Seminaris Curs 2009-2010.

14/10/2009 Iván Torrecilla.

Lattice schemes for an elliptic SPDE with fractional noise.

Abstract: We consider an elliptic SPDE on a bounded domain with smooth boundary $D \subset \mathbb{R}^k$, driven by a fractional Brownian field with Hurst parameter $H \in [\frac{1}{2}, 1[^k$. Firstly we define the stochastic convolution derived from the Green kernel. Using monotonicity methods, we prove existence and uniqueness of solution along with regularity of the sample paths. Finally, we use the finite difference method on the domain $D =]0, 1[^k$ to give a sequence of lattice approximations and prove its convergence to the solution of the SPDE at a given rate. This is a joint work with Marta Sanz-Solé.

21/10/2009 Omiros Papaspiliopoulos.

Importance sampling techniques for estimation of diffusion models.

Abstract: The context of the talk is statistical inference for diffusion processes by means of Monte Carlo methods. In the talk I present a general methodology for constructing importance samplers for conditioned diffusion processes. Two main ingredients of the approach is a change of measure for diffusion processes and a generalized formulation of importance sampling. Special attention is paid to a by-product of this technology in yielding identities for the diffusion transition density, which allow for the estimation of the latter by Monte Carlo techniques.

The talk is based primarily on a SEMSTAT book chapter, written jointly with Gareth O. Roberts.

11/11/2009 Noèlia Viles Cuadros.

Continuity with respect to the Hurst parameter of the law of the Russo-Vallois symmetric integral.

Abstract: Consider the laws in $\mathscr{C}([0,T])$ of the family of fractional Brownian motions $\{B^H,\ H \in (0,1)\}$ with Hurst parameter $H \in (0,1)$. It is easily seen that these laws converge weakly to that of B^{H_0}, when H tends to $H_0 \in (0,1)$.

It is interesting to study whether certain functionals of fractional Brownian motion conserve this property. That is, we ask if their law (in $\mathscr{C}([0,T])$) remains near to that of the corresponding functional for B^{H_0} when H is near to H_0. This kind of result justifies the use of $B^{\hat{H}}$ as a model in applied situations where the true value of the Hurst parameter is unknown and $\hat{H}$ is some estimation of it.

We prove that, under certain conditions, the law (in the space of continuous functions) of the

Russo–Vallois symmetric integral with respect to the fractional Brownian motion with Hurst parameter $H > 1/2$, converges weakly to that of the corresponding integral with respect to B^{H_0}, when H approaches to $H_0 \in [1/2, 1)$.

This is a joint work with M. Jolis.

18/11/2009 Eulàlia Nualart.
Estricta positivitat de la densitat d'equacions en derivades parcials estocàstiques no lineals espacialment homogènies
Abstract: En aquest seminari, considerarem un sistema de k equacions de segon ordre en derivades parcials estocàstiques no lineals amb dimensió espacial $d \geq 1$, perturbades per un soroll Gaussià k-dimensional, blanc en temps i amb una certa covariància en espai. Demostrarem existència, regularitat, i estricta positivitat de la densitat de la llei de la solució d'aquest sistema d'equacions en el conjunt on la matriu de difusió és invertible. Per demostrar aquest resultat utilitzarem tècniques de càlcul de Malliavin. Aplicarem aquest resultat pel cas de l'equació del calor en dimensió espacial arbitrària, i pel cas de l'equació d'ones en dimensió $d \in \{1, 2, 3\}$, amdós amb una covariància definida com el nucli de Riesz. Finalment, estudiarem l'estricta positivitat de la densitat pel cas d'una sola equació (i.e. $k = 1$).

25/11/2009 Jorge León.
Cálculo de Malliavin para procesos de Lévy.
Abstract: En esta charla estudiaremos un cálculo de Malliavin para procesos de Lévy basado en una familia de operadores de derivada. Este enfoque incluye los cálculos de Malliavin para el proceso de Wiener y para el proceso de Poisson introducido por Carlen y Pardoux. Finalmente analizaremos la existencia de una densidad para la solución de una ecuación diferencial estocástica gobernada por un proceso de Lévy.

02/12/2009 Maria Jolis.
Sobre la integral de Wiener respecte processos amb increments estacionaris.
Abstract: We prove that for any second order stochastic process X with stationary increments with continuous paths and continuous variance function, there exists a tempered measure μ (for which we give an explicit expression) related with the domain of the Wiener integral with respect to X as follows: the space of tempered distributions f such that the Fourier transform of f is square integrable with respect to μ is always a dense subset of the domain of the Wiener integral. Moreover, we provide sufficient conditions on μ in order that the domain of the integral is exactly this space of distributions. We apply our results to the fractional Brownian

motion. In particular, it is proved that the domain of the Wiener integral with respect to the fractional Brownian motion with Hurst parameter $H > 1/2$ contains distributions that are not given by locally integrable functions. We have also considered the example of the process given by Ornstein and Uhlenbeck as a model for the position of a Brownian particle.

09/12/2009 Josep Vives.
Two-sided estimates for stock price distribution densities in jump-diffusion models.
Abstract: We consider uncorrelated Stein-Stein, Heston, and Hull-White models and their perturbations by compound Poisson processes with jump amplitudes distributed according to a double exponential law. Similar perturbations of the Black-Scholes model were studied by S. Kou. For perturbed stochastic volatility models, we obtain two-sided estimates for the stock price distribution density and compare the tail behavior of this density before and after perturbation.

It is shown that if the value of the parameter, characterizing the right tail of the double exponential law, is small, then the stock price density in the perturbed model decays slower than the density in the original model. On the other hand, if the value of this parameter is large, then there are no significant changes in the behavior of the stock price distribution density.

21/01/2010 Khalifa Sebaiy.
Occupation densities for certain processes related to fractional Brownian motion.
Abstract: In this paper the authors establish the existence of a square integrable occupation density for two classes of stochastic processes. First they consider a Gaussian process with an absolutely continuous random drift, and secondly they handle the case of a (Skorohod) integral with respect to the fractional Brownian motion with Hurst parameter H¿ 1/2. The proof of these results uses a general criterion for the existence of a square integrable local time, which is based on the techniques of Malliavin calculus.

04/02/2010 Ely Merzbach.
The Set Indexed Levy Process.
Abstract: We present a satisfactory definition of the important class of Levy Processes indexed by a general collection of sets (SILP: Set-Indexed Levy Process). This class is characterized by the Levy-Khintchine representation. The notion of increment stationarity for set-indexed processes is discussed. Another characterization is presented using projections on flows. Connection between SILP and the set-indexed Markov property will be given. Finally the Set-Indexed Compound process will be studied as a typical example of SILP.

17/02/2010 José Fajardo.

Implied Volatility Skew and New Smile Factor.

Abstract: In Lévy market models it is possible to relate different skewness measures, as Fajardo and Mordecki (2009) have shown. In this paper we concentrate on the implied volatility skewness, finding conditions for slope downwards and upwards. Moreover, we find a new smile factor and test it with some empirical exercises.

17/02/2010 Carles Rovira.

Equacions diferencials estocàstiques amb retard dirigides per un moviment Brownià fraccionari amb parameter $H > 1/2$.

Abstract: Demostrem l existència i unicitat de solució per aquesta classe d equacions i per equacions amb reflexió. Estudiem també la convergència de les solucions quan el retard va cap a zero. La integral estocàstica que apareix respecte al moviment Brownià fraccionari s interpreta com una integral trajectorial de Riemann-Stieltjes.

10/03/2010 Robert Dalang.

Stochastic integrals for spde's: a comparison.

Abstract: We present the Walsh theory of stochastic integrals with respect to martingale measures, alongside of the Da Prato and Zabczyk theory of stochastic integrals with respect to Hilbert-space-valued Wiener processes and some other approaches to stochastic integration, and we explore the links between these theories. We then show how each theory can be used to study stochastic partial differential equations, with an emphasis on the stochastic heat and wave equations driven by spatially homogeneous Gaussian noise that is white in time. We compare the solutions produced by the different theories. This talk is based on a joint paper with Lluis Quer-Sardanyons.

24/03/2010 Vlad Bally.

Regularity of probability laws using the Riesz transform and Sobolev spaces techniques.

Abstract: We give a criterion of regularity for a probability measure μ on $\mathbb{R}^d$ based on integration by parts formulas. The standard way to deal with this problem is to use a Fourier transform argument. Here we give an alternative approach using the Riesz transform and the machinery of the Sobolev spaces associated to μ. Finally, we apply this criterion in order to improve the classical regularity criterion for functionals on the Wiener space due to Malliavin. The basic gain is that we need less regularity for the functionals at hand. The talk is based on

a joint paper with Lucia Caramellino.

07/04/2010 Lluís Quer.

Optimal Gaussian density estimates for a class of stochastic equations with additive noise.

Abstract: We establish optimal lower and upper Gaussian bounds for the density of the solution to a class of stochastic integral equations driven by an additive spatially homogeneous Gaussian random field. The proof is based on the techniques of the Malliavin calculus and a density formula obtained by Nourdin and Viens. Then, the main result is applied to the mild solution of a general class of SPDEs driven by a Gaussian noise which is white in time and has a spatially homogeneous correlation. In particular, this covers the case of the stochastic heat and wave equations in $\mathbb{R}^d$ with $d \geq 1$ and $d \leq 3$, respectively. The upper and lower Gaussian bounds have the same form and are given in terms of the variance of the stochastic integral term in the mild form of the equation. This is a joint work with David Nualart (University of Kansas).

12/05/2010 Antonio Bru.

Dinámica del crecimiento tumoral y papel de los neutrófilos en la respuesta inmunológica antitumoral.

Abstract: Todos los tumores sólidos poseen una dinámica común de crecimiento que está bien caracterizada a partir de la aplicación de una serie de técnicas de escala basadas en la geometría fractal. En esta presentación se expondrán dichas técnicas y se interpretará la ecuación en derivadas parciales estocástica (SPDE) que rige el crecimiento de los tumores. Se explicará como se logra obtener un marco formal a partir de ella, mediante el cual se pueden explicar las numerosas características de los procesos tumorales así como surge la propuesta de una nueva estrategia terapéutica, que propone la estimulación del sistema inmunológico para la obtención de una respuesta inflamatoria innata aguda, cuyos actores determinantes son los neutrófilos. Mediante las interpretaciones biológicas de la ecuación matemática, se demuestra una vez más la potencialidad de lo que suponen los trabajos interdisciplinares, en este caso entre el Cálculo Estocástico y la Biología.

19/05/2010 Esko Valkeila.

Insider information in pricing models with jumps.

Abstract: Enlargement of filtrations is a classical topic in the general theory of stochastic processes. This theory has been applied to stochastic finance in order to analyze models with insider information. We study initial enlargement in a Markov chain market model, introduced by R.Norberg. Surprisingly, in this model two things can happen. Firstly, the insider

has no arbitrage possibilities strictly before the expiration date, and this the case with a jump-model introduced by A.Kohatsu-Higa. Secondly, the insider can also have arbitrage possibilities strictly before the expiration date. Moreover, the jump times can have accessible part in the enlarged filtration.

The talk is based on joint work with Dario Gasbarra and Jose Igor Morlanes.

26/05/2010 Peter Spreij.
The Stochastic Wave Equation with Fractional Noise: a random field approach.
Abstract: We consider a continuous-time stochastic volatility model. The model contains a stationary volatility process, the multivariate density of the finite dimensional distributions of which we aim to estimate. We assume that we observe the process at discrete instants in time. The sampling times will be equidistant with vanishing distance. A multivariate Fourier-type deconvolution kernel density estimator based on the logarithm of the squared processes is proposed to estimate the multivariate volatility density. An expansion of the bias and a bound on the variance are derived. (joint work with Bert van Es).

02/06/2010 Raluca Balan.
Multivariate volatility density estimation.
Abstract: We consider the linear stochastic wave equation with spatially homogenous Gaussian noise, which is fractional in time with index $H > 1/2$. We show that the necessary and sufficient condition for the existence of the solution is a relaxation of the condition obtained when the noise in white in time ($H = 1/2$). Similar results are obtained for the heat equation. Unlike the white noise case, the condition that we obtain for the heat equation is different than the one obtained for the wave equation. (Joint work with Ciprian Tudor).

02/06/2010 Rahul Roy.
Learning from neighbours.
Abstract: Consider a discrete time stochastic process on the integer line where on each integer point is situated a particle which is one of two types– red or blue. At each instance a particle tosses a coin and based on the success/failure of its own coin, as well as the colours and tosses of the neighbouring coins it decides whether to retain its colour or change to the other colour. We study the limiting behaviour of the resultant Markov chain. This model has been studied by economists to model diffusion of technologies.

03/06/2010 Murad S. Taqqu.

Self-similarity and computer network traffic.

Abstract: Ethernet local area network traffic appears to be approximately statistically self-similar. This discovery, made about twenty years ago, has had a profound impact on the field. I will try to explain what statistical self-similarity means, how it is detected and indicate how one can construct random processes with that property by aggregating a large number of "on-off"renewal processes. If the number of replications grows to infinity then, after rescaling, the limit turns out to be the Gaussian self-similar process called fractional Brownian motion. But if one looks at very large time scales, then one obtains instead a Levy stable motion which is a process with independent increments, infinite variance and heavy tails.

16/06/2010 Martin Keller-Ressel.

Moment Explosions and Long-Term Behavior of Affine Stochastic Volatility Models.

Abstract: We consider a class of asset pricing models, where the risk-neutral joint process of log-price and its stochastic variance is an affine process in the sense of Duffie, Filipovic and Schachermayer (2003). We present results on the long-term behavior of the model, including an expression for the stationary distribution of the stochastic variance process. These results can be linked – through a saddlepoint type approximation – to the shape of the implied volatility surface for large time-to-maturity. We also study moment explosions of the price process, and provide explicit expressions for the first time a moment of given order explodes. Again, the results on moment explosions can be used to obtain asymptotics of the implied volatility surface, in this case for extreme strike prices. Our results apply to several well-known stochastic volatility models, such as the Heston model with and without additional jumps, a model of Bates and the Barndorff-Nielsen-Shephard model.