A TYPE OF FRACTIONAL NOISE THAT REGULARIZES FLOWS OF SDES WITH DISCONTINUOUS COEFFICIENTS

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ABSTRACT. In this work we define a type of noise of fractional nature that is neither a Markov process, nor a semimartingale, it is not Hölder continuous and has no finite p-variation. This noise possesses a strong regularizing effect when considered on SDEs with irregular coefficients and it seems to be the right noise in order to generate smooth stochastic flows.

More concretely, let $H = \{H_n\}_{n=1}^{\infty} \subset (0, 1/2)$ be a sequence such that $\lim_n H_n = 0$ and $\lambda = \{\lambda_n\}_{n=1}^{\infty} \subset \mathbb{R}$ such that $\lambda_n > 0$ for all $n \geq 1$ and $\sum_{n=1}^{\infty} |\lambda_n| < \infty$. This noise is defined as

$$\mathbb{B}_t^H = \sum_{n=1}^{\infty} \lambda_n B_t^{H_n,n}, \quad P-\text{a.s.},$$

where $B^{H_n,n}$ are independent fBm's with Hurst parameters H_n.

Then we show existence and uniqueness of global strong solutions of the SDE

$$dX_t = b(t, X_t)dt + d\mathbb{B}_t^H, \quad t \in [0,T], \quad X_0 = x \in \mathbb{R}^d,$$

where the equation is interpreted in such a way that if a solution exists then

$$X_t = x + \int_0^t b(s, X_s)ds + \mathbb{B}_t^H,$$

whenever it makes sense. Here, the time-dependent vector field $b : [0,T] \times \mathbb{R}^d \to \mathbb{R}^d$ just belongs to the class $L^q\left([0,T], L^p(\mathbb{R}^d, \mathbb{R}^d)\right)$ for arbitrary $p, q \in (2, \infty]$, including essentially bounded coefficients.

Furthermore, we show that the mapping $x \mapsto X_t$ is P-a.s. infinitely many times differentiable. In is known that in the Wiener case with irregular coefficients, the associated flow is just once Sobolev differentiable. Hence, this noise seems to be the right noise to consider in order to produce C^{∞}-flows.

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