundary. Here, stability amounts to the nonnegativity of the principal eigenvalue of the linearized equation. We will show that stable solutions are globally Hölder continuous when $n \leq 9$. This dimension is optimal, since there are examples of unbounded stable solutions for $n \geq 10$. Before our work, the only optimal result [1] involved the model operator, the Laplacian, and needed a $C^3$ regularity assumption on the domain.

To prove the global result, flattening out the boundary, it will suffice to establish a priori estimates on half-balls. Our bounds there will be independent of the nonlinearity $f \in C^1$, which we assume to be nonnegative, nondecreasing, and convex. For the extension of these estimates to $C^{1,1}$ domains, it is essential to make the constants in our bounds depend on specific norms of the coefficients, namely, the $C^{0,1}$ norm of $a_{ij}$ and the $L^\infty$ norm of $b_i$.

## References

[1] X. Cabré, A. Figalli, X. Ros-Oton, J. Serra, *Stable solutions to semilinear elliptic equations are smooth up to dimension 9*. Acta Math. **224** (2020) 187-252.

[2] I.U. Erneta, *Stable solutions to semilinear elliptic equations for operators with variable coefficients*. To appear in Commun. Pure Appl. Anal. (2022).

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