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Nonlocal Concave-Convex Critical problems with Mixed Boundary Conditions

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In this talk we introduce some existence results for nonlocal critical problems with concave-convex nonlinearities,

$$(P_\lambda) \qquad \left\{ \begin{array}{ll} (-\Delta)^s u = \lambda u^q + u^{2_s^*-1}, \quad u>0 & \text{in } \Omega, \\ B(u)=0 & \text{on } \partial\Omega, \end{array} \right.$$

with $\frac{1}{2}<s<1$ and $0<q<2_s^*-1$, $q\neq 1$, being $2_s^*=\frac{2N}{N-2s}$; $\lambda>0$ and $\Omega\subset\mathbb{R}^N$, $N>2s$, is a smooth bounded domain with mixed Dirichlet-Neumann boundary conditions

$$B(u)=u\chi_{\Sigma_{\mathcal{D}}}+\frac{\partial u}{\partial\nu}\chi_{\Sigma_{\mathcal{N}}},$$

where χ_A is the characteristic function of a set A, $\Sigma_{\mathcal{D}}$ and $\Sigma_{\mathcal{N}}$ are smooth $(N-1)$-dimensional submanifolds of $\partial\Omega$ such that $\Sigma_{\mathcal{D}}$ is closed with measure $|\Sigma_{\mathcal{D}}|=\alpha\in(0,|\partial\Omega|)$; $\Sigma_{\mathcal{D}}\cap\Sigma_{\mathcal{N}}=\emptyset$, $\Sigma_{\mathcal{D}}\cup\Sigma_{\mathcal{N}}=\partial\Omega$ and $\Sigma_{\mathcal{D}}\cap\overline{\Sigma}_{\mathcal{N}}=\Gamma$ is a smooth $(N-2)$-dimensional submanifold.

Using variational and topological methods we prove:

Theorem 0.1 *Let $0<q<1$ and $N>2s$. Then, there exists $0<\Lambda<\infty$ such that the problem (P_λ)*

1. *has no solution for $\lambda>\Lambda$,*
2. *has a minimal solution for any $0<\lambda<\Lambda$. Moreover, the family of minimal solutions is increasing in λ,*
3. *has at least one solution for $\lambda=\Lambda$,*
4. *has at least two solutions for $0<\lambda<\Lambda$.*

Theorem 0.2 *Let $1<q<2_s^*-1$ and $N>2s\left(1+\frac{1}{q}\right)$. The problem (P_λ) has at least one solution for any $\lambda>0$.*

The existence of a second solution in Theorem 0.1 crucially relies on a Strong Maximum Principle for mixed fractional problems that will be also discussed in the talk. In particular, let u be the solution to

$$\left\{ \begin{array}{ll} (-\Delta)^s u = f & \text{in } \Omega, \\ B(u)=0 & \text{on } \partial\Omega, \end{array} \right.$$

with $f\in L^\infty(\Omega)$, $f\gneq 0$ and v be the solution to

$$\left\{ \begin{array}{ll} (-\Delta)^s v = g & \text{in } \Omega, \\ B(u)=0 & \text{on } \partial\Omega, \end{array} \right.$$

with $g\in L^p(\Omega)$, $p>\frac{N}{s}$ and $g\gneq 0$. Then, the following holds.

Theorem 0.3 *There exists a constant $C>0$ such that*

$$\left\| \frac{v}{u} \right\|_{L^\infty(\Omega)} \leq C\|g\|_{L^p(\Omega)}.$$

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