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Concentration versus Simplification in Aggregation-Diffusion Equations

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Over the last two decades, intense work has been devoted to the Aggregation-Diffusion equation

$$\partial_t \rho = \operatorname{div}\Big(\rho \nabla (U'(\rho) + V + W * \rho)\Big).$$

This family of problem model, amongst other phenomena, the mean-field limit of systems with a large number of interacting particles arising in biology. It includes, for example, the famous model by Keller-Segel for chemotaxis.

From the mathematical point of view, they benefit from having a gradient flow structure. There is a long literature discussing the existence of gradient-flow solutions, characterising the existence/non-existence of minimisers of the associated free-energy functional and, in particular, discussing the existence or not of delta Deltas. The presence of a Delta is usually described as a concentration phenomena. In the absence of candidate stationary state, the solutions diffuse. We will discuss some of the key elements of the theory.

The aim of this talk is to present two works at opposite ends of the spectrum. On the one hand, the asymptotic formation of Dirac deltas in the case of Fast Diffusion [**CarrilloGomezCastroVazquez2022JMPA**]. On the other, a general result of asymptotic simplification to the heat kernel when W is bounded, with suitably integrable derivatives [**CarrilloGomezCastroYaoZeng**].

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