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Blowing up solutions to elliptic problems with natural growth in the gradient

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In the talk we will discuss the following model:

$$\begin{cases} -\Delta u = \lambda u + \mu(x)|\nabla u|^q + f(x), & x \in \Omega, \\ u = 0, & x \in \partial\Omega. \end{cases}$$

Here, Ω is a bounded domain of \mathbb{R}^N with smooth boundary, $\lambda \in \mathbb{R}$, $1 < q \leq 2$, μ is a non-negative bounded function and f is a non-negative $L^p(\Omega)$ function for $p > N$.

In this general framework, the problem is nowadays well-understood for $\lambda < 0$. More precisely, there exists a unique bounded weak solution for every $\lambda < 0$, and the family of solutions in terms of λ constitutes a continuum (see [1] and references therein). Moreover, if the problem does not admit a solution for $\lambda = 0$, then the continuum blows up as $\lambda \rightarrow 0^-$ and the asymptotic behavior is characterized in terms of the so-called ergodic problem (see [4]).

If, on the contrary, there is a solution for $\lambda = 0$, then the behavior of the branch of solutions for $\lambda \geq 0$ is not completely understood. It is conjectured that the branch bifurcates from infinity still at zero, meaning that there is a continuum of blowing up solutions as $\lambda \rightarrow 0^+$. However, this conjecture has been

proven only in particular cases where, typically, restrictions depending on q , N or μ appear (see [3] and references therein).

In the talk we will give an overview of the results that are known for the range $\lambda > 0$. We will also present some new results in this direction as part of the joint work [2] with J. Carmona and P.J. Martínez-Aparicio.

References

- [1] D. Arcoya, C. De Coster, L. Jeanjean, K. Tanaka. *Continuum of solutions for an elliptic problem with critical growth in the gradient*. J. Funct. Anal. **268** (2015) 2298-2335.
- [2] J. Carmona, S. López-Martínez, P.J. Martínez-Aparicio. *A priori estimates for non-coercive Dirichlet problems with suquadratic gradient terms*. Submitted.
- [3] S. López-Martínez. *A singularity as a break point for the multiplicity of solutions to quasilinear elliptic problems*. Adv. Nonlinear Anal. **9** (2020), no. 1, 1351-1382.
- [4] A. Porretta, *The “ergodic limit” for a viscous Hamilton-Jacobi equation with Dirichlet conditions*. Rend. Lincei Mat. Appl. **21** (2010) 59-78.

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