

EUROPEAN ROAD TRANSPORT REGULATION No. 561/2006

Regulation 561 concerns activity sequences of truck drivers recorded by tachographs. Formally: finite words with letters d, r, w representing activities **driving**, **resting** and **other work** per minute. E.g. $dddwwrrr$ means 3 minutes driving, followed by 2 minutes other work and 3 minutes resting. In practice, these words are very long. One needs an **algorithm** that checks legality.

Regulation 561 is a complex set of laws. Depending on accumulated durations of e.g. driving, it requires various kinds of resting periods, namely **breaks**, **daily rest periods** and **weekly rest periods**, each of which can be **regular** or **reduced**. E.g. Article 8.6 requires:

In any two consecutive weeks a driver shall take at least two regular weekly rest periods, or one regular weekly rest period and one reduced weekly rest period of at least 24 hours. However, the reduction shall be compensated by an equivalent period of rest taken en bloc before the end of the third week following the week in question.

STOPWATCH AUTOMATA

A **stopwatch automaton** is given by finitely many **states**, **transitions** and **stopwatches**. At each time-point of a computation the automaton resides in a state. It can either stay in this state or take a transition and switch to another state instantaneously. At each time-point the stopwatches show values, up to some **bound**. In a given state each stopwatch is **active** or not. When staying in a state the active stopwatches increment their value, the unactive ones keep their value.

Each transition has a guard and an action. The **guard** specifies a property of stopwatch values that must be met for the transition to be taken, e.g. certain upper or lower bounds of certain stopwatch values. The **action** updates stopwatch values, e.g. resets some to value 0. States are labeled by d, w or r . The automaton **reads** the label for each time unit spent in the state. The automaton **accepts** a word w if there is a computation reading w and leads from the state *start* to the state *accept*.

CONSTRUCTION

We construct a stopwatch automaton that accepts a word if and only if the represented activity sequence is legal according to Regulation 561.

ARCHITECTURE

The automaton uses the 11 states shown in the figure plus the state *start*.

Letters labeling states are shown in yellow. The label of *week* and *accept* do not matter.

Below each state there is the list of stopwatches active in it.

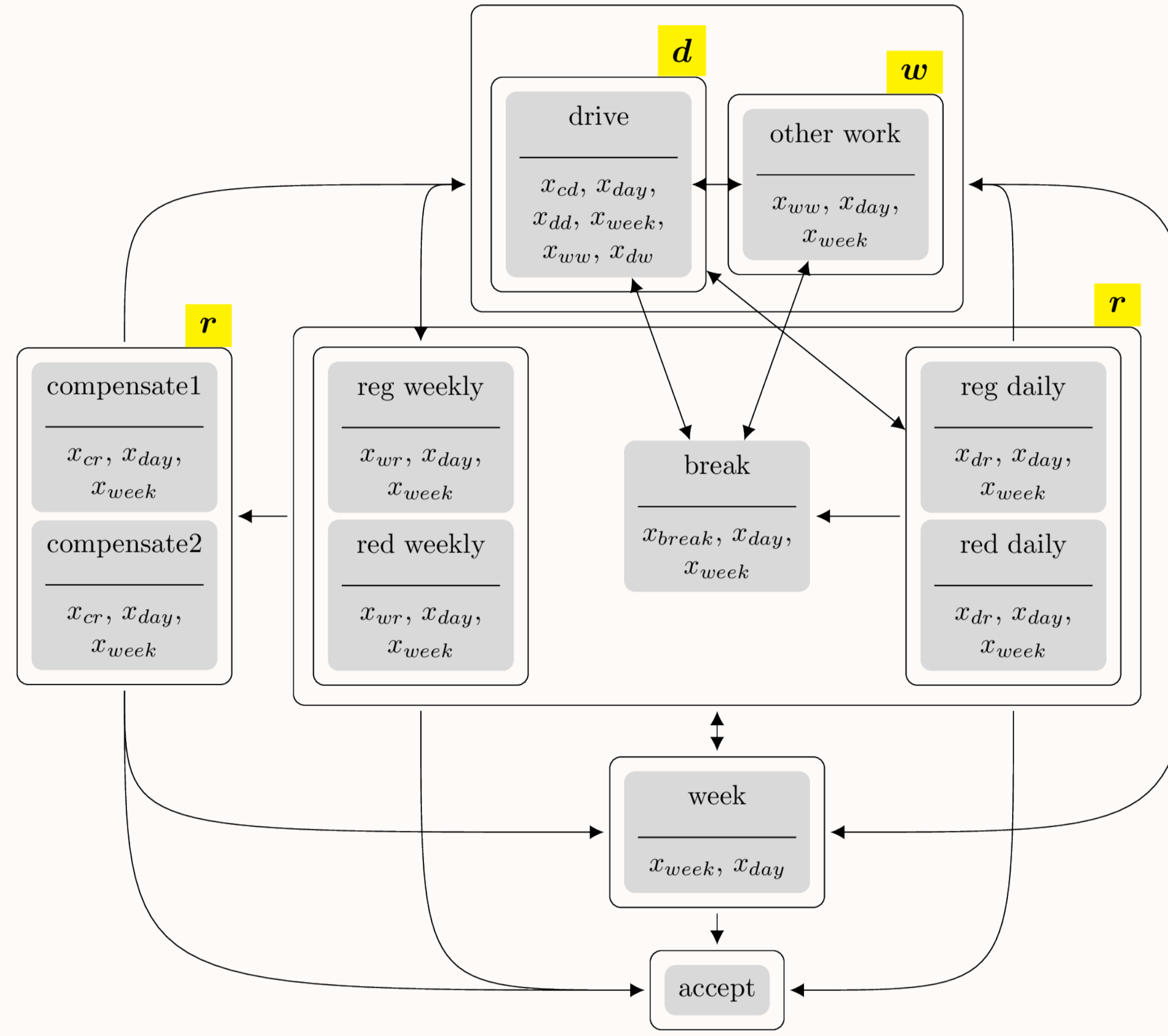
Their bounds are

$x_{start}: 1$ $x_{cd}: 271$ $x_{week}: 10081$
 $x_{break}: 540$ $x_{day}: 1440$ $x_{ww}: 3601$
 $x_{dr}: 660$ $x_{dd}: 601$ $x_{dw}: 3361$
 $x_{wr}: 2700$ $x_{cr}: 1260$

Additionally, the automaton operates 16 bits, 4 counters and 3 registers – these are stopwatches that are nowhere active.

The automaton has 102 transitions. An arrow between states indicates possibly many transitions. An arrow between boxes indicates transitions between all states within the boxes. The figure omits transitions from state *start* to all other states.

The table below lists the transitions excluding those leaving *start*, *compensate1* and *compensate2*.



TRANSITIONS

There is a transition from *break* to *red. daily*. Its guard checks that the stopwatch x_{day} has value at most $15h = 900$, that the counter c_{rd} has value at most 3 and that the bit b_{rb} has value 0. Its action resets the stopwatch x_{cd} to 0 and increments the counter c_{cd} by 1.

State	Guard	Action	State	
drive	$x_{cd} \leq 4.5h, x_{dd} \leq 9h$		break	
	$x_{cd} \leq 4.5h, x_{dd} \leq 9h$		other work	
	$x_{cd} \leq 4.5h, x_{day} \leq 15h, x_{dd} \leq 9h, c_{rd} \leq 3, b_{rb} = 0$	$c_{rd} := c_{rd} + 1, x_{cd} := 0$	red. daily	
	$x_{cd} \leq 4.5h, x_{day} \leq 15h, 9 < x_{dd} \leq 10h, c_{dd} \leq 2, c_{rd} \leq 3, b_{rb} = 0$	$c_{rd} := c_{rd} + 1, x_{cd} := 0, c_{dd} := c_{dd} + 1$	reg. daily	
	$x_{cd} \leq 4.5h, x_{day} \leq 13h, x_{dd} \leq 9h, b_{rb} = 0$	$x_{cd} := 0, c_{dd} := c_{dd} + 1$	reg. daily	
	$x_{cd} \leq 4.5h, x_{dd} \leq 9h, x_{week} \leq 6 \cdot 24h, x_{cd} \leq 4.5h, x_{day} \leq 13h, x_{dd} \leq 9h, b_{used} = 0, b_{rb} = 0$	$x_{cd} := 0, b_{used} := 1, b_{wr} := 1, b_{wr} := 0$	red. weekly	
	$x_{cd} \leq 4.5h, x_{dd} \leq 9h, x_{week} \leq 6 \cdot 24h, x_{cd} \leq 4.5h, x_{day} \leq 13h, x_{dd} \leq 9h, b_{wr} = 0, b_{rb} = 0$	$x_{cd} := 0, b_{wr} := 1$	red. weekly	
	$x_{cd} \leq 4.5h, x_{dd} \leq 9h, x_{week} \leq 6 \cdot 24h, x_{cd} \leq 4.5h, x_{day} \leq 13h, x_{dd} \leq 9h, b_{wr} = 0, b_{rb} = 0$	$x_{cd} := 0, b_{wr} := 1$	reg. weekly	
	$x_{cd} \leq 4.5h, x_{dd} \leq 9h, x_{week} \leq 6 \cdot 24h, x_{cd} \leq 4.5h, x_{day} \leq 13h, x_{dd} \leq 9h, b_{wr} = 0, b_{rb} = 0$	$x_{cd} := 0, b_{wr} := 0$	reg. weekly	
	$x_{cd} \leq 4.5h, x_{dd} \leq 9h, x_{day} \leq 56h, x_{ww} \leq 60h, x'_{dw} + x_{dd} \leq 90h, b_{wr} = 1, c_{c1} \leq 3, c_{c2} \leq 3$	$b_{drive} := 1$	week	
	break	$x_{break} \geq 45$	$x_{break} := 0, x_{cd} := 0$	drive
		$15 \leq x_{break} < 45$	$b_{rb} := 1, x_{break} := 0$	drive
$b_{rb} = 1, x_{break} \geq 30$		$b_{rb} := 0, x_{cd} := 0, x_{break} := 0$	other work	
$x_{break} < 15$		$x_{break} := 0$	other work	
$x_{break} \geq 45$		$x_{break} := 0, x_{cd} := 0$	other work	
$15 \leq x_{break} < 45$		$b_{rb} := 1, x_{break} := 0$	other work	
$b_{rb} = 1, x_{break} \geq 30$		$b_{rb} := 0, x_{cd} := 0, x_{break} := 0$	other work	
$x_{break} < 15$		$x_{break} := 0$	other work	
$x_{day} \leq 15h, c_{rd} \leq 3, b_{rb} = 0$		$c_{rd} := c_{rd} + 1, x_{cd} := 0$	red. daily	
$x_{day} \leq 13h, b_{rb} = 0$		$x_{cd} := 0$	reg. daily	
$x_{break} \geq 9h$			compensate1	
$x_{break} \geq 9h$			compensate2	
other work	$x_{dw} \leq 56h, x_{ww} \leq 60h, x'_{dw} + x_{dd} \leq 90h, b_{wr} = 1, c_{c1} \leq 3, c_{c2} \leq 3$	$b_{break} := 1$	week	
			drive	
			break	
	$x_{day} \leq 15h, c_{rd} \leq 3, b_{rb} = 0$	$c_{rd} := c_{rd} + 1, x_{cd} := 0$	other work	
	$x_{day} \leq 13h, b_{rb} = 0$	$x_{cd} := 0$	compensate1	
	$x_{week} \leq 6 \cdot 24h, x_{day} \leq 13h, b_{wr} = 0, b_{rb} = 0$	$x_{cd} := 0, b_{used} := 1, b_{wr} := 1, b_{wr} := 0$	compensate2	
	$x_{week} \leq 6 \cdot 24h, x_{day} \leq 13h, b_{wr} = 0, b_{rb} = 0$	$x_{cd} := 0, b_{wr} := 1$	week	
	$x_{week} \leq 6 \cdot 24h, x_{day} \leq 13h, b_{wr} = 0, b_{rb} = 0$	$x_{cd} := 0, b_{used} := 1, b_{wr} := 1, b_{wr} := 0$	drive	
	$x_{dw} \leq 56h, x_{ww} \leq 60h, x'_{dw} + x_{dd} \leq 90h, b_{wr} = 1, c_{c1} \leq 3, c_{c2} \leq 3$	$b_{otherwork} := 1$	drive	
			break	
			other work	
	red. daily	$11h > x_{dr} > 9h$	$x_{dr} := 0, x_{day} := 0, b_{dr} := 0, x_{dd} := 0$	break
$11h > x_{dr} > 9h$		$x_{dr} := 0, x_{day} := 0, b_{dr} := 0, x_{dd} := 0$	other work	
$11h > x_{dr} > 9h$		$x_{dr} := 0, x_{day} := 0, b_{dr} := 0, x_{dd} := 0$	compensate1	
$11h > x_{dr} > 9h$		$x_{dr} := 0, x_{day} := 0, b_{dr} := 0, x_{dd} := 0$	compensate2	
$x_{dw} \leq 56h, x_{ww} \leq 60h, x'_{dw} + x_{dd} \leq 90h, b_{wr} = 1, c_{c1} \leq 3, c_{c2} \leq 3$		$b_{red.daily} := 1, x_{cd} := 0$	week	
$x_{dr} \geq 11h$		$x_{dr} := 0, x_{day} := 0, b_{dr} := 0, x_{dd} := 0$	drive	
$3h \leq x_{dr} < 11h, b_{dr} = 0$		$x_{dr} := 0, b_{dr} := 1, x_{dd} := 0$	drive	
$x_{dr} \geq 9h, b_{dr} = 1$		$x_{dr} := 0, x_{day} := 0, b_{dr} := 0, x_{dd} := 0$	break	
$x_{dr} \geq 11h$		$x_{dr} := 0, x_{day} := 0, b_{dr} := 0, x_{dd} := 0$	other work	
$x_{dr} \geq 11h$		$x_{dr} := 0, x_{day} := 0, b_{dr} := 0, x_{dd} := 0$	compensate1	
$x_{dr} \geq 11h$		$x_{dr} := 0, x_{day} := 0, b_{dr} := 0, x_{dd} := 0$	compensate2	
$x_{dw} \leq 56h, x_{ww} \leq 60h, x'_{dw} + x_{dd} \leq 90h, b_{wr} = 1, c_{c1} \leq 3, c_{c2} \leq 3$		$b_{reg.daily} := 1, x_{cd} := 0$	week	
reg. weekly	$24h \leq x_{wr} < 45h, x_{c1} > 0, x_{c2} = 0$	$x_{wr} := 0, c_{rd} := 0, b_{used} := 0, x_{c2} := 45h - x_{wr}$	drive	
	$24h \leq x_{wr} < 45h, x_{c1} = 0, x_{c2} = 0$	$x_{wr} := 0, c_{rd} := 0, b_{used} := 0, x_{c1} := 45h - x_{wr}$	drive	
	$24h \leq x_{wr} < 45h, x_{c1} > 0, x_{c2} = 0$	$x_{wr} := 0, c_{rd} := 0, b_{used} := 0, x_{c2} := 45h - x_{wr}$	other work	
	$24h \leq x_{wr} < 45h, x_{c1} = 0, x_{c2} = 0$	$x_{wr} := 0, c_{rd} := 0, b_{used} := 0, x_{c1} := 45h - x_{wr}$	other work	
	$x_{week} = 168h, b_{otherwork} = 1$	$x_{week} := 0, c_{dd} := 0, b_{otherwork} := 0, x_{dw} := 0, x_{dw} := x_{dw}, c_{c1} := c_{c1} + \text{sgn}(x_{c1}), c_{c2} := c_{c2} + \text{sgn}(x_{c2})$	other work	
	$x_{week} = 168h, b_{reg.daily} = 1, c_{rd} \leq 3, b_{rb} = 0$	$c_{rd} := c_{rd} + 1, x_{cd} := 0, x_{week} := 0, c_{dd} := 0, b_{red.daily} := 1, x_{dw} := 0, x_{dw} := x_{dw}, c_{c1} := c_{c1} + \text{sgn}(x_{c1}), c_{c2} := c_{c2} + \text{sgn}(x_{c2})$	red. daily	
	$x_{week} = 168h, b_{reg.daily} = 1, b_{rb} = 0$	$x_{cd} := 0, x_{week} := 0, c_{dd} := 0, b_{reg.daily} := 0, x_{dw} := 0, x_{ww} := 0, x'_{dw} := x_{dw}, c_{c1} := c_{c1} + \text{sgn}(x_{c1}), c_{c2} := c_{c2} + \text{sgn}(x_{c2})$	reg. daily	
	$x_{week} = 168h, b_{red.weekly} = 1, x_{week} \leq 6 \cdot 24h, b_{used} = 1, b_{wr} = 0, b_{rb} = 0$	$x_{week} := 0, c_{dd} := 0, b_{red.weekly} := 0, x_{dw} := 0, x_{ww} := 0, x'_{dw} := x_{dw}, b_{used} := 0, b_{wr} := 1, b_{wr} := 1, c_{c1} := c_{c1} + \text{sgn}(x_{c1}), c_{c2} := c_{c2} + \text{sgn}(x_{c2})$	red. weekly	
	$x_{week} = 168h, b_{reg.weekly} = 1, x_{week} \leq 6 \cdot 24h, b_{used} = 0, b_{rb} = 0$	$x_{week} := 0, c_{dd} := 0, b_{reg.weekly} := 0, x_{dw} := 0, x_{ww} := 0, x'_{dw} := x_{dw}, b_{used} := 1, b_{wr} := 1, b_{wr} := 1, c_{c1} := c_{c1} + \text{sgn}(x_{c1}), c_{c2} := c_{c2} + \text{sgn}(x_{c2})$	reg. weekly	
	$x_{week} = 168h, b_{compensate1} = 1$	$x_{week} := 0, c_{dd} := 0, b_{compensate1} := 0, x_{dw} := 0, x_{dw} := x_{dw}, c_{c1} := c_{c1} + \text{sgn}(x_{c1}), c_{c2} := c_{c2} + \text{sgn}(x_{c2})$	compensate1	
	$x_{week} = 168h, b_{compensate2} = 1$	$x_{week} := 0, c_{dd} := 0, b_{compensate2} := 0, x_{dw} := 0, x_{dw} := x_{dw}, c_{c1} := c_{c1} + \text{sgn}(x_{c1}), c_{c2} := c_{c2} + \text{sgn}(x_{c2})$	compensate2	
	$x_{day} \leq 24h, x_{week} \leq 168h, x_{week} = 168h, x_{dw} \leq 56h, x_{ww} \leq 60h, x'_{dw} + x_{dd} \leq 90h$	$x_{week} := 0, c_{dd} := 0, x_{dw} := 0, x_{ww} := 0, x'_{dw} := x_{dw}, c_{c1} := c_{c1} + \text{sgn}(x_{c1}), c_{c2} := c_{c2} + \text{sgn}(x_{c2})$	accept	



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